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NUMERICAL SOLUTION OF PARABOLIC PARTIAL DIFFERENTIAL EQUATIONS AND DIGITAL SIMULATION OF PETROLEUM RESERVOIRS

BY

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled NUMERICAL SOLUTION OF PARABOLIC PARTIAL DIFFERENTIAL EQUATIONS AND DIGITAL SIMULATION OF PETROLEUM RESERVOIRS, submitted by Sambasiva Rao Allada, B.Sc.(Hons)., M.Sc. Tech., in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

ABSTRACT

The second order parabolic partial differential equations, arising from diffusion processes, are currently integrated by two types of numerical procedures: the explicit and the implicit. These various procedures have been interpreted by a unified approach based on matrix theory. By discretizing the space derivatives, the second order partial differential equation can be converted into a first order matrix differential equation. The existing numerical methods have been shown to result from different splittings of the coefficient matrix in the above equation. Further, the stability of solution has been examined in terms of the properties of the integrating matrix of each method. The application of matrix theory offers a more general method of stability analysis of numerical procedures than the von Neumann technique.

The alternating direction explicit procedure (ADEP) was extended to the three-dimensional case. The accuracy and the computational speed of ADEP was compared with the best alternating direction implicit procedure.

The alternating direction implicit procedures

(ADIP's) suitable for linear problems, become considerably more involved in solving nonlinear parabolic problems. The number of integration stages increases with the dimensionality of the problem. On the other hand, the solution of multi-dimensional nonlinear problems by the ADEP has been shown to be not significantly more difficult than the solution of the linear case. Iterative procedure required for this case involves variable or variables at one grid point only. Further, even in a three-dimensional problem, the number of integration stages is only two. In the solution of nonlinear and large linear systems, ADEP is computationally faster than any of the ADIP's.

The application of ADEP has been demonstrated in solving three parabolic problems of progressively increasing complexity, namely, 1) a linear mathematical model of an oil reservoir, 2) a nonlinear model of a gas reservoir, and 3) simulation of two-phase flow in a water-injected oil reservoir, which is characterized by two simultaneous nonlinear parabolic partial differential equations. The solution of the linear

problem by ADEP was in good agreement with that of ADIP of Peaceman and Rachford. In the gas reservoir problem, though the model was nonlinear, simple relation between the properties of the gas permitted explicit solution by ADEP. In the case of two-phase flow, the simultaneous nonlinear algebraic equations given by ADEP were solved iteratively by the Newton-Raphson procedure. The solution converged in 3-4 iterations.

In all the three cases, the behavior of the reservoirs predicted by ADEP has been found to be consistent with that encountered in practice.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF TABLES (cont'd)

LIST OF FIGURES

I. INTRODUCTION

Ever since Newton invented Calculus in the seventeenth century, differential equations have served to describe precisely and adequately many natural phenomena. A large number of physical systems, being multi-dimensional in geometry and nonlinear in behavior, are characterized by nonlinear partial differential equations.

Unfortunately, however, most partial differential equations, particularly the nonlinear type, have proved to be most difficult to solve by classical methods. Among these, parabolic partial differential equations constitute an important class. Until very recently, the solution of real problems involving these equations were either not attempted or could be obtained only on reducing the equations to simpler approximate forms and solving them by conventional methods. Analytical solutions of many of these idealized versions, available in standard texts [6, 11, 3], are mainly of academic interest.

In keeping with the general trend of sophistication in science and technology in recent times, a more realistic representation of physical systems is being

insisted upon. In a number of practical problems this meant preservation of one or more of the following features:

1. Non-homogeneity of the medium, leading to partial differential equations with variable coefficients;
2. Nonlinear behavior of the system;
3. Irregular geometry of the system boundary.

The advent of high-speed computing machines and the recent advances in numerical analysis have brought great promise in finding precise solutions of involved differential equations. The basis of this approach is to divide a multi-dimensional system, irrespective of its geometry, into a suitable network of small elemental regions or cells and to reduce the differential equation to a set of algebraic equations, each of which approximates the behavior of the individual cell. Given a sound technique, the solution of such a system of algebraic equations is routine on a modern high-speed computer.

This study is concerned in general with the

numerical integration of the second order parabolic partial differential equations of the form:

$$g \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(K \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(K \frac{\partial T}{\partial z}\right) + S\,(T, x, y, z, t) \qquad \text{(I-1)}$$

where

$T = T\,(x,y,z,t)$

x,y,z = space coordinates

t = time

g = capacity

S = source

K = rate coefficient

This equation covers the two cases mentioned above, namely 1) the linear equation with variable coefficients when g and K are functions of x,y,z, and t only, and S is at most a linear function of T, and 2) the nonlinear equation when g and K are also functions of T.

The choice of parabolic equations, in particular,

was motivated by two specific considerations. Firstly, a number of diffusion processes are characterized by this equation, and development of a suitable method of solution to equation (I-1) would be, hopefully, a contribution in these areas in general. These processes include transient heat conduction, mass diffusion with chemical source, fluid flow through porous media, and time-dependent neutron transport in a nuclear reactor. The second consideration was the growing demand for efficient mathematical modelling of petroleum reservoirs whose behavior, as will be seen subsequently, is well represented by equation (I-1).

Broadly, the first part of the thesis deals with the development of a general numerical technique suitable for solving parabolic equations of the type (I-1). The proposed method has been found to be particularly advantageous for solving nonlinear equations. The application of this technique in the study of three types of petroleum reservoirs constitutes the second part. These types are associated with progressively complex equations namely; 1) a linear partial

differential equation representing a liquid petroleum reservoir, 2) a nonlinear parabolic equation characterizing a gas reservoir, and finally, 3) a set of simultaneous nonlinear partial differential equations which mathematically simulate the dynamics of two-phase flow in a water-injected petroleum reservoir. The predicted performance of these reservoirs has been found to be consistent with their known normal behavior.

II. GENERAL THEORY AND LITERATURE REVIEW

The general theory of numerical solution of parabolic partial differential equations is perhaps best discussed by reviewing some important numerical methods and analyzing them. Of fundamental importance in the numerical solution of partial differential equations are the concepts of convergence and stability. These are defined and applied in the analysis of some finite difference schemes. For convenience, consideration of the numerical methods in this chapter is confined to the linear two-dimensional case of equation (I-1) although it can be extended to the three-dimensional case as well.

The physical system under consideration is defined by a region R bounded by a closed curve C, as shown in Figure 1. The behavior of the interior of R is governed by,

$$g\,(x,y)\,\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\,[K(x,y)\,\frac{\partial T}{\partial x}] + \frac{\partial}{\partial y}\,[K(x,y)\,\frac{\partial T}{\partial y}] + S(x,y,t) \qquad \text{(II-1)}$$

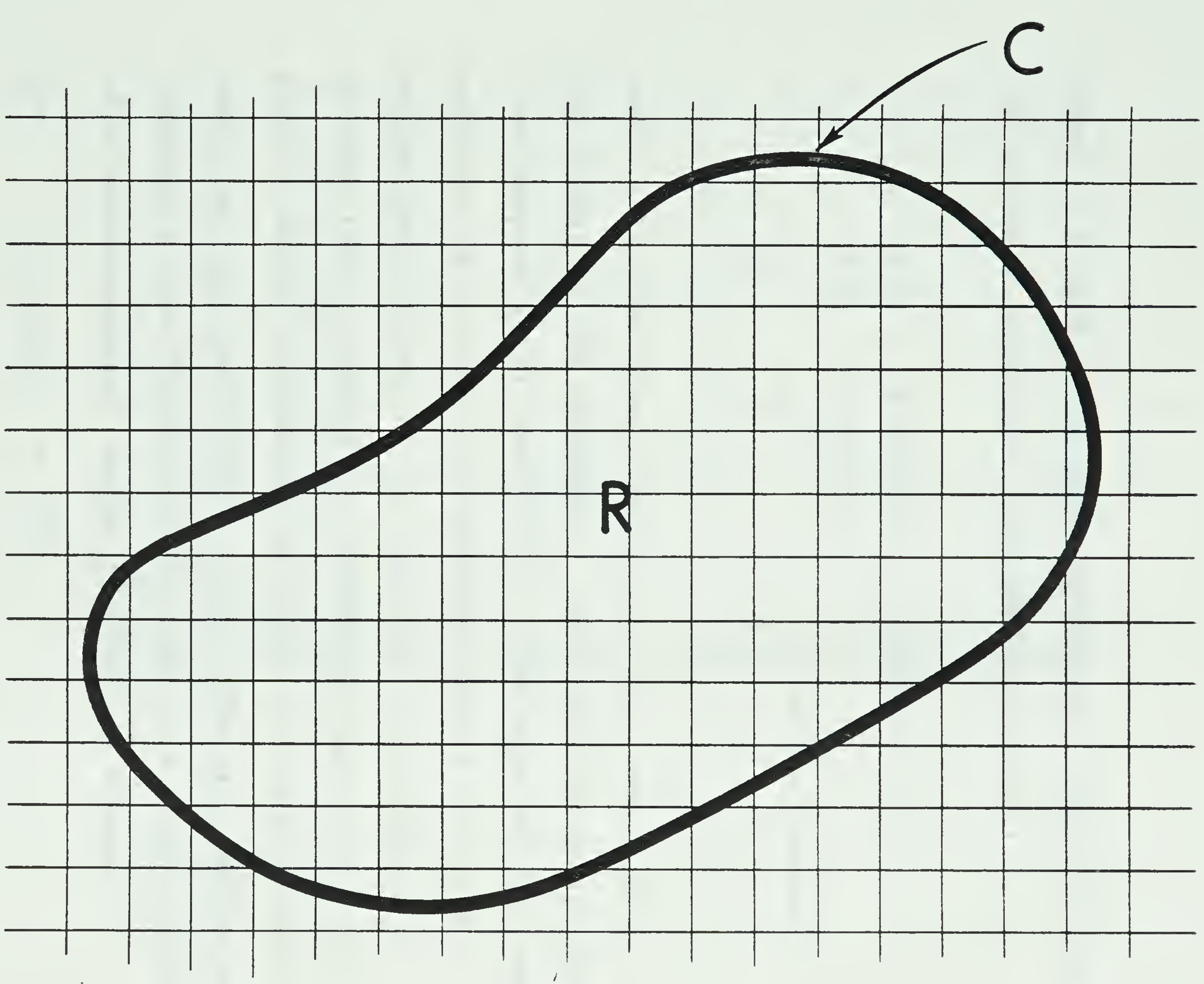

FIG. 1 TWO-DIMENSIONAL MODEL OF A PHYSICAL SYSTEM

where g, K, and S are known functions. Typical initial and boundary conditions for the problem are:

$$\begin{array}{llll} \text{I.C.} & T(x,y,o) & = & f(x,y) \\ \text{B.C.} & T(x,y,t) & = & \chi \\ \text{or} & \dfrac{\partial T}{\partial \gamma} & = & 0 \end{array} \left| \ \text{on the boundary C} \quad \text{(II-2)} \right.$$

where χ is some known function of x,y, and t, and γ is the normal to the boundary. The objective is to determine T satisfying (II-1) and (II-2). The approach of numerical solution is to find T at discrete points in the x-y region and in time t. To this end, the region R is divided into a network of elemental regions or cells, each of which, designated as r_i, is situated around the point (x_i, y_i) and of size $\Delta x\ \Delta y$. The grid points are numbered in the "typewriting" order. A simple network of a two-dimensional region is shown in Figure 2.

Let $\quad T\ (x,y,t) \ = \ T_i^{(n)}$

To find $T_i^{(n)}$ at any grid point, equation (II-1) is integrated over the corresponding cell r_i. On

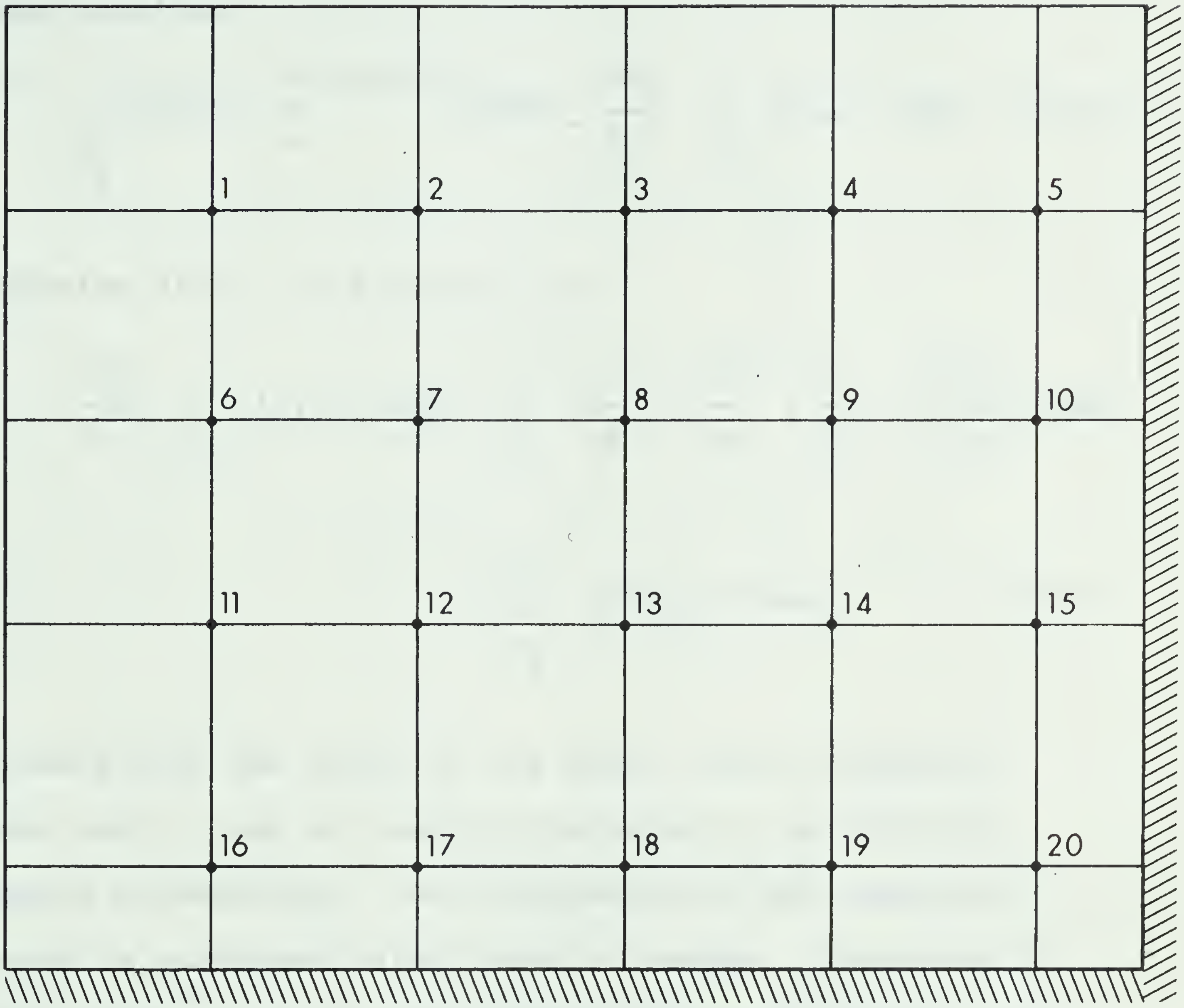

FIG. 2 TWO — DIMENSIONAL NETWORK

approximating

$$\iint_{r_i} g(x,y)\,\frac{\partial T}{\partial t}(x,y,t)\,dxdy \doteq \frac{dT_i}{dt}\iint_{r_i} g(x,y)\,dxdy \qquad \text{(II-3)}$$

equation (II-1) is rewritten as:

$$\frac{dT_i}{dt}\iint_{r_i} g(x,y)\,dxdy = \iint_{r_i}\left[\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right)\right]dxdy$$

$$+\iint_{r_i} S(x,y,t)\,dxdy \qquad \text{(II-4)}$$

Since g(x,y) and S(x,y,t) are known, their integrals over each r_i may be readily evaluated to any desired degree of accuracy. The integration on the remaining terms is performed using Green's Theorem. According to this theorem for any two differentiable functions $F_1(x,y)$ and $F_2(x,y)$ defined in r_i,

$$\iint_{r_i}\left(\frac{\partial F_1}{\partial x}-\frac{\partial F_2}{\partial y}\right)dxdy = \int_{c_i}(F_1\,dy + F_2\,dx) \qquad \text{(II-5)}$$

where c_i is the boundary of the cell r_i (Figure 3). Then, equation (II-4) becomes:

$$\frac{dT_i}{dt} \iint_{r_i} g(x,y)\ dxdy = \int_{c_i} [K \frac{\partial T}{\partial x} dy - K \frac{\partial T}{\partial y} dx] + \iint_{r_i} S(x,y,t)\ dxdy \qquad \text{(II-6)}$$

The line integral of (II-6) is taken along the boundary of the cell in the positive sense (Figure 3). On approximating each derivative in (II-6) by a central difference, and using uniform grid sizes,

$$\int [K \frac{\partial T}{\partial x} dy - K \frac{\partial T}{\partial y} dx] \doteq$$

$$\Delta y \left[K(x_i + \frac{\Delta x}{2}, y_i) \left[\frac{T(x_i+\Delta x, y_i) - T(x_i,y_i)}{\Delta x}\right] + K(x_i - \frac{\Delta x}{2}, y_i) \left[\frac{T(x_i - \Delta x, y_i) - T(x_i, y_i)}{\Delta x}\right]\right]$$

$$+ \Delta x \left[K(x_i, y_i + \frac{\Delta y}{2}) \left[\frac{T(x_i, y_i + \Delta y) - T(x_i, y_i)}{\Delta y}\right] + K(x_i, y_i - \frac{\Delta y}{2}) \left[\frac{T(x_i, y_i - \Delta y) - T(x_i, y_i)}{\Delta y}\right]\right] \qquad \text{(II-7)}$$

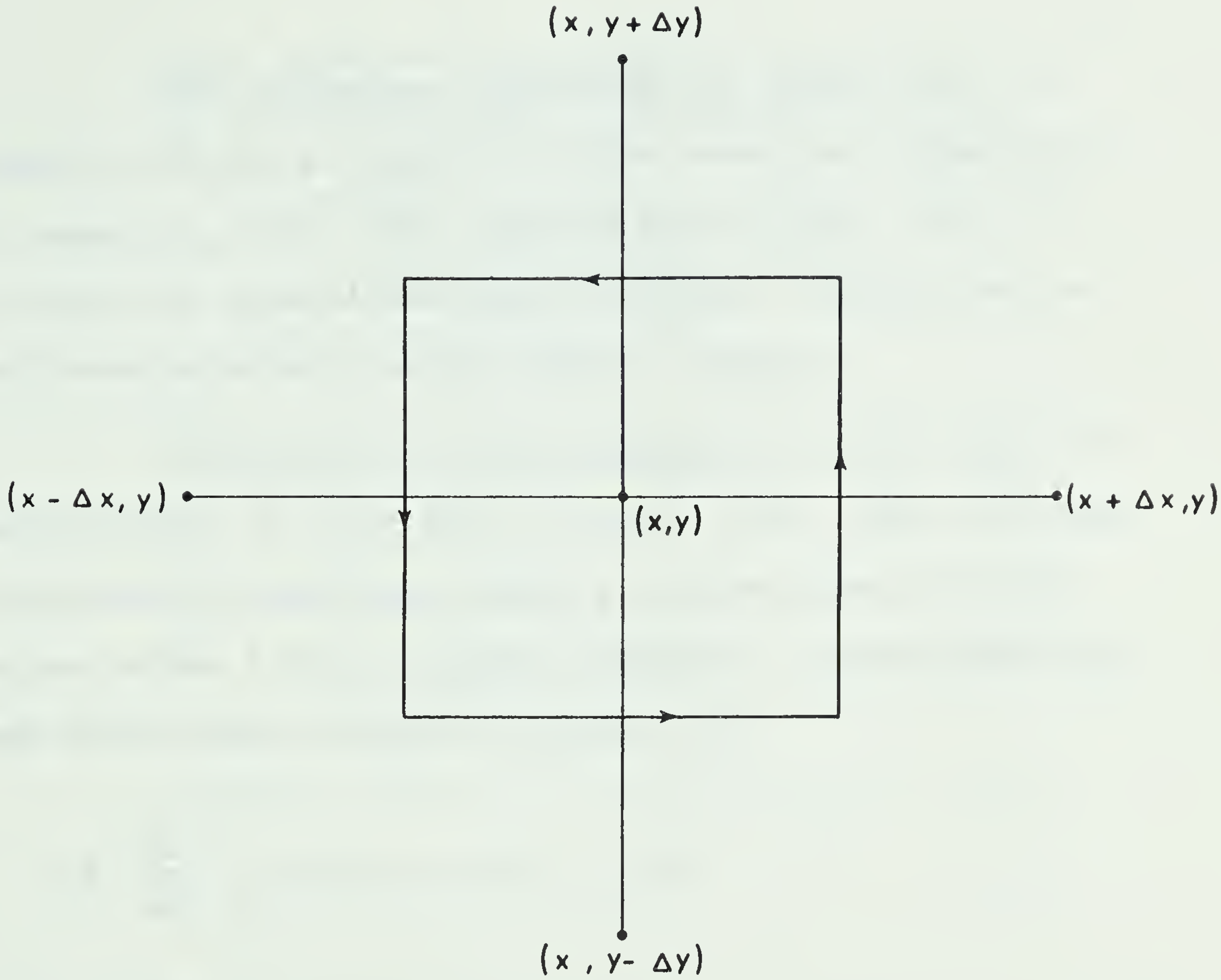

FIG. 3 INTEGRATION OVER A CELL

The procedure, described by Varga [39], is general and can be applied to the numerical integration of equation (II-4) over any polygonal region [27]. In the case of three-dimensional problems, integration is performed similarly using Stokes' Theorem.

Application of this procedure to all the grid points leads to a set of m^2 linear, first order ordinary differential equations, where m is the number of grid points along a row in either direction. These equations may be written in matrix notation as:

$$\underline{G} \frac{d\underline{T}}{dt} = \underline{A}\ \underline{T}(t) + \underline{S}(t) + \underline{\tau}(t) \quad t > 0$$

$$\text{or} \quad \frac{d\underline{T}}{dt} = \underline{G}^{-1}\ \underline{A}\ \underline{T}(t) + \underline{G}^{-1}\ \underline{S}(t) + \underline{G}^{-1}\ \underline{\tau}(t) \qquad \text{(II-8)}$$

$$\text{and} \quad \underline{T}(0) = \underline{T}_o$$

where $\underline{T}$ = solution vector for all grid points at time t.

$\underline{G}$ = diagonal matrix whose diagonal entries are the integrals on the right hand side of (II-3).

$\underline{A}$ = linear matrix operator of order $m^2 \times m^2$

$\underline{S}$ = vector whose elements are integral averages of $S(x,y,t)$; points adjacent to the boundary are affected by the boundary conditions.

$\underline{T}_o$ = initial condition vector whose elements are integral averages of $f(x,y)$ over each cell.

$\underline{\tau}$ = vector of truncation errors resulting from discretization of space derivatives.

Assuming that the truncation error is small,

$$\underline{G} \frac{d\underline{u}}{dt} = \underline{A}\ \underline{u}\ (t) + \underline{S}\ (t) \qquad \text{(II-9)}$$

where $\underline{u}(t)$ approximates $\underline{T}(t)$.

Matrix $\underline{G}$ is diagonal and positive definite. Matrix $\underline{A}$ has negative diagonal entries and non-negative off-diagonal entries. Further, $\underline{A}$ is diagonally dominant, symmetric and hence, a negative definite matrix.

Integration of this equation in the semi-discrete form in the classical manner to yield a closed

form solution is known and will be referred to later. The main concern here is, however, with the solution of a completely discrete form of (II-9).

Perhaps the most convenient and also common approximation of the derivative on the left hand side of (II-9) is by forward difference:

$$\frac{d\underline{u}}{dt} \doteq \frac{\underline{u}^{(n+1)} - \underline{u}^{(n)}}{\Delta t} \qquad \text{(II-10)}$$

where $\underline{u}^{(n)}$ designates $\underline{u}$ at time $t = n\Delta t$. This may be considered as an approximation of $\left(\frac{d\underline{u}}{dt}\right)$ at some time between $(n+1)\,\Delta t$ and $n\Delta t$. It is reasonable therefore that the term $\underline{A}\ \underline{u}$ in (II-9) is also expressed as a weighted average of $\underline{u}^{(n)}$ and $\underline{u}^{(n+1)}$. This is usually accomplished by splitting the matrix $\underline{A}$ into two, one of which premultiplies $\underline{u}^{(n)}$, and the other premultiplies $\underline{u}^{(n+1)}$.

If $\qquad \underline{A} = \underline{M} + \underline{N}$

the numerical integration of (II-9) then leads to:

$$\underline{G}\,\frac{\underline{u}^{(n+1)}-\underline{u}^{(n)}}{\Delta t} = [\underline{M}\,\underline{u}^{(n+1)}+\underline{N}\,\underline{u}^{(n)}] + \frac{1}{\Delta t}\int_{n\Delta t}^{(n+1)\Delta t}\underline{S}(t)\,dt \qquad \text{(II-11)}$$

Rearranging,

$$\underline{u}^{(n+1)} = [\underline{I} - \Delta t\,\underline{G}^{-1}\underline{M}]^{-1}[\underline{I} + \Delta t\,\underline{G}^{-1}\underline{N}]\,\underline{u}^{(n)} + [\underline{I} - \Delta t\,\underline{G}^{-1}\underline{M}]^{-1}\underline{G}^{-1}\int_{n\Delta t}^{(n+1)\Delta t}\underline{S}(t)\,dt \qquad \text{(II-12)}$$

$$\begin{aligned} \text{Let } \underline{P} &= [\underline{I} - \Delta t\,\underline{G}^{-1}\underline{M}] \quad \text{(assumed nonsingular)} \\ \underline{R} &= [\underline{I} + \Delta t\,\underline{G}^{-1}\underline{N}] \end{aligned} \qquad \text{(II-13)}$$

In reviewing the existing numerical methods, an attempt is made to show how different matrix splittings of $\underline{A}$ give rise to different schemes and also to show their effect on the numerical solution. Besides the above physical argument, the choice of any splitting of matrix $\underline{A}$, is guided by four considerations which have an important bearing on the numerical solution:

1. Consistency and convergence
2. Stability

3. Accuracy

4. Simplicity of the method of solution.

The first two considerations are of fundamental importance and provide the primary basis for discussing different finite difference approximations and their consideration follows. The remaining two features will be discussed subsequently for some selected schemes.

<u>Convergence and Stability</u>

The basic notion of an approximation is that of a scheme whereby the error can be made as small as one desires so that the error will actually tend to zero in the limit. The concepts of consistency and convergence are concerned with the substance of this notion and are interrelated. The following formal definitions are from Lax and Richtmyer [25].

The linear parabolic equation (II-1) is approximated in general by the finite difference analog (II-12) which may be rewritten as:

$$\underline{u}^{(n+1)} = \underline{B}\ (\Delta t, \Delta x, \Delta y, \ldots)\ \underline{u}^{(n)} \qquad \text{(II-14)}$$

where

$\underline{B} = \underline{P}^{-1}\underline{R}$ = a linear matrix operator,

Δt, Δx, Δy = increments in time and space

The integral term on the right hand side of (II-12) is omitted. Assuming Δt, Δx, Δy are related by:

$$\Delta x = g_1(\Delta t)$$

$$\Delta y = g_2(\Delta t)$$

then,

$$\underline{u}^{(n+1)} = \underline{B}\,[\Delta t,\ g_1(\Delta t),\ g_2(\Delta t),\ldots]\ \underline{u}^{(n)}$$

$$= \underline{C}\,(\Delta t)\ \underline{u}^{(n)} \qquad \text{(II-15)}$$

From equations (II-10) and (II-15), it is seen that $\underline{G}\left[\dfrac{\underline{C}(\Delta t)-\underline{I}}{\Delta t}\right]\underline{u}$ is indirectly an approximation of the right hand side of equation (II-1). Calling the latter $\underline{A}'\ \underline{T}$, equation (II-14) is a consistent approximation if:

$$\lim_{\Delta t\to 0} \left|\left|\ \underline{G}\left[\frac{\underline{C}(\Delta t)-\underline{I}}{\Delta t}\right]\underline{u} - \underline{A}'\ \underline{T}\ \right|\right| = 0$$

uniformly in t. In other words, a finite difference approximation to a differential equation is consistent

if the former approaches the true solution of the differential equation when the increments of the independent variables (Δt, and hence Δx, and Δy) vanish. For linear problems, consistency of an approximation can be ascertained by examining the truncation error.

In considering convergence, the initial condition is also taken into account. Let E(t) be an operator such that the solution at any finite time t, $\underline{u}(t) = \underline{E}(t)\,\underline{T}_o$, $\underline{T}_o$ being the initial condition. Approximation (II-14) is convergent for the initial value problem if for any initial condition $\underline{T}_o$, and for any sequence $\Delta_\ell t$, n_ℓ such that $\Delta_\ell t \to 0$ and $n_\ell \Delta_\ell t \to t$, then,

$$\left\| \left[\underline{C}(\Delta_\ell t) \right]^{n_\ell} \underline{T}_o - \underline{E}(t)\,\underline{T}_o \right\| \to 0$$

It is seen from the foregone that consistency and convergence are concerned with the question, "Does the finite difference solution $\underline{u}^{(n)}$ minus the true solution $\underline{T}(n\Delta t)$ tend to zero as Δt, Δx, and Δy vanish

for a fixed time t?" On the other hand, the concept of stability is concerned with the question, "Is there a limit to $|\underline{u}^{(n)} - \underline{T}(n\Delta_\ell t)|$ for any n, for fixed $\Delta_\ell t$, Δx, Δy and $\ell = 1, 2, 3, \ldots$?"

More than one formal definition of stability exists in the literature [31,36,20]. O'Brien, Hyman, and Kaplan [31] relate stability to growth of rounding errors, whereas Richtmyer [36] defines it in terms of properties of the matrix operator in the finite difference approximation. Following the latter, the essence of stability is that there should be a limit to the extent to which any component of an initial function can be amplified in the numerical procedure. If, in a sequence of calculations with $\Delta_\ell t \to 0$, each calculation is carried out from $t = 0$ to $t = t_1$, then the operators used in equation (II-15) are,

$$\underline{C}^n \ (\Delta_\ell t) \qquad\qquad 0 < n\Delta t \leq t_1$$
$$\ell = 1,2,3, \ldots \qquad \text{(II-16)}$$

all applied to the initial condition vector $\underline{T}_o$. The approximation $\underline{C}(\Delta_\ell t)$ is said to be stable if $||\underline{C}^n \ (\Delta_\ell t)||$

is bounded for any sequence $\Delta_\ell t > 0$, $\Delta_\ell t \to 0$ as $\ell \to \infty$. This is ensured if the spectral norm $\| \underline{C}(\Delta_\ell t) \|$ (or the spectral radius $\rho(\underline{C}(\Delta_\ell t))$, if $\underline{C}$ is symmetric) is less than unity. It may be noted that the concept of stability has no reference to the differential equation, but is an intrinsic property of the difference scheme.

In the practical analysis of difference approximations, it would suffice if only two of these three criteria are satisfied since, according to Lax's Equivalence Theorem [25], "given a properly posed initial value problem and a finite difference approximation $\underline{C}(\Delta t)$ to it, that satisfies the consistency condition, stability is a necessary and sufficient condition to ensure that $\underline{C}(\Delta t)$ is a convergent approximation".

The analysis of a finite difference approximation for stability may be carried out either by matrix methods according to the above definition or by the von Neumann technique. The latter, which is more commonly used in the literature, is based on expanding

an individual error at a grid point in a finite trigonometric sum and studying the decay or growth of the time-dependent term. At this point, a brief review of the literature on convergence and stability relevant to parabolic equations is in order.

Much of the work on the concepts of convergence and stability is confined to linear systems. Perhaps the earliest work on convergence of finite difference schemes is by Courant, Friedricks, and Lewy [10]. Essentially equivalent results on convergence and also on stability were obtained by J. von Neumann and were reported at length by O'Brien, Hyman, and Kaplan [31]. These authors detailed the Neumann technique of stability analysis of the classical explicit and implicit finite difference schemes for parabolic as well as hyperbolic equations. This technique is limited to linear systems. Its application to the case of a linear equation with variable coefficients is not straight forward. On the other hand, the matrix method of stability analysis is more general and can determine the stability criterion for

the case of variable as well as constant coefficients. In some cases, however, it may require the determination of the maximum eigenvalue of the integrating matrix. The method automatically takes boundary conditions into account. The concept of stability and its interrelation to convergence were variously interpreted in the literature. The interpretation given by O'Brien et al [31] and Richtmyer [36] were stated earlier. Evans, Brousseau, and Keirstead [20] contended that the ideas of stability, convergence, and error propagation were intrinsically unrelated. Specifically, they showed how an unstable scheme diverged even if the rounding-off errors were zero. Douglas [14] dealt with this subject at length and concluded that "for a wide class of difference analogues of parabolic partial differential equations with time-independent co-efficients, stability implied convergence in the mean". The stability analysis of a limited class of nonlinear problems was described by Douglas [14].

In summary, the von Neumann technique of

stability analysis is adequate for the linear system with constant coefficients. If the coefficients are variable, the application of the technique is not straight forward. The matrix method of stability analysis is more general. Techniques applicable to the nonlinear systems need to be developed.

Review of Numerical Methods

The existing numerical methods may now be examined with reference to the generalized approximation represented by equation (II-12). In this review, two practically important considerations are borne in mind:

1. stability of the solution which is ensured if the spectral norm of the coefficient matrix ($\underline{P}^{-1}\underline{R}$) in (II-12) is less than unity;
2. Computational ease of inverting matrix $\underline{P}$.

The survey article by Douglas [18] provides an

excellent source on the numerical methods for parabolic partial differential equations. Important among the finite difference approximations of equation (II-1) are the Forward Difference Explicit (FDE) Method, the Backward Difference Implicit (BDI) Method, the Crank-Nicolson procedure, the Alternating Direction Implicit Procedure (ADIP), and the Alternating Direction Explicit Procedure (ADEP). The structure of these schemes may be visualized from the computational models shown in Figure 4.

If the integral in (II-12) is represented by $\underline{S}'$, the generalized approximation may then be re-written as:

$$\underline{G}\left[\frac{\underline{u}^{(n+1)} - \underline{u}^{(n)}}{\Delta t}\right] = [\underline{M}\ \underline{u}^{(n+1)} + \underline{N}\ \underline{u}^{(n)}] + (\Delta t)^{-1}\underline{S}'$$

or

$$\underline{u}^{(n+1)} = [\underline{I}-\Delta t\ \underline{G}^{-1}\underline{M}]^{-1}[\underline{I}+\Delta t\ \underline{G}^{-1}\underline{N}]\ \underline{u}^{(n)}$$

$$+ [\underline{I}-\Delta t\ \underline{G}^{-1}\underline{M}]^{-1}G^{-1}\underline{S}' \qquad \text{(II-17)}$$

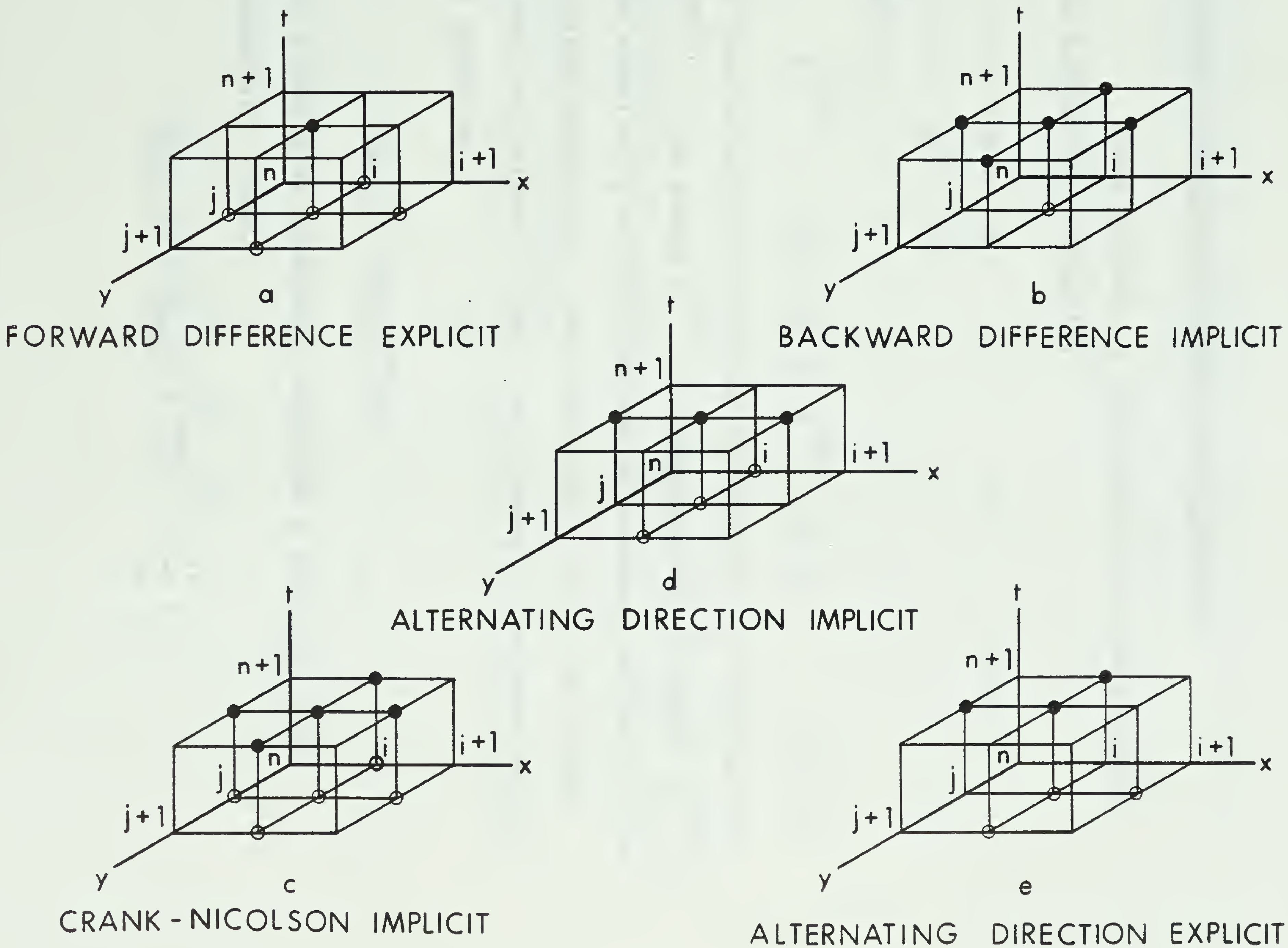

FIGURE 4 COMPUTATIONAL MODELS

Forward Difference Explicit (FDE) Method

This method is computationally very simple. The matrix splitting is:

$$\underline{N} = \underline{A}$$

$$\underline{M} = \underline{0}$$

and

$$\underline{u}^{(n+1)} = [\underline{I}+\Delta t\ \underline{G}^{-1}\underline{N}]\ \underline{u}^{(n)} + \underline{G}^{-1}\underline{S}' \qquad \text{(II-18)}$$

so that no matrix inversion is involved in solving for $\underline{u}^{(n+1)}$. The known $\underline{u}^{(n)}$ is extrapolated to obtain $\underline{u}^{(n+1)}$. The simplicity of this method is severely offset by its limited stability. Let each matrix $(\underline{P}^{-1}\underline{R})$ be called after the method. Then, the forward difference matrix is given by,

$$\underline{B}_{FDE} = [\underline{I}+\Delta t\ \underline{G}^{-1}\underline{A}]$$

Let K and g be constant; matrices $\underline{A}$ and $\underline{B}_{FDE}$ are symmetric. Then,

$$||\underline{B}_{FDE}|| = \rho(\underline{B}_{FDE}) = \max_i \left| 1+\frac{\Delta t}{g}\lambda_i \right|$$

where λ_i are the eigenvalues of $\underline{A}$. Stability requires that:

$$1 \geqslant 1 + \frac{\Delta t}{g} \lambda_i \geqslant -1 \qquad \text{for all } i$$

Since $\underline{A}$ is negative definite, the left hand side inequality is trivially satisfied. The most stringent condition on the right hand side is given by:

$$1 - \frac{\Delta t}{g} |\lambda_i|_{max} \geqslant -1$$

On substituting $|\lambda_i|_{max} \leqslant 4k(\frac{1}{\overline{\Delta x}^2} + \frac{1}{\overline{\Delta y}^2})$, this condition

leads to the criterion:

$$\Delta t_{max} \frac{k}{g} (\frac{1}{\overline{\Delta x}^2} + \frac{1}{\overline{\Delta y}^2}) = 1/2$$

Similarly, the criterion for the three-dimensional case may be shown to be:

$$\Delta t_{max} \frac{k}{g} (\frac{1}{\overline{\Delta x}^2} + \frac{1}{\overline{\Delta y}^2} + \frac{1}{\overline{\Delta z}^2}) = 1/2$$

Backward Difference Implicit (BDI) Method

In contrast to the FDE Method, the matrix splitting in the BDI Method is:

$$\underline{M} = \underline{A}$$

$$\underline{N} = \underline{O}$$

and

$$\underline{u}^{(n+1)} = [\underline{I}-\Delta t\ \underline{G}^{-1}\underline{M}]^{-1}\underline{u}^{(n)} + [\underline{I}-\Delta t\ \underline{G}^{-1}\underline{M}]\ \underline{G}^{-1}\underline{S}' \qquad \text{(II-19)}$$

Since $\underline{M}$ has five non-zero entries, five unknowns are coupled in this equation at each grid point, leading to a set of simultaneous equations. The solution involves for each time step the inversion of a matrix of order $m^2 \times m^2$ in a two-dimensional problem, where m is the number of grid points in the system in each direction. The computational problems are therefore enormous for any but a small value of m. The superior feature of this method is its unlimited stability. The inherent stability of the method is shown by the BDI matrix:

$$\underline{B}_{BDI} = [\underline{I}-\Delta t\ \underline{G}^{-1}\underline{A}]^{-1}$$

Since matrix $\underline{A}$ is negative definite and $\underline{G}$ positive definite,

$$\rho(B_{BDI}) = \left[\frac{1}{1-\Delta t\lambda(\underline{G}^{-1}\underline{A})}\right]_{max.} < 1 \text{ for any } \Delta t > 0.$$

The matrix splittings in the FDE and the BDI methods are limiting cases since one of the matrices $\underline{M}$ and $\underline{N}$ is taken as $\underline{0}$. It is therefore interesting to examine the nature of their solutions. The truncation error for the FDE approximation may be obtained from Taylor series expansion of $T^{(n)}(x-\Delta x,y)$, $T^{(n)}(x+\Delta x,y)$, $T^{(n)}(x,y)$, $T^{(n)}(x,y-\Delta y)$, $T^{(n)}(x,y+\Delta y)$, and $T^{(n+1)}(x,y)$ about $T^{(n+1/2)}(x,y)$.

$$\text{Let} \quad \Delta_\beta^2 T = \frac{[T(\beta-\Delta\beta) - 2T(\beta) + T(\beta+\Delta\beta)]}{\overline{\Delta\beta}^2}$$

and assume $k/g = 1$, for simplicity.

Assuming that $T(x,y,t)$ is sufficiently differentiable with respect to t,x, and y, the truncation error for the FDE method is given by:

$$\tau_{FDE} \equiv [T_t - T_{xx} - T_{yy}] - \left[\frac{T^{(n+1)}(x,y) - T^{(n)}(x,y)}{\Delta t} - \Delta_x^2 T^{(n)} - \Delta_y^2 T^{(n)}\right]$$

$$= \frac{\Delta t}{2}(T_{txx} + T_{tyy}) + \frac{\overline{\Delta x}^2}{12} T_{xxxx} + \frac{\overline{\Delta y}^2}{12} T_{yyyy} + \ldots$$

where the derivatives on the right hand side are evaluated at $(x,y,t+\frac{\Delta t}{2})$. Similarly, the truncation error of the BDI method is given by:

$$\tau_{BDI} = -\frac{\Delta t}{2}(T_{txx} + T_{tyy}) + \frac{\overline{\Delta x}^2}{12} T_{xxxx} + \frac{\overline{\Delta y}^2}{12} T_{yyyy} + \ldots$$

Note that the leading terms of the truncation errors in the two methods are of the same magnitude but of opposite sign. The actual errors are also of opposite sign as shown by the typical solutions of the two methods (Figure 5). The FDE Method approaches the true solution from below, whereas the BDI solution approaches it from above [38].

Crank-Nicolson Procedure

The above truncation errors suggest that a combination of the FDE and the BDI schemes should give an improved solution, since the truncation error will

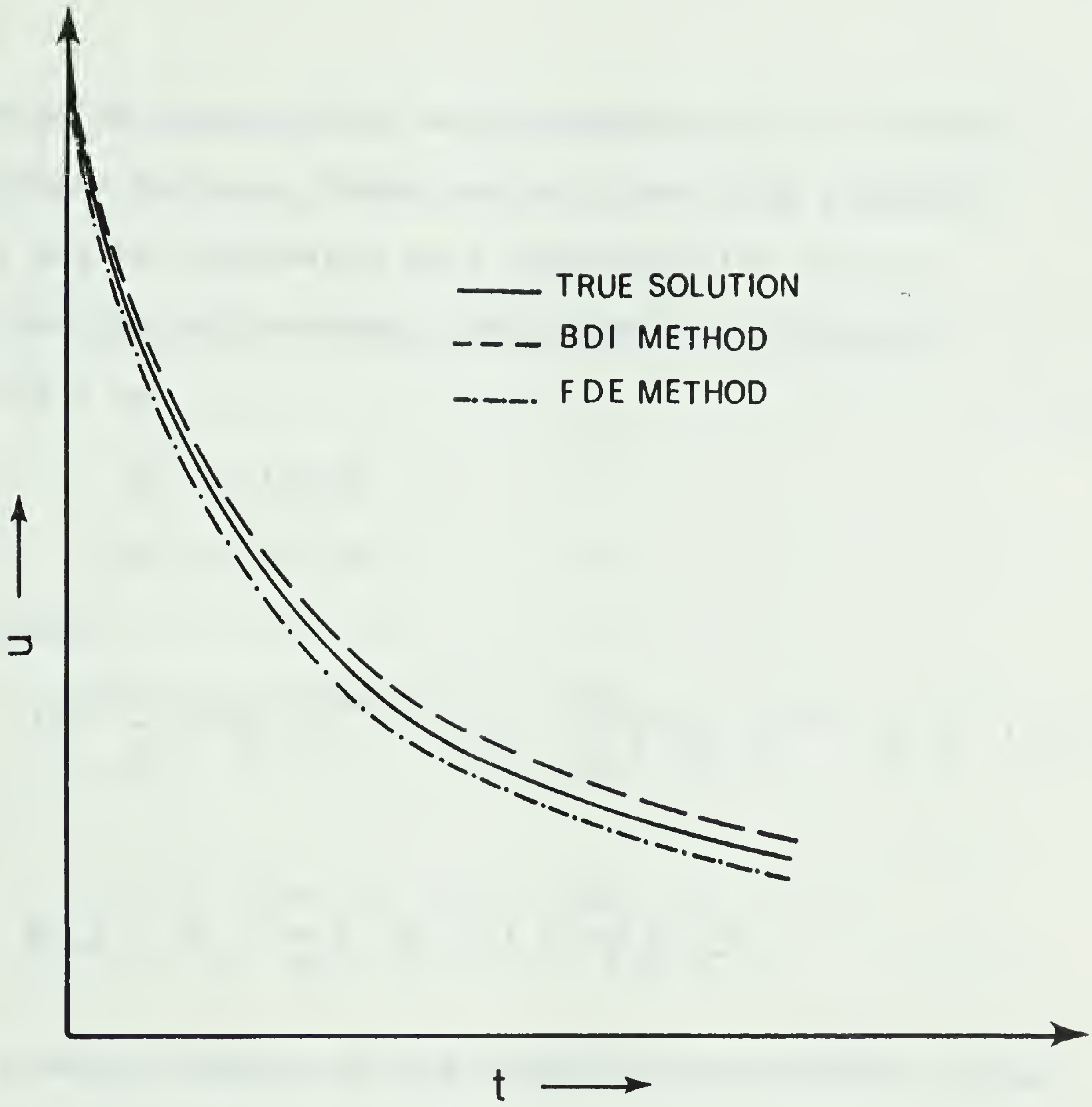

FIGURE 5: IMPLICIT & EXPLICIT METHODS

then be of second order with respect to Δt. Taking advantage of this, Crank and Nicolson [12] proposed what may be considered as a superposition of the FDE and the BDI schemes. This meant splitting the matrix $\underline{A}$ as:

$$\underline{M} = 1/2\ \underline{A}$$

$$\underline{N} = 1/2\ \underline{A}$$

so that,

$$(\underline{I} - \frac{\Delta t}{2} \underline{G}^{-1}\underline{A}]\ \underline{u}^{(n+1)} = [\underline{I} + \frac{\Delta t}{2} \underline{G}^{-1}\underline{A}]\ \underline{u}^{(n)} + \underline{G}^{-1}\underline{S}' \quad \text{(II-20)}$$

and

$$\underline{B}_{C-N} = [\underline{I} - \frac{\Delta t}{2} \underline{G}^{-1}\underline{A}]^{-1} [\underline{I} + \frac{\Delta t}{2} \underline{G}^{-1}\underline{A}]$$

The spectral radius of the Crank-Nicolson matrix, given by;

$$\rho(\underline{B}_{C-N}) = \left| \frac{1 + \frac{\Delta t}{2} \lambda(\underline{G}^{-1}\underline{A})}{1 - \frac{\Delta t}{2} \lambda(\underline{G}^{-1}\underline{A})} \right|_{max}$$ max, is less than unity for all $\Delta t > 0$, since $\underline{A}$ is negative definite. The C-N procedure is, therefore, unconditionally stable despite

the inclusion of $\underline{u}^{(n)}$. The method is implicit and has the same computational problems as the BDI scheme.

Alternating Direction Implicit Procedure (ADIP)

In view of the attractive feature of unlimited stability of the BDI scheme, considerable research effort was devoted towards a better splitting of matrix $\underline{A}$ so that the order and nature of the matrix to be inverted are simpler. This has given rise to a class of alternating direction implicit procedures. The original procedure developed by Peaceman and Rachford [32] primarily for the two-dimensional problems, is typical of this class and is discussed here in detail.

Peaceman and Rachford, in effect, split the matrix $\underline{A}$ in such a way that all the entries of $\underline{A}$ that arise from discretizing the x-directional derivative of T are incorporated in one matrix, $\underline{H}$, and all those that arise from discretizing the y-directional derivative are represented by another matrix, $\underline{V}$, so that the matrix splitting in ADIP is:

$$\underline{A} = \underline{H} + \underline{V}$$

or

$$\underline{A} = \underline{V} + \underline{H}$$

Matrices $\underline{H}$ and $\underline{V}$ are of the same order as $\underline{A}$, $m^2 \times m^2$. They have each no more than three non-zero entries in any row. Both are diagonally dominant matrices with negative diagonal entries. Both $\underline{H}$ and $\underline{V}$ are therefore negative definite. The method of integration consists of two stages for a two-dimensional problem. Each stage uses a half time step.

In Stage one,

$$\underline{M} = \underline{H}$$

$$\underline{N} = \underline{V}$$

so that,

$$[\underline{I} - \frac{\Delta t}{2} \underline{G}^{-1}\underline{H}]\underline{u}^{(n+1/2)} = [\underline{I} + \frac{\Delta t}{2} \underline{G}^{-1}\underline{V}]\underline{u}^{(n)} + \underline{G}^{-1}\underline{S}' \qquad \text{(II-21)}$$

In Stage two, the choice of $\underline{M}$ and $\underline{N}$ is reversed:

$$\underline{M} = \underline{V}$$

$$\underline{N} = \underline{H}$$

so that,

$$[\underline{I} - \frac{\Delta t}{2}\,\underline{G}^{-1}\underline{V}]\underline{u}^{(n+1)} = [\underline{I} + \frac{\Delta t}{2}\,\underline{G}^{-1}\underline{H}]\underline{u}^{(n+1/2)} + \underline{G}^{-1}\underline{S}' \qquad \text{(II-22)}$$

The matrix $[\underline{I} - \frac{\Delta t}{2}\,\underline{G}^{-1}\underline{H}]$ in stage one and the matrix $[\underline{I} - \frac{\Delta t}{2}\,\underline{G}^{-1}\underline{V}]$ in stage two are both tridiagonal and can be inverted by an efficient method due to Thomas to be described later. The above matrices are of order $m^2 \times m^2$. In practice, taking advantage of the boundary condition, the matrix equation (II-21) can be broken into m small tridiagonal matrix equations; the matrix associated with each of them is of m x m order and corresponds to a row of grid points. So, equation (II-21) may be solved for a row of points at a time. Similarly, equation (II-22) may be solved for a column of points at a time. Computationally, the two stages involve the inversion of an m x m tridiagonal matrix, 2m times for each time step. This is significantly simpler than inverting an $m^2 \times m^2$ matrix once as in the BDI method.

The tridiagonal matrix equation resulting from (II-21) or (II-22) is solved by an algorithm

outlined by Bruce et al [5] as due to Thomas. The equation is of the form:

$$\underline{W}\ \underline{Z} = \underline{Q}$$

Triangular decomposition of the tridiagonal matrix $\underline{W}$ yields two bidiagonal matrices $\underline{L}$ and $\underline{U}$, whose inversion is very easy.

Then, $\underline{W}\ \underline{Z} = \underline{L}\ \underline{U}\ \underline{Z} = \underline{Q}$

If $\underline{U}\ \underline{Z} = \underline{Y}$, then, $\underline{L}\ \underline{Y} = \underline{Q}$

and $\underline{Y} = \underline{L}^{-1}\underline{Q}$ and $\underline{Z} = \underline{U}^{-1}\underline{Y}$

Solving for $\underline{Y}$ and $\underline{Z}$ in that order is, therefore, computationally simple.

To examine the stability of ADIP, equations (II-21) and (II-22) may be combined to yield:

$$\underline{u}^{(n+1)} = \underline{B}_{P-R}\ \underline{u}^{(n)} \qquad \text{(II-23)}$$

where the constant is deleted and

$$\underline{B}_{P-R} = [\underline{I} - \frac{\Delta t}{2}\underline{G}^{-1}\underline{V}]^{-1}[\underline{I} + \frac{\Delta t}{2}\underline{G}^{-1}\underline{H}][\underline{I} - \frac{\Delta t}{2}\underline{G}^{-1}\underline{H}]^{-1}$$

$$[\underline{I} + \frac{\Delta t}{2}\underline{G}^{-1}\underline{V}]$$

is the Peaceman-Rachford matrix. Assuming that $\underline{H}$ and $\underline{V}$ commute, Varga [39], showed that $\underline{B}_{P-R}$ is similar to the Crank-Nicolson matrix which is unconditionally stable.

Douglas [18] reported that for the three-dimensional case, ADIP was unstable for any useful values of Δt. As a result, a number of modifications of this procedure were attempted. Of these the Douglas-Rachford method and its subsequent modification by Brian proved to be useful. The basic approach of these methods is, nevertheless, similar to that of ADIP and involves in each stage, the inversion of a tridiagonal matrix arising from a row or a column of points.

<u>Douglas-Rachford Method</u> [15]

The essential difference between this method and ADIP is: whereas ADIP calculated $\underline{u}^{(n+1/2)}$ followed by $\underline{u}^{(n+1)}$, this method makes a first estimate, $\overset{*}{\underline{u}}^{(n+1)}$, in the first stage. This is further improved to obtain

$\underline{u}^{(n+1)}$ in one more stage if it is a two-dimensional problem or in two more stages in a three-dimensional problem. The two stages in the former case are as follows:

Stage one,

$$\underline{M} = \underline{H}$$

$$\underline{N} = \underline{V}$$

so that,

$$[\underline{I} - \Delta t\ \underline{G}^{-1}\underline{H}]\overset{*}{\underline{u}}^{(n+1)} = [\underline{I} + \Delta t\ \underline{G}^{-1}\underline{V}]\ \underline{u}^{(n)} \qquad \text{(II-24)}$$

In stage two, the matrix splitting is not a reversal of that in stage one as in ADIP. The approximation is given by:

$$\underline{G}\left[\frac{\underline{u}^{(n+1)} - \overset{*}{\underline{u}}^{(n+1)}}{\Delta t}\right] = \underline{V}\ [\underline{u}^{(n+1)} - \underline{u}^{(n)}]$$

Therefore,

$$[\underline{I} - \Delta t\ \underline{G}^{-1}\underline{V}]\underline{u}^{(n+1)} = \overset{*}{\underline{u}}^{(n+1)} - \Delta t\ \underline{G}^{-1}\underline{V}\ \underline{u}^{(n)}.$$

On substituting for $\overset{*}{\underline{u}}^{(n+1)}$ from (II-24),

$$\underline{u}^{(n+1)} = [\underline{I} - \Delta t\ \underline{G}^{-1}\underline{V}]^{-1}\{(\underline{I} - \Delta t\ \underline{G}^{-1}\underline{H})^{-1}(\underline{I} + \Delta t\ \underline{G}^{-1}\underline{V})$$

$$- \Delta t\ \underline{G}^{-1}\underline{V}\ \}\ \underline{u}^{(n)} \qquad \text{(II-25)}$$

where the constant term is omitted. The method was shown to be stable for any $\Delta t > 0$.

Brian's Scheme [4]

This scheme combines the approach of successive estimations of the Douglas-Rachford method with the ADIP's calculation of intermediate values (i.e. at $(n+1/2)\ \Delta t$). The advantage of this scheme is that it achieves a higher order accuracy with practically the same computational time. This procedure is to the Douglas-Rachford procedure what Crank-Nicolson method is to the BDI method. The method is identical to the ADIP in the two-dimensional case. The details of this scheme in the three-dimensional case will be given in the next chapter.

A similar alternating direction implicit procedure was also proposed by Douglas [18], which might be considered as a perturbation of the Crank-

Nicolson procedure. Notably, in all the above multi-stage procedures the storage requirements of the function values are as many times the number of function values as the dimensionality of the problem.

Alternating Direction Explicit Procedure (ADEP)

An interesting new method of splitting the matrix $\underline{A}$ was achieved in what may be called an alternating direction explicit procedure. First suggested by Saul'yev [37] for the solution of one-dimensional diffusion equation and later extended by Larkin [24] to the two-dimensional case, this method bisects the matrix $\underline{A}$ along the diagonal. The procedure consists of two stages irrespective of the dimensionality of the problem.

Let $$\underline{A} = \underline{E} + \underline{F} - \underline{D}$$

where $\underline{D}$ is a diagonal matrix with $-a_{ii}$ as the diagonal elements. $\underline{E}$ and $\underline{F}$ are strictly lower and upper triangular matrices whose entries are those of $\underline{A}$ respectively below and above the main diagonal of $\underline{A}$.

Let $\underline{D}$ be further disected along the diagonal into two halves, $\underline{D_1}$ and $\underline{D_2}$, with the restriction that $\underline{D_1}$ does not contain components of K not present in $\underline{E}$; Similarly $\underline{D_2}$ does not contain components of K not present in $\underline{F}$. Note that, for a homogeneous medium (K - constant) $\underline{D_1} = \underline{D_2}$ and $\underline{E} = \underline{F}^T$. Referring to equation (II-17), the matrix splitting may be described as follows:

Stage one,

$$\underline{M} = \underline{E} - \underline{D}_1$$

$$\underline{N} = \underline{F} - \underline{D}_2$$

so that,

$$[\underline{I} + \frac{\Delta t}{2}\underline{G}^{-1}(\underline{D}_1 - \underline{E})]\ \underline{u}^{(n+1/2)} = [\underline{I} - \frac{\Delta t}{2}\underline{G}^{-1}(\underline{D}_2 - \underline{F})]\underline{u}^{(n)} + \underline{G}^{-1}\ \underline{S}' \qquad \text{(II-26)}$$

Stage two, the matrix splitting is now reversed;

$$\underline{M} = \underline{F} - \underline{D}_2$$

$$\underline{N} = \underline{E} - \underline{D}_1$$

so that,

$$[\underline{I}+ \frac{\Delta t}{2} \underline{G}^{-1}(\underline{D}_2 - \underline{F})]\underline{u}^{(n+1)}=[\underline{I}- \frac{\Delta t}{2} \underline{G}^{-1}(\underline{D}_1 - \underline{E})]\underline{u}^{(n+1/2)}$$

$$+ \underline{G}^{-1} \underline{S}' \qquad \text{(II-27)}$$

Since $\underline{G}$, $\underline{D}_1$, and $\underline{D}_2$ are diagonal matrices, these equations may be rearranged to yield respectively:

$$[\underline{D}_1' - \frac{\Delta t}{2} \underline{G}^{-1}\underline{E}]\underline{u}^{(n+1/2)} = [\underline{D}_2' + \frac{\Delta t}{2} \underline{G}^{-1}\underline{F}]\underline{u}^{(n)} + \underline{G}^{-1}\underline{S}' \qquad \text{(II-28)}$$

and

$$[\underline{D}_2'' - \frac{\Delta t}{2} \underline{G}^{-1}\underline{F}]\underline{u}^{(n+1)} = [\underline{D}_1'' + \frac{\Delta t}{2} \underline{G}^{-1}\underline{E}]\underline{u}^{(n+1/2)} + \underline{G}^{-1}\underline{S}' \qquad \text{(II-29)}$$

where

$$\underline{D}_1' = \underline{I} + \frac{\Delta t}{2} \underline{G}^{-1}\underline{D}_1$$

$$\underline{D}_1'' = \underline{I} - \frac{\Delta t}{2} \underline{G}^{-1}\underline{D}_1$$

$$\underline{D}_2' = \underline{I} - \frac{\Delta t}{2} \underline{G}^{-1}\underline{D}_2$$

$$\underline{D}_2'' = \underline{I} + \frac{\Delta t}{2} \underline{G}^{-1}\underline{D}_2$$

Each of these is obviously a diagonal matrix; $\underline{D}_1'$ and $\underline{D}_2''$, which are contained in the matrices to be inverted, are, in addition, positive definite.

In the set of algebraic equations (II-28), in general, three unknowns are coupled in any one equation and the matrix equation appears to be implicit. However, $\underline{P} = (\underline{D}_1' - \frac{\Delta t}{2}\,\underline{G}^{-1}\underline{E})$ being a lower triangular matrix, the equations can be solved explicitly, if the computation is started from the top. Physically, this means that, of the three unknowns coupled in one equation, two are known from the two boundary conditions, if this point is at a corner. The equation is solved for the remaining one unknown explicitly. For the next point in the row, of the three unknowns, one is just computed, another is known from one boundary condition still available, leaving the third as the only unknown. For an interior grid point, two of the three unknowns are always known from the previous computation provided computation is performed in the increasing order of the number of the grid point.

In the reverse sweep, $\underline{P} = [\underline{D}_2'' - \frac{\Delta t}{2}\underline{G}^{-1}\underline{F}]$ being an upper triangular matrix, the computation is started at the corner point with the highest number. This is obviously the point where the forward sweep ended. Equations (II-28) and (II-29) are in effect, explicit and may be written as:

$$\underline{D}_1' \, \underline{u}^{(n+1/2)} = \frac{\Delta t}{2}\underline{G}^{-1}\underline{E} \, \underline{u}^{(n+1/2)} + [\underline{D}_2' + \frac{\Delta t}{2}\underline{G}^{-1}\underline{F}]\underline{u}^{(n)} + \underline{G}^{-1}\underline{S}' \qquad \text{(II-30)}$$

and

$$\underline{D}_2'' \, \underline{u}^{(n+1)} = \frac{\Delta t}{2}\underline{G}^{-1}\underline{F} \, \underline{u}^{(n+1)} + [\underline{D}_1'' + \frac{\Delta t}{2}\underline{G}^{-1}\underline{E}]\underline{u}^{(n+1/2)} + \underline{G}^{-1}\underline{S}' \qquad \text{(II-31)}$$

The ADEP, as will be shown in the next chapter, is similar to the "Point Successive Symmetric Over-relaxation" method developed by Frankel [21] for the iterative solution of elliptic equations. The latter method was shown by Varga [39] to be stable for $0 \leqslant \omega \leqslant 2$, where ω is the relaxation factor. This range of ω

corresponds to $0 \leqslant \Delta t \leqslant \infty$. The unconditional stability of ADEP may also be proved by the von Neumann technique.

The advantage of ADEP over ADIP in respect of computational time is obvious. While ADIP involves the inversion of an m x m matrix 2m times, ADEP involves the explicit solution of one unknown at a time, m^2 times. Larkin [24] gave the ratio of computational times of ADIP and ADEP as 5:1. The accuracy of ADEP was comparable to that of ADIP. A recent article by Coats and Terhume [8] claimed better accuracy for ADIP. They also reported that in a particular reservoir problem the ADEP solution was non-conservative. These points will be discussed subsequently.

The closed form solution of the semi-discrete equation (II-9), referred to earlier, was obtained by Darsi and Quon [13]. The method is based on transforming $\underline{u}$ into another vector using eigenvectors of the coefficient matrix $\underline{A}$. Its application was demonstrated for a linear problem. A limitation of this method is that the evaluation of eigenvectors

and eigenvalues takes excessive time if the coefficient matrix is of a large order.

In all the above methods, the space derivative is approximated by a three-point central difference and the time derivative by two-point forward difference. However, attention has also been given [18, 33] to higher order approximations using more than two time levels for the time derivative and more than three points for the space derivative . This approach has not gained wide application because of three disadvantages: larger memory requirements, difficulty of treating points close to the boundary, and greater computing effort. Nevertheless, the higher order accuracy possible in this approach may eventually prove to be an overriding advantage.

In summary, although there are a number of methods proposed to numerically integrate equation (II-1), only two of them are practically useful, namely, the alternating direction implicit procedures typified by the method of Peaceman and Rachford and

the alternating direction explicit procedure. Both the methods are unconditionally stable. Being an explicit method, ADEP holds special significance for large and nonlinear systems.

III. ALTERNATING DIRECTION EXPLICIT PROCEDURE

The alternating direction explicit procedure (ADEP), discussed in the previous chapter, combines the stability of the implicit methods and the simplicity of the explicit methods. These two features are especially desirable in the solution of large and nonlinear systems. In this chapter, the extension of this method to the three-dimensional parabolic problem is discussed. In an example problem the accuracy of the proposed method is compared with the best implicit procedure. This is followed by a comparison of their computational times and also their application to nonlinear problems.

The new method, ADEP, is extended to the three-dimensional problem defined by equation (III-1). As stated earlier, the number of integration stages in ADEP is independent of the dimensionality of the problem. Therefore, the two stages of ADEP given by equation (II-26) and (II-27) or by (II-28) and (II-29) for the two-dimensional case, are also valid for the three-dimensional problem. All the features associated with matrices $\underline{D}$, $\underline{E}$, $\underline{F}$, $\underline{D}_1'$, $\underline{D}_1''$, $\underline{D}_2'$, and $\underline{D}_2''$

in the above equations, also hold good. The order of these matrices is now $m^3 \times m^3$ instead of $m^2 \times m^2$, where m is the number of grid points in each direction.

In the matrix equation (II-28), $\underline{P} = \underline{D}_1' - \frac{\Delta t}{2} \underline{G}^{-1}\underline{E}$ is a lower triangular matrix. Each of the equations in this set is solved explicitly starting from the top and proceeding in the increasing order of the grid number. The procedure is similar to the two-dimensional case except that the number of unknowns coupled in an algebraic equation are four in a three-dimensional case compared to three in a two-dimensional case. But the number of boundary conditions available is now three. Therefore in the three-dimensional case three of the four quantities at $(n+1/2)\Delta t$ are known from the boundary conditions or from previous computation. The reverse sweep, using equation (II-29) is similar. $\underline{P} = \underline{D}_2'' - \frac{\Delta t}{2} \underline{G}^{-1}\underline{F}$ is an upper triangular matrix and the point by point evaluation of $\underline{u}^{(n+1)}$ starts from the bottom equation of (II-29). This obviously corresponds to the grid point with the

highest index. Computation proceeds along the same path as in the forward sweep but in the opposite direction.

The unconditional stability of ADEP in the three-dimensional case can be proved as in the two-dimensional case.

Comparison with Iterative Methods for Solving Laplace's Equation

Under certain conditions, the ADEP may be shown to be identical to the Point Successive Symmetric Overrelaxation (PSOR) Method developed by Frankel [21] for solving Laplace's equation. Let $\Delta t/\overline{\Delta x}^2 = \theta$ and assume $\underline{G} = \frac{1}{4}\underline{D}$. Then, the forward sweep of ADEP (II-26) is given by:

$$\frac{1}{4\theta}\underline{D}\,[\underline{u}^{(n+1)} - \underline{u}^{(n)}] = [\underline{E} - \underline{D}_1]\,\underline{u}^{(n+1)} + [\underline{F} - \underline{D}_2]\,\underline{u}^{(n)}$$

Adding $\underline{D}[\underline{u}^{(n+1)} - \underline{u}^{(n)}]$ to both sides and rearranging,

$$\underline{D}[\underline{u}^{(n+1)} - \underline{u}^{(n)}] = [\underline{D}^{-1}\underline{D}_1 + \frac{1}{4\theta}\ \underline{I}]^{-1}[\underline{E}\ \underline{u}^{(n+1)} + (\underline{F}-\underline{D})\underline{u}^{(n)}]$$

In the case of homogeneous medium, $\underline{D}_1 = \underline{D}_2 = \frac{1}{2}\ \underline{D}$.
On substituting for $\underline{D}_1$ and $\underline{D}_2$, and rearranging, the above equation becomes:

$$[\underline{D} - \omega\ \underline{E}]\underline{u}^{(n+1)} = [(1-\omega)\ \underline{D} + \omega\ \underline{F}]\ \underline{u}^{(n)}$$

where $\omega = \dfrac{4\theta}{1+2\theta}$ is the relaxation factor. This is identical to the forward sweep of PSOR. Likewise, the reverse sweeps of ADEP and PSOR also may be shown to be similar. The PSOR matrix was shown by Varga [39] to be convergent for $0 \leqslant \omega \leqslant 2$ which corresponds to $0 \leqslant \Delta t \leqslant \infty$.

The Brian-Douglas-Rachford (B-D-R) method [4] for the three-dimensional problem was shown to be the most efficient method and also to have the higher order accuracy of the Crank-Nicolson procedure. The latter has a truncation error of $0[\overline{\Delta t}^2 + \overline{\Delta x}^2]$. The accuracy of the ADEP is compared with that of the B-D-R method. Before doing this, the truncation errors in the two

stages of ADEP are examined. For the sake of simplicity, the normalized form of the three-dimensional diffusion equation is considered:

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \qquad \text{(III-1)}$$

Let $\qquad T(x,y,z,t) = T_{i,j,k}^{(n)}$

and $\qquad \Delta_x T_{i,j,k}^{(n)} = T_{i+1,j,k}^{(n)} - T_{i,j,k}^{(n)}$

The truncation error, τ, reckoned as the differential equation minus its difference analogue, is given below for the forward as well as the reverse sweeps.

$$\tau_F \equiv [T_t - T_{xx} - T_{yy} - T_{zz}] - \left[\frac{T_{i,j,k}^{(n+1/2)} - T_{i,j,k}^{(n)}}{\frac{\Delta t}{2}} \right.$$
$$- \frac{\Delta_x T_{i,j,k}^{(n)} - \Delta_x T_{i-1,j,k}^{(n+1/2)}}{\overline{\Delta x}^2} - \frac{\Delta_y T_{i,j,k}^{(n)} - \Delta_y T_{i,j-1,k}^{(n+1/2)}}{\overline{\Delta y}^2}$$
$$\left. - \frac{\Delta_z T_{i,j,k}^{(n)} - \Delta_z T_{i,j,k-1}^{(n+1/2)}}{\overline{\Delta z}^2} \right]$$

$$= \left[\frac{\Delta t}{2\Delta x} T_{tx} + \frac{\Delta t}{2\Delta y} T_{ty} + \frac{\Delta t}{2\Delta z} T_{tz}\right]$$

$$+ \left[\frac{\overline{\Delta t}^2}{24} T_{ttt}\right] - \left[\frac{\overline{\Delta x}^2}{12} T_{xxxx} + \frac{\overline{\Delta y}^2}{12} T_{yyyy} + \frac{\overline{\Delta z}^2}{12} T_{zzzz}\right]$$

$$+ \left[\frac{\overline{\Delta t}^2}{8\Delta x} T_{ttx} + \frac{\overline{\Delta t}^2}{8\Delta y} T_{tty} + \frac{\overline{\Delta t}^2}{8\Delta z} t_{ttz}\right] + \ldots \quad \text{(III-2)}$$

$$\tau_R \equiv [T_t - T_{xx} - T_{yy} - T_{zz}] - \left[\frac{T_{i,j,k}^{(n+1)} - T_{i,j,k}^{(n+1/2)}}{\Delta t/2}\right.$$

$$- \frac{\Delta_x T_{i,j,k}^{(n+1)} - \Delta_x T_{i-1,j,k}^{(n+1/2)}}{\overline{\Delta x}^2} - \frac{\Delta_y T_{i,j,k}^{(n+1)} - \Delta_y T_{i,j-1,k}^{(n+1/2)}}{\overline{\Delta y}^2}$$

$$\left. - \frac{\Delta_z T_{i,j,k}^{(n+1)} - \Delta_z T_{i,j,k-1}^{(n+1/2)}}{\overline{\Delta z}^2}\right]$$

$$= -\left[\frac{\Delta t}{2\Delta x} T_{tx} + \frac{\Delta t}{2\Delta y} T_{ty} + \frac{\Delta t}{2\Delta z} T_{tz}\right] + \left[\frac{\overline{\Delta t}^2}{24} T_{ttt}\right]$$

$$- \left[\frac{\overline{\Delta x}^2}{12} T_{xxxx} + \frac{\overline{\Delta y}^2}{12} T_{yyyy} + \frac{\overline{\Delta z}^2}{12} T_{zzzz} \right]$$

$$- \left[\frac{\overline{\Delta t}^2}{8\Delta x} T_{ttx} + \frac{\overline{\Delta t}^2}{8\Delta y} T_{tty} + \frac{\overline{\Delta t}^2}{8\Delta z} T_{ttz} \right] + \ldots \qquad \text{(III-3)}$$

The derivatives on the right hand side of (III-2) are at $(n\Delta t)$ and those on the right hand side of (III-3) are at $(n + 1/2)\Delta t$. Note that the first and the last terms of τ_F have approximately the same magnitude but opposite sign as the corresponding terms of τ_R. On alternate application of the forward and the reverse sweeps of ADEP, these error terms, presumably, tend to cancel each other. This improves the accuracy of the two-stage solution compared to the accuracy of the individual stages.

The Brian-Douglas-Rachford procedure in its simplified form may be written as follows.

Let

$$\Delta_x^2 \ u_{i,j,k}^{(n)} = \frac{u_{i-1,j,k}^{(n)} - 2u_{i,j,k}^{(n)} + u_{i+1,j,k}^{(n)}}{\overline{\Delta x}^2}$$

$$\frac{\overset{*}{u}{}^{(n+1/2)}_{i,j,k} - u^{(n)}_{i,j,k}}{\Delta t/2} = \Delta_x^2 \overset{*}{u}{}^{(n+1/2)}_{i,j,k} + \Delta_y^2 u^{(n)}_{i,j,k} + \Delta_z^2 u^{(n)}_{i,j,k} \qquad \text{(III-4)}$$

$$\frac{\overset{**}{u}{}^{(n+1/2)}_{i,j,k} - \overset{*}{u}{}^{(n+1/2)}_{i,j,k}}{\Delta t/2} = \Delta_y^2 \overset{**}{u}{}^{(n+1/2)}_{i,j,k} - \Delta_y^2 u^{(n)}_{i,j,k} \qquad \text{(III-5)}$$

$$\frac{u^{(n+1)}_{i,j,k} + u^{(n)}_{i,j,k} - 2\overset{**}{u}{}^{(n+1/2)}_{i,j,k}}{\Delta t/2} = \Delta_z^2 u^{(n+1)}_{i,j,k} - \Delta_z^2 u^{(n)}_{i,j,k} \qquad \text{(III-6)}$$

Equation (III-4), when applied to a row of grid points along x direction, yields a tridiagonal set of equations. These are solved by the Thomas method described in the previous chapter. The procedure is repeated to cover all the rows in the net work. Equations (III-5) and (III-6), in that order, are similarly applied along the rows in y and z directions respectively.

Example Problem

The three methods - ADEP, ADIP, and B-D-R method-

were tested on a simple parabolic problem namely the three-dimensional transient heat conduction [2]. The problem is defined by equation (III-1) and the initial and boundary conditions given below:

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \quad 0 < x,y,z < 1$$

I.C. $T(x,y,z,0) = 1.0$ for all x,y,z

B.C. $t > 0$: $T(1,y,z,t) = T(x,1,z,t) = T(x,y,1,t) = 0$

$$\frac{\partial T}{\partial x} = 0 \text{ at } x = 0$$

$$\frac{\partial T}{\partial y} = 0 \text{ at } y = 0 \qquad \text{(III-7)}$$

$$\frac{\partial T}{\partial z} = 0 \text{ at } z = 0$$

This is one of the three-dimensional problems for which an analytical solution is available. For certain types of initial and boundary conditions [7], which

include (III-7), analytical solution of the three dimensional problem can be expressed as the product of the solutions of the three one-dimensional problems [30]. The analytical solution of the problem (III-1) and (III-7) is given by:

$$T(x_1, x_2, x_3, t) = T_1(x_1, t) * T_2(x_2, t) * T_3(x_3, t)$$

where

$$T_\ell(x_\ell, t) = \frac{4}{\pi} \sum_{m=0}^{\infty} \frac{(-1)^m}{(2m+1)} \text{Exp}\left[-\pi^2 t \left(\frac{2m+1}{2}\right)^2\right]$$

$$\text{Cos}\left[\left(\frac{2m+1}{2}\right) \pi x_\ell\right]$$

$$\ell = 1,2,3. \qquad \text{(III-8)}$$

is the solution of the one-dimensional diffusion equation in the x_ℓ direction. Since the system is symmetric with respect to physical geometry and the boundary condition, the solution of the one-dimensional problem is identical in all the directions. In computing

$T_\ell(x_\ell,t)$, the series on the right hand side of (III-8) is terminated when the absolute value of the term is less than 10^{-10}. The length of the system in each direction was divided into 10.5 increments with a half increment at the insulated boundary, giving Δx $(=\Delta y=\Delta z)$ = 0.0952.

The ADEP, the ADIP, and the B-D-R method were programmed on IBM 7040 computer. The source listings of the programmes for these methods and for the analytical method are given in Appendix D.

The stability of ADEP, ADIP, and B-D-R scheme was tested using Δt up to 20 times Δt_{max} where Δt_{max} is the maximum permissible time step in the classical explicit scheme. ADIP quickly became unstable and no further consideration was given to this scheme.

The solutions of the problem (III-1) and (III-7) by the analytical method, the ADEP, and the B-D-R method are presented in Tables 1, 2, and 3 for

three selected points along the diagonal of the system. The results are obtained by using three time steps which are approximately 1.25, 2.5, and 5 times Δt_{max}. The corresponding error tabulations are given in Tables 4, 5, and 6. For $\Delta t = 1.25\ \Delta t_{max}$, the ADEP and the B-D-R solutions are of the same accuracy with perhaps a slight advantage to ADEP. At double this time step, the accuracies are comparable. However, at $\Delta t = 5\ \Delta t_{max}$, the B-D-R solution is on the whole somewhat better than the ADEP solution. Examining the truncation errors, τ_F and τ_R of ADEP, it is seen that the coefficients of the first and the fourth terms of τ_F are at the time level $n\Delta t$ whereas the corresponding terms in τ_R are at time level $(n+1/2)\Delta t$. When Δt is small, the magnitudes of these coefficients are close to each other and the first and the fourth terms of τ_F and the corresponding terms of τ_R tend to cancel each other more or less completely in each cycle. The accuracy of ADEP is the same as that of B-D-R scheme. When Δt is large, the magnitudes of the coefficients may differ sufficiently and the resulting cancellation may be incomplete and hence somewhat lesser accuracy of ADEP compared to that of B-D-R scheme.

TABLE 1

COMPARISON OF B-D-R METHOD AND ADEP: $\Delta t = 0.00198$

No. of Cycles n	Location*	Analytical	ADEP	B-D-R
		SOLUTION		
24	A	0.9832	0.9788	0.9791
	B	0.8207	0.8143	0.8140
	C	0.0995	0.1005	0.1001
48	A	0.8359	0.8305	0.8305
	B	0.5297	0.5287	0.5279
	C	0.0386	0.0388	0.0387
72	A	0.6346	0.6316	0.6312
	B	0.3551	0.3547	0.3542
	C	0.0215	0.0215	0.0216
96	A	0.4599	0.4586	0.4581
	B	0.2449	0.2446	0.2443
	C	0.0138	0.0138	0.0138
120	A	0.3273	0.3268	0.3264
	B	0.1709	0.1708	0.1706
	C	0.0093	0.0093	0.0093

* Location:	x = y = z
A	0.143
B	0.429
C	0.810

TABLE 2

COMPARISON OF B-D-R METHOD AND ADEP: $\Delta t = 0.00396$

No. of Cycles n	Location*	SOLUTION		
		Analytical	ADEP	B-D-R
12	A	0.9832	0.9778	0.9791
	B	0.8207	0.8155	0.8143
	C	0.0995	0.1015	0.0999
24	A	0.8359	0.8307	0.8306
	B	0.5297	0.5309	0.5279
	C	0.0386	0.0388	0.0387
36	A	0.6346	0.6330	0.6313
	B	0.3551	0.3562	0.3542
	C	0.0215	0.0215	0.0216
48	A	0.4599	0.4601	0.4581
	B	0.2449	0.2456	0.2443
	C	0.0138	0.0137	0.0138
60	A	0.3273	0.3281	0.3264
	B	0.1709	0.1714	0.1706
	C	0.0093	0.0093	0.0093

* (As in Table 1)

TABLE 3

COMPARISON OF B-D-R METHOD AND ADEP: $\Delta t = 0.00792$

No. of Cycles n	Location*	Analytical	ADEP	B-D-R
6	A	0.9832	0.9742	0.9791
	B	0.8207	0.8191	0.8154
	C	0.0995	0.1063	0.0994
12	A	0.8359	0.8313	0.8309
	B	0.5297	0.5396	0.5279
	C	0.0386	0.0390	0.0386
18	A	0.6346	0.6380	0.6314
	B	0.3551	0.3621	0.3541
	C	0.0215	0.0214	0.0215
24	A	0.4599	0.4661	0.4582
	B	0.2449	0.2493	0.2443
	C	0.0138	0.0136	0.0138
30	A	0.3273	0.3333	0.3264
	B	0.1709	0.1738	0.1706
	C	0.0093	0.0091	0.0093

(The heading "S O L U T I O N" spans the Analytical, ADEP and B-D-R columns.)

* Location (Same as in Table 1)

TABLE 4

ERROR TABULATION: $\Delta t = 0.00198$

No. of Cycles n	x=y=z=0.143		x=y=z=0.429		x=y=z=0.810	
	ADEP	B-D-R	ADEP	B-D-R	ADEP	B-D-R
24	-0.0044	-0.0041	-0.0063	-0.0067	0.0010	0.0006
48	-0.0054	-0.0055	-0.0010	-0.0018	0.0002	0.0001
72	-0.0030	-0.0034	-0.0004	-0.0009	0.0000	0.0001
96	-0.0003	-0.0018	-0.0003	-0.0006	0.0000	0.0000
120	-0.0005	-0.0009	-0.0001	-0.0003	0.0000	0.0000

TABLE 5

ERROR TABULATION: $\Delta t = 0.00396$

No. of Cycles n	x=y=z=0.143 ADEP	x=y=z=0.143 B-D-R	x=y=z=0.429 ADEP	x=y=z=0.429 B-D-R	x=y=z=0.810 ADEP	x=y=z=0.810 B-D-R
12	-0.0054	-0.0041	-0.0052	-0.0064	0.0020	0.0004
24	-0.0052	-0.0053	-0.0012	-0.0018	-0.0002	0.0001
36	-0.0016	-0.0033	-0.0011	-0.0009	0.0000	0.0001
48	0.0002	-0.0018	0.0007	-0.0006	-0.0001	0.0000
60	0.0008	-0.0009	0.0005	-0.0003	0.0000	0.0000

TABLE 6

ERROR TABULATION: $\Delta t = 0.00792$

No. of cycles	x=y=z=0.143		x=y=z=0.429		x=y=z=0.810	
n	ADEP	B-D-R	ADEP	B-D-R	ADEP	B-D-R
6	-0.0090	-0.0041	-0.0016	-0.0053	0.0068	-0.0001
12	-0.0046	-0.0050	-0.0001	-0.0018	-0.0004	0.0000
18	0.0036	-0.0032	0.0070	-0.0010	0.0001	0.0010
24	0.0062	-0.0017	0.0044	-0.0006	-0.0002	0.0000
30	0.0060	-0.0009	0.0028	-0.0003	0.0002	0.0000

An estimate of the computing times of the two methods may be obtained by examining the number of arithmatic operations performed by each method in advancing the solution by one time step. The average number of operations for a linear problem are shown in Table 7. The grid size is assumed to be identical in all three directions. Assuming that the performance of multiplication and division takes about the same computing time, it is seen that the B-D-R method takes nearly five times the computational work required by ADEP. The saving in computing time is therefore significant in a three-dimensional problem. The storage requirements of function values is only twice the number of function values in ADEP, compared to three times the number of function values in any ADIP.

It is of practical interest to know which of the two methods is more suitable and under what conditions. Since both ADEP and B-D-R methods are unconditionally stable, they have to be judged on the basis of accuracy and computational simplicity.

TABLE 7

ARITHMETIC OPERATIONS OF ADEP AND B-D-R METHOD:

LINEAR PROBLEM

	Average number of operations per grid point per cycle		
	Addition/Subtraction	Multiplication	Division
ADEP	12	2	2
B-D-R	19	13	6

Considering first the linear case of the three-dimensional problems, for small Δt, ADEP is recommended over the B-D-R method since their accuracies are equivalent and ADEP is faster. At larger time steps, each method has an advantage, namely, ADEP is faster while B-D-R method is somewhat more accurate. A reasonable basis for evaluating the two methods is to use a smaller time step in ADEP so that it has approximately the same computing time as the B-D-R method. The relative accuracies of the two methods should determine suitability of the one over the other.

Nonlinear Case

The case of nonlinear parabolic equations is of great interest in many practical problems. Nonlinearities in equation (I-1) may arise in g and K, in the source term S and in the boundary conditions. In any of these cases the matrix equation (II-28) is no longer explicit, since the matrices $\underline{D}_1'$, $\underline{G}$, $\underline{E}$, and $\underline{S}'$ are then functions of $\underline{u}$. The advantage of ADEP lies

in the fact that each of the equations in the set (II-28), though nonlinear, involves only one unknown. The solution of one nonlinear equation for one unknown can be obtained simply by a standard iterating technique such as Newton-Raphson. Point by point evaluation of all the grid points is the same as in the linear problem. On the other hand, any alternating direction implicit procedure leads to a set of m nonlinear coupled equations in m unknowns, where m is the number of grid points in a row or a column. In terms of computational task, this certainly compares with ADEP more unfavorably than in the linear case. To advance the solution of the whole net work by one time step, ADEP involves solving one nonlinear equation for one unknown m^3 times. Remembering that any ADIP is a three stage process, now m nonlinear equations are simultaneously solved for m unknowns $3m^2$ times. Further, when m is large, problems of convergence of iterative solution are more likely to arise.

In summary, the alternating direction explicit procedure combines the simplicity of the classical

explicit methods with the unconditional stability of the implicit procedures. In the case of linear problems, the use of ADEP is recommended at smaller time steps. At larger time steps, the fastness of ADEP has to be weighed against the somewhat better accuracy of the B-D-R method. In the case of nonlinear problems, the computational advantages of ADEP over the B-D-R method are even more marked. ADEP is more likely to prove advantageous to use.

IV. TWO-DIMENSIONAL MODEL OF AN OIL RESERVOIR

A petroleum reservoir is a highly complex physical system. Its complexity arises from a number of sources. The geometrical configuration of the reservoir is irregular and its magnitude huge. The structure of the rock which constitutes the reservoir may vary considerably. There are, in general, more than one fluid phase coexisting in the reservoir. The gas phase is highly compressible whereas the liquid phases - oil and water - are only slightly compressible. Added to these, the information available on a reservoir before its exploitation is usually inadequate.

A practical approach in solving reservoir problems is to develop a mathematical model of the reservoir by isolating the important factors from those which are less relevant to the problem. In the present study, three types of reservoir models are considered with varying degrees of complexity, namely, a liquid reservoir, a gas reservoir and a water-injected oil reservoir. The development of a mathematical model for the liquid reservoir is presented in this chapter.

The question of primary interest regarding a petroleum reservoir is to determine the optimum conditions for producing petroleum. Of basic importance in answering this question is an understanding of the reservoir behavior as oil is withdrawn from selected locations in the reservoir. Rigorous formulation of a mathematical model for the transient behavior of the reservoir has not been possible until recently because the mathematical equations arising from realistic formulation are too complex to permit solution by classical methods. The early approach to this problem had been to hypothesize a reservoir model simple enough for analytical solution [28, 9]. Many practical problems, however, require more sophisticated treatment. Recently attempts have also been made to understand the reservoir behavior by physically simulating the reservoir such as by potentiometric and electrolytic models [26, 29]. These methods, besides being extremely costly, are reportedly handicapped by instrumentation problems.

Recent advances in digital computers have greatly enhanced the hope of simulating petroleum reservoirs more precisely. Current literature shows that digital simulation of petroleum reservoirs is being increasingly used in gaining greater engineering control of reservoir operations. Being a system of often complex geometry, the petroleum reservoir requires a large number of grid points for its simulation. Hence the cost of digital simulation becomes an important factor. The need for fast and efficient numerical techniques of solving mathematical models is therefore obvious.

It was shown in Chapter II that in the solution of parabolic partial differential equations, the alternating direction implicit procedure (ADIP) of Peaceman and Rachford, considerably reduced the computational time compared to the backward difference implicit method. This procedure has proved to be the most successful numerical method employed in solving two-dimensional reservoir models. This method however, cannot be applied to three-dimensional models. It has

been shown in the previous chapter that the alternating direction explicit procedure is an efficient technique in solving multi-dimensional parabolic problems. It has also an advantage over ADIP in respect of computational speed. An aim of this study was to test this new method on a hypothetical oil field and to compare the results with those of ADIP. The development of a mathematical model for an oil reservoir and the underlying assumptions are described first.

Formulation of Mathematical Model

In general, a petroleum reservoir is three-dimensional and occurs in wide variety of geometry. The magnitude of the system as well as its properties vary in all three directions. Therefore, rigorously, a reservoir problem should be treated as three-dimensional. As a simplification, however, the dimension in one direction is usually treated as a parameter rather than as an independent variable. This approximation is reasonable where the magnitude of the system as well as the variation of its properties in this

direction are small compared to those in the other two directions. Assuming these conditions are satisfied, the present study is confined to a two-dimensional model, although this is not a limitation on the applicability of ADEP. The reservoir of interest is a two-dimensional bounded region exemplified by that shown in Figure 1. Given the properties of formation and oil, it is required, to determine the pressure distribution in the reservoir as a function of time, as oil is withdrawn at a known rate from a number of arbitrarily located wells. The fundamental formulation of flow behavior in a reservoir is based on combining the principle of mass conservation, an appropriate flow rate relation, and an equation of state for the fluid. Assuming single phase, namely oil, and that the saturation of oil remains constant throughout the period of interest, the principle of mass conservation applied to a differential volume element yields,

$$\phi(x,y)h(x,y)S \frac{\partial \rho}{\partial t} = - \frac{\partial}{\partial x} [h(x,y)m_x] - \frac{\partial}{\partial y} [h(x,y)m_y] - w(x,y,t) \qquad \text{(IV-1)}$$

where

m_x, m_y = mass fluxes in x and y directions respectively

$h(x,y)$ = formation height

$\phi(x,y)$ = porosity of formation

s = saturation of oil

$w(x,y,t)$ = mass rate of withdrawal of oil

ρ = density of oil

It is assumed here that the gravitational effect is negligible. Under viscous flow conditions, which normally prevail in an oil reservoir, the flow behavior is adequately described by Darcy's Law, the differential form of which, expressed in mass units, is given by:

$$m_x = - \frac{k(x,y)}{\mu} \rho \frac{\partial P}{\partial x} \qquad \text{(IV-2)}$$

where

P = pressure

$k(x,y)$ = permeability at (x,y)

μ = viscosity.

Assuming the reservoir to be isotropic in nature, and substituting for mass fluxes according to (IV-2), equation (IV-1) becomes:

$$\phi h\, s\rho c \frac{\partial P}{\partial t} = \frac{\partial}{\partial x}\left(\frac{kh}{\mu}\,\rho\,\frac{\partial P}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{kh}{\mu}\,\rho\,\frac{\partial P}{\partial Y}\right) - w(x,y,t) \qquad \text{(IV-3)}$$

where $c = \frac{1}{\rho}\frac{\partial \rho}{\partial P}$ = compressibility of oil, which also defines the equation of state for isothermal conditions in differential form

This equation is nonlinear since ρ depends, though mildly, on P. For a slightly compressible liquid, c is small and

$$\frac{\partial}{\partial x}\left(\frac{kh}{\mu}\,\frac{\partial P}{\partial x}\right) >> c\,\frac{kh}{\mu}\left(\frac{\partial P}{\partial x}\right)^2$$

and

$$\frac{\partial}{\partial y}\left(\frac{kh}{\mu}\,\frac{\partial P}{\partial y}\right) >> c\,\frac{kh}{\mu}\left(\frac{\partial P}{\partial y}\right)^2 \qquad \text{(IV-4)}$$

The conditions under which this assumption is appropriate are quantitatively defined by Dranchuk and Quon [19].

Then, equation (IV-3) simplifies to:

$$cs\,\phi h \frac{\partial P}{\partial t} = \frac{\partial}{\partial x}\left(\frac{kh}{\mu}\frac{\partial P}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{kh}{\mu}\frac{\partial P}{\partial y}\right) - q(x,y,t) \qquad \text{(IV-5)}$$

where $q(x,y,t) = w(x,y,t)/\rho =$ volume rate of production per unit area.

If the viscosity can be assumed independent of pressure, equation (IV-5) is a linear second order parabolic partial differential equation with variable coefficients. Typical initial and boundary conditions are:

I.C. $P(x,y,o) = F(x,y)$ (IV-6)

B.C. $\frac{\partial P}{\partial \gamma} = 0$ where γ is the normal to the boundary.

Equation (IV-5) was integrated numerically by both ADEP and ADIP. Preparatory to finite difference approximation, let $x_i = i\Delta x$, $y_j = j\Delta y$, and $t_n = n\Delta t$. For a typical point (i,j), the finite difference analogues of equation (IV-5) according to ADEP, are given below.

Forward Sweep:

$$cs\ (\phi h)_{i,j}\left[\frac{P_{i,j}^{(n+1/2)} - P_{i,j}^{(n)}}{\Delta t/2}\right]$$

$$= \frac{\left(\frac{kh}{\mu}\right)_{i-1/2,j}\left[P_{i-1,j}^{(n+1/2)} - P_{i,j}^{(n+1/2)}\right] + \left(\frac{kh}{\mu}\right)_{i+1/2,j}\left[P_{i+1,j}^{(n)} - P_{i,j}^{(n)}\right]}{\bar{\Delta}x^2}$$

$$+ \frac{\left(\frac{kh}{\mu}\right)_{i,j-1/2}\left[P_{i,j-1}^{(n+1/2)} - P_{i,j}^{(n+1/2)}\right] + \left(\frac{kh}{\mu}\right)_{i,j+1/2}\left[P_{i,j+1}^{(n)} - P_{i,j}^{(n)}\right]}{\bar{\Delta}y^2}$$

$$- q_{i,j}^{(n+1/4)} \qquad \text{(IV-7)}$$

Reverse Sweep:

$$cs(\phi h)_{i,j}\left[\frac{P_{i,j}^{(n+1)} - P_{i,j}^{(n+1/2)}}{\Delta t/2}\right]$$

$$= \frac{\left(\frac{kh}{\mu}\right)_{i-1/2,j}\left[P_{i-1,j}^{(n+1/2)} - P_{i,j}^{(n+1/2)}\right] + \left(\frac{kh}{\mu}\right)_{i+1/2,j}\left[P_{i+1,j}^{(n+1)} - P_{i,j}^{(n+1)}\right]}{\bar{\Delta}x^2}$$

$$+ \frac{(\frac{kh}{\mu})_{i,j-1/2}[P_{i,j-1}^{(n+1/2)} - P_{i,j}^{(n+1/2)}] + (\frac{kh}{\mu})_{i,j+1/2}[P_{i,j+1}^{(n+1)} - P_{i,j}^{(n+1)}]}{\overline{\Delta y}^2}$$

$$- q_{i,j}^{(n+3/4)} \qquad \text{(IV-8)}$$

On letting $\Delta x = \Delta y$, and rearranging, equations (IV-7) and (IV-8) may be rewritten respectively as:

$$- (\frac{kh}{\mu})_{i-1/2,j} P_{i-1,j}^{(n+1/2)} - (\frac{kh}{\mu})_{i,j-1/2} P_{i,j-1} + [cs(\phi h)_{i,j} 2\frac{\overline{\Delta x}^2}{\Delta t} +$$

$$+ (\frac{kh}{\mu})_{i-1/2,j} + (\frac{kh}{\mu})_{i,j-1/2}] \; P_{i,j}^{(n+1/2)}$$

$$= (\frac{kh}{\mu})_{i+1/2,j}[P_{i+1,j}^{(n)} - P_{i,j}^{(n)}] + (\frac{kh}{\mu})_{i,j+1/2}[P_{i,j+1}^{(n)} - P_{i,j}^{(n)}]$$

$$+ \; cs(\phi h)_{i,j} \; 2 \; \frac{\overline{\Delta x}^2}{\Delta t} \; P_{i,j}^{(n)} - Q_{i,j}^{(n+1/4)} \qquad \text{(IV-9)}$$

and

$$-(\frac{kh}{\mu})_{i+1/2,j} P_{i+1,j}^{(n+1)} - (\frac{kh}{\mu})_{i,j+1/2} P_{i,j+1}^{(n+1)} + [cs(\phi h)_{i,j} \; 2 \; \frac{\overline{\Delta x}^2}{\Delta t}$$

$$+ \left(\frac{kh}{\mu}\right)_{i+1/2,j} + \left(\frac{kh}{\mu}\right)_{i,j+1/2}\Big] \; P_{i,j}^{(n+1)}$$

$$= \left(\frac{kh}{\mu}\right)_{i-1/2,j}\left[P_{i-1,j}^{(n+1/2)} - P_{i,j}^{(n+1/2)}\right] + \left(\frac{kh}{\mu}\right)_{i,j-1/2}\left[P_{i,j-1}^{(n+1/2)} - P_{i,j}^{(n+1/2)}\right]$$

$$+ cs(\phi h)_{i,j} \; 2 \; \frac{\overline{\Delta x}^2}{\Delta t} \; P_{i,j}^{(n+1/2)} - Q_{i,j}^{(n+3/4)} \qquad \text{(IV-10)}$$

where $$Q_{i,j}^{(n+\delta)} = \int_{x_i-\frac{\Delta x}{2}}^{x_i+\frac{\Delta x}{2}} \int_{y_i-\frac{\Delta y}{2}}^{y_i+\frac{\Delta y}{2}} q_{(\eta,\beta)}^{(n+\delta)} \, d\eta \, d\beta \qquad \text{(IV-11)}$$

Integration of equation (IV-5) by ADIP is similarly carried out in two stages for each time increment.

<u>x-directional Sweep:</u>

$$cs(\phi h)_{i,j} \; \frac{P_{i,j}^{(n+1/2)} - P_{i,j}^{(n)}}{\Delta t/2}$$

$$= \frac{\left(\frac{kh}{\mu}\right)_{i-1/2,j}\left[P_{i-1,j}^{(n+1/2)} - P_{i,j}^{(n+1/2)}\right] + \left(\frac{kh}{\mu}\right)_{i+1/2,j}\left[P_{i+1,j}^{(n+1/2)} - P_{i,j}^{(n+1/2)}\right]}{\overline{\Delta x}^2}$$

$$+ \frac{(\frac{kh}{\mu})_{i,j-1/2}[P^{(n)}_{i,j-1}-P^{(n)}_{i,j}]+(\frac{kh}{\mu})_{i,j+1/2}[P^{(n)}_{i,j+1}-P^{(n)}_{i,j}]}{\overline{\Delta y}^2}$$

$$- q^{(n+1/4)}_{i,j} \qquad \text{(IV-12)}$$

<u>y-directional Sweep:</u>

$$cs(\phi h)_{i,j} \frac{P^{(n+1)}_{i,j} - P^{(n+1/2)}_{i,j}}{\Delta t/2}$$

$$= \frac{(\frac{kh}{\mu})_{i-1/2,j}[P^{(n+1/2)}_{i-1,j} - P^{(n+1/2)}_{i,j}]+(\frac{kh}{\mu})_{i,j+1/2}[P^{(n+1/2)}_{i,j+1} - P^{(n+1/2)}_{i,j}]}{\overline{\Delta x}^2}$$

$$+ \frac{(\frac{kh}{\mu})_{i,j-1/2}[P^{(n+1)}_{i,j-1}-P^{(n+1)}_{i,j}]+(\frac{kh}{\mu})_{i,j+1/2}[P^{(n+1)}_{i,j+1}- P^{(n+1)}_{i,j}]}{\overline{\Delta y}^2}$$

$$- q^{(n+3/4)}_{i,j} \qquad \text{(IV-13)}$$

Bringing all the unknowns to the left, equations (IV-12) and (IV-13) are rewritten as:

$$- (\frac{kh}{\mu})_{i-1/2,j} P_{i-1,j}^{(n+1/2)} + [cs(\phi h)_{i,j}\ 2\ \frac{\overline{\Delta x}^2}{\Delta t} + (\frac{kh}{\mu})_{i-1/2,j}$$

$$+ (\frac{kh}{\mu})_{i+1/2,j}] P_{i,j}^{(n+1/2)} - (\frac{kh}{\mu})_{i+1/2,j} P_{i+1,j}^{(n+1/2)}$$

$$= (\frac{kh}{\mu})_{i,j-1/2}[P_{i,j-1}^{(n)} - P_{i,j}^{(n)}] + (\frac{kh}{\mu})_{i,j+1/2} [P_{i,j+1}^{(n)} - P_{i,j}^{(n)}]$$

$$+ cs\ (\phi h)_{i,j}\ 2\ \frac{\overline{\Delta x}^2}{\Delta t} P_{i,j}^{(n)} - Q_{i,j}^{(n+1/4)} \qquad \text{(IV-14)}$$

and

$$- (\frac{kh}{\mu})_{i,j-1/2} P_{i,j-1}^{(n+1)} + [cs(\phi h)_{i,j}\ 2\ \frac{\overline{\Delta x}^2}{\Delta t} + (\frac{kh}{\mu})_{i,j-1/2}$$

$$+ (\frac{kh}{\mu})_{i,j+1/2}] P_{i,j}^{(n+1)} - (\frac{kh}{\mu})_{i,j+1/2} P_{i,j+1}^{(n+1)}$$

$$= (\frac{kh}{\mu})_{i-1/2,j}[P_{i-1,j}^{(n+1/2)} - P_{i,j}^{(n+1/2)}] + (\frac{kh}{\mu})_{i+1/2,j}[P_{i+1,j}^{(n+1/2)} - P_{i,j}^{(n+1/2)}]$$

$$+ cs(\phi h)_{i,j}\ 2\ \frac{\overline{\Delta x}^2}{\Delta t} P_{i,j}^{(n+1/2)} - Q_{i,j}^{(n+3/4)} \qquad \text{(IV-15)}$$

These two methods were tested [34] on a hypothetical oil field of geometrical configuration shown in Figure 6.

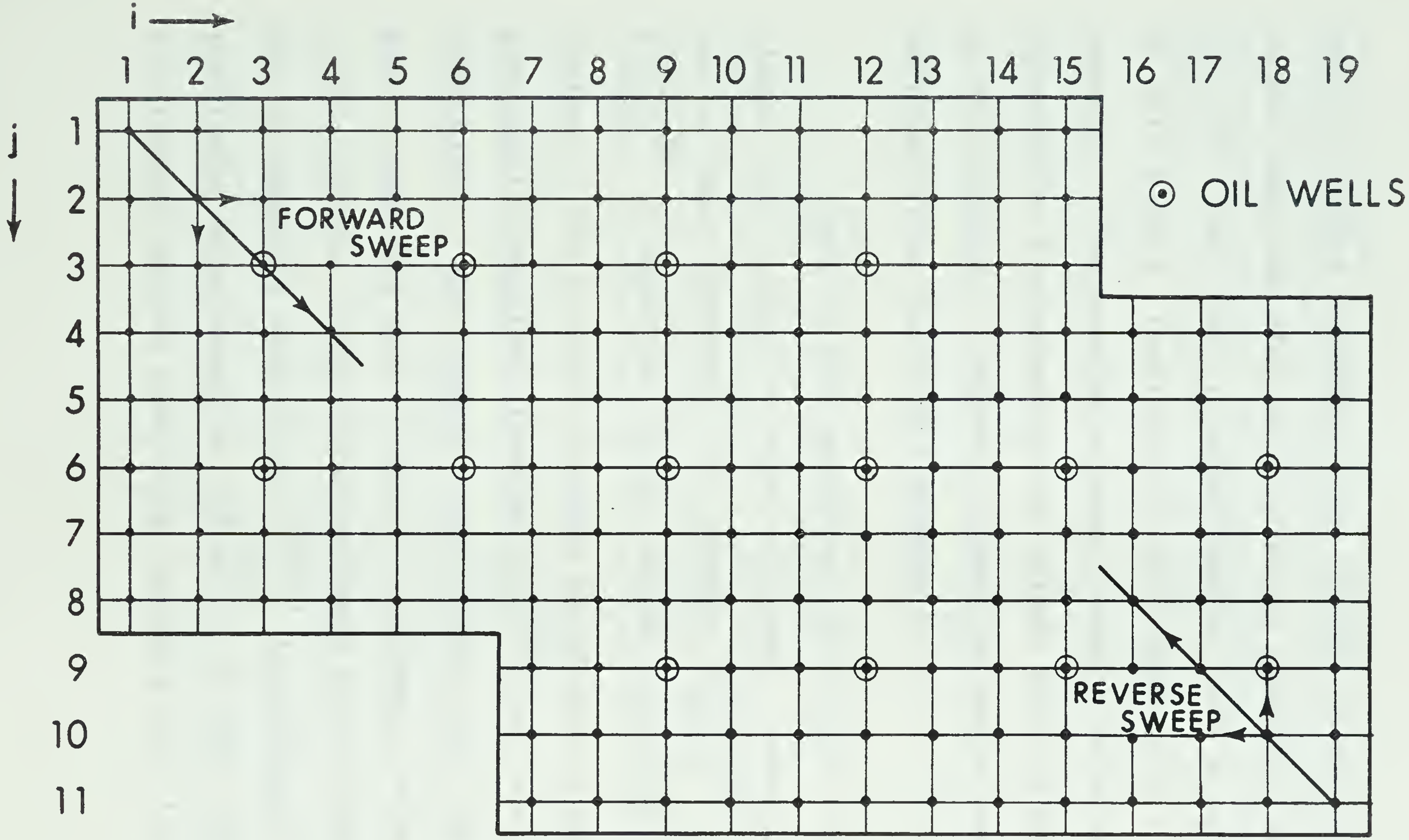

FIG. 6 TWO-DIMENSIONAL MODEL OF OIL RESERVOIR

While straight boundaries are chosen in this example, curved ones can also be handled by standard techniques [23,40]. The field is superimposed with a square network of grid points, leaving half-grid spacing all along the boundary. The no-flux boundary condition is approximated by equating the pressures on either side of and adjacent to the boundary. Fourteen producing wells are located arbitrarily in the field (Figure 6).

Each of the equations (IV-9) and (IV-10) is linear and coupled in three unknowns. Nevertheless, two of these unknowns, for example, $P_{i-1,j}^{(n+1/2)}$ and $P_{i,j-1}^{(n+1/2)}$ in equation (IV-9), either disappear by virtue of the no-flux boundary condition or have been previously computed, leaving $P_{i,j}^{(n+1/2)}$ to be computed explicitly. The forward sweep starts with the north-west corner and ends with the south-east corner. In the reverse sweep which proceeds in the same path but in opposite direction, $P_{i,j}^{(n+1)}$ is evaluated similarly using equation (IV-10). In ADIP, the tridiagonal matrix equations, arising from (IV-14)

as well as from (IV-15), are solved by the algorithm of Thomas described in Chapter II. In both methods, the viscosity at the last known pressure was used in computing the pressure at the next time level.

The two methods were programmed on an IBM 7040 computer. The source listings of the programmes are included in Appendix D. The data on the reservoir and the oil are given in Appendix A. The properties of the formation (ϕh, kh) are described in the form of two matrices (Table A-1, and Table A-2). Computation was performed by both methods using a wide range of time steps. ADEP gave stable solutions for time steps as high as 200 days, which is over 600 times Δt_{max} (Appendix A), where Δt_{max} is the maximum time step consistent with stability for the classical explicit method. Results for two time steps, $\Delta t = 10$ days and $\Delta t = 20$ days, are presented in Tables A-3 through A-7. These include the production history, and the pressure distribution in the reservoir. To enable quick comparison of the pressures predicted by

the two methods, values taken from these tables are presented in Table 8 for a number of points approximately along a diagonal of the reservoir.

For a total decline of approximately 430 psia, the average difference between the pressures predicted by ADEP and ADIP is about 14 psia. The check is considered good. No significance should be placed on the fact that the pressures predicted by ADEP are lower than those predicted by ADIP. For a different time increment, they turned out to be higher.

Table 9 shows the effect of time step size on the ADEP solution after 1000 days. From the close agreement between the three sets of values, it is reasonable to assume convergence of the solution.

Coats and Terhune [8] reported that in a reservoir problem with a producing well located at a corner, the solution by ADEP gave material balance deviations of up to 35%. To examine this, computation of the pressure distribution was repeated with eight additional wells located at the eight corners of the

TABLE 8

PRESSURE DISTRIBUTION PREDICTED BY ADIP AND ADEP

Δt = 20 days

Grid Point Location (i,j)	Pressure at the end of 1000 days, psia ADIP	ADEP
1,1	636	623
2,2	634	621
3,3	628	615
4,4	628	615
6,5	624	611
8,6	622	609
10,7	622	608
12,8	621	606
14,9	622	607
16,10	625	611
18,11	627	613

TABLE 9

EFFECT OF TIME STEP SIZE ON ADEP SOLUTION

Grid Point Location (i,j)	Pressure at the end of 1000 days, psia $\Delta t = 5$	$\Delta t = 10$	$\Delta t = 20$
1,1	622	622	623
2,2	619	619	621
3,3	613	613	615
4,4	613	614	615
6,5	609	609	611
8,6	607	607	609
10,7	606	606	608
12,8	604	604	606
14,9	605	605	607
16,10	608	609	611
18,11	611	611	613

the reservoir. The overall material balance of the results was computed at the end of every cycle. Estimated and actual volumes of oil depleted from the reservoir were calculated as follows:

Estimated volume of oil depleted from reservoir (STB)

$$= \Delta x \ \Delta y \ \sum_{ij}\sum (\phi h)_{i,j} \left[\frac{1}{B(P_{i,j})_{\text{initial}}} - \frac{1}{B(P_{i,j})_{\text{final}}} \right]$$

Actual volume of oil depleted from reservoir (STB)

$$= \text{Time} * \sum_i \sum_j Q'_{i,j}$$

B(P) is the formation volume factor (Appendix A). Table A-10 shows some representative values of the ratio of the actual and the calculated depletion of oil from the reservoir. The production history and the predicted pressure distribution are given in Tables A-8 and A-9, respectively.

The value of the ratio differed from the correct value of unity by a maximum of 1.8 percent

after the third cycle. The value subsequently attains a steady value very close to unity. The time step used was 10 days per cycle which is about 33 times Δt_{max}. When the time step was doubled, the maximum material balance deviation was less than 2.4 percent. In either case, the deviations are far less than those reported by Coats and Terhune, at least in this reservoir problem. Further, these deviations will be even less for more reasonable time steps of 10 to 15 times Δt_{max}.

In summary, the alternating direction explicit procedure was successfully applied to an oil reservoir problem. Reservoir behavior predicted by this method was consistent with known normal behavior. Although a two-dimensional model was considered here, the method can be applied to a three-dimensional case.

V. TWO-DIMENSIONAL NONLINEAR MODEL OF A GAS RESERVOIR

In the previous chapter, the application of the alternating direction explicit procedure (ADEP) was successfully demonstrated for a linear two-dimensional model of an oil reservoir. However, as stated earlier, the technique of ADEP is especially suitable for the solution of nonlinear parabolic equations. Accordingly, this procedure was applied to the analysis of a gas reservoir whose behavior is inherently nonlinear.

In the following development, the reservoir considered is similar to the oil reservoir, but the fluid phase consists entirely of gas. The geometry of the reservoir is such that a two-dimensional model is adequate.

The basic equation of continuity given by equation (IV-1) is valid for any fluid phase in the reservoir. On incorporating Darcy's law, this equation becomes:

$$\phi h\, S_g \frac{\partial \rho}{\partial t} = \frac{\partial}{\partial x}\left[\frac{k_g h \rho}{\mu} \frac{\partial P}{\partial x}\right] + \frac{\partial}{\partial y}\left[\frac{k_g h \rho}{\mu} \frac{\partial P}{\partial y}\right] - w(x,y,t) \quad \text{(V-1)}$$

where k_g is the effective permeability. The equation of state for a real gas is given by:

$$\rho = \frac{PM}{ZRT} \tag{V-2}$$

Substituting this in (V-1),

$$\phi h\, S_g \frac{\partial}{\partial t}\left[\frac{PM}{ZRT}\right] = \frac{\partial}{\partial x}\left[\frac{k_g hPM}{\mu ZRT}\,\frac{\partial P}{\partial x}\right] + \frac{\partial}{\partial y}\left[\frac{k_g hPM}{\mu ZRT}\,\frac{\partial P}{\partial y}\right] - w(x,y,t) \tag{V-3}$$

For an isothermal reservoir, this may be rearranged to yield,

$$\phi h\, S_g \frac{\partial}{\partial t}\left[\frac{P}{Z}\right] = \frac{\partial}{\partial x}\left[k_g h\left(\frac{P}{\mu Z}\right)\frac{\partial P}{\partial x}\right] + \frac{\partial}{\partial y}\left[k_g h\left(\frac{P}{\mu Z}\right)\frac{\partial P}{\partial y}\right] - \left(\frac{RT}{M}\right) w(x,y,t) \tag{V-4}$$

where μ and Z are function of P. Equation (V-4) is a nonlinear, second order parabolic partial differential equation. It is subject to the following set of initial and boundary conditions:

$$\text{I.C.} \quad P(x,y,t) = f(x,y) \text{ at some time } t{=}to$$

$$\text{B.C.} \quad \frac{\partial P}{\partial \gamma} = 0 \qquad \text{(V-5)}$$

where γ is the normal to the boundary

Al-Hussainy, Ramey, and Crawford [1] reviewed the various analytical and numerical solutions of this transient reservoir problem. Among the recent approaches, two methods appear to be significant: the first method is based on the concept of 'real gas pseudo-pressure' and the second uses an implicit numerical procedure based on either linearization or iteration.

Al-Hussainy et al [1] transformed the nonlinear equation (V-4) into a quasilinear form by a change of variable. They defined the 'real gas pseudo-pressure' $\alpha(P)$ as:

$$\alpha(P) = 2 \int_{P_\alpha}^{P} \frac{\beta}{\mu(\beta)\ z(\beta)}\ d\beta \qquad \text{(V-6)}$$

Substitution of the derivatives of $\alpha(P)$ into (V-4) leads to:

$$\phi h\, S_g\, C_g(p)\, \mu(P) \frac{\partial \alpha}{\partial t} = \frac{\partial}{\partial x}\left[k_g h \frac{\partial \alpha}{\partial x}\right] + \frac{\partial}{\partial y}\left[k_g h \frac{\partial \alpha}{\partial y}\right]$$

$$- \frac{RT}{M} w(x,y,t) \qquad \text{(V-7)}$$

The nonlinearity is now confined to the capacity term. Computationally, the integration of (V-7) requires, presumably, less time than the original equation. However, an additional interpolation subroutine is required to transform $\alpha(P)$ to P after every time step.

The second approach is illustrated by the work of Henderson, Dempsey, and Nelson [22]. Their model is identical to that given by (V-4). It was approximated by the ADIP of Peaceman and Rachford. The two finite difference equations for the two stages are similar to (II-21) and (II-22) and may be written as:

$$[\underline{I} - \frac{\Delta t}{2} \underline{G}_1^{-1} \underline{H}]\underline{p}^{(n+1/2)} = [\underline{I} + \frac{\Delta t}{2} \underline{G}_1^{-1} \underline{V}]\ \underline{p}^{(n)} - \frac{\Delta t}{2} \underline{G}_1^{-1} \underline{q} \qquad \text{(V-8)}$$

and

$$(\underline{I} - \frac{\Delta t}{2}\underline{G}_1^{-1}\underline{V})\underline{p}^{(n+1)} = [\underline{I} + \frac{\Delta t}{2}\underline{G}_1^{-1}\underline{H}]\underline{p}^{(n+1/2)} - \frac{\Delta t}{2}\underline{G}_1^{-1}\underline{q} \quad \text{(V-9)}$$

where $\underline{q}$ is the vector of production rates; $\underline{G}_1$ is a function of Z, while $\underline{H}$ and $\underline{V}$ are functions of P, μ , and Z. Rigorously, each matrix equation in this form is a set m nonlinear algebraic equations in m unknowns and may be solved by an iterative procedure. Henderson et al, however, suggest that matrices $\underline{G}_1$, $\underline{H}$, and $\underline{V}$ be treated as functions of P, μ, and Z, but at one time step behind. Hence, equation (V-8) and (V-9) can be solved as in the linear case.

The advantage of ADEP is that no further approximation need be made in solving equation (V-4). The two-stage approximation of ADEP leads to two nonlinear algebraic equations apparently implicit as in ADIP. For an interior point (i,j) of a rectangular network, these are as follows:

Forward Sweep:

$$(\phi h)_{i,j} \quad S_g \left[\frac{\left(\frac{P}{Z}\right)_{i,j}^{(n+1/2)} - \left(\frac{P}{Z}\right)_{i,j}^{(n)}}{\Delta t/2} \right] =$$

$$\frac{(k_g h)_{i-1/2,j} \left(\frac{P}{\mu Z}\right)_{i-1/2,j}^{(n+1/2)} \left[P_{i-1,j}^{(n+1/2)} - P_{i,j}^{(n+1/2)}\right]}{\overline{\Delta x}^2}$$

$$+ \frac{(k_g h)_{i+1/2,j} \left(\frac{P}{\mu Z}\right)_{i+1/2,j}^{(n)} \left[P_{i,j+1}^{(n)} - P_{i,j}^{(n)}\right]}{\overline{\Delta x}^2}$$

$$+ \frac{(k_g h)_{i,j-1/2} \left(\frac{P}{\mu Z}\right)_{i,j-1/2}^{(n+1/2)} \left[P_{i,j-1}^{(n+1/2)} - P_{i,j}^{(n+1/2)}\right]}{\overline{\Delta y}^2}$$

$$+ \frac{(k_g h)_{i,j+1/2} \left(\frac{P}{\mu Z}\right)_{i,j+1/2}^{(n)} \left[P_{i,j+1}^{(n)} - P_{i,j}^{(n)}\right]}{\overline{\Delta y}^2} - \left(\frac{RT}{M}\right) w_{i,j}^{(n+1/4)} \qquad \text{(V-10)}$$

Reverse Sweep:

$$(\phi h)_{i,j} \quad S_g \left[\frac{\left(\frac{P}{Z}\right)_{i,j}^{(n+1)} - \left(\frac{P}{Z}\right)_{i,j}^{(n+1/2)}}{\Delta t/2} \right] =$$

$$\frac{(k_g h)_{i-1/2,j} \left(\frac{P}{\mu Z}\right)^{(n+1/2)}_{i-1/2,j} [P^{(n+1/2)}_{i-1,j} - P^{(n+1/2)}_{i,j}]}{\overline{\Delta x}^2}$$

$$+ \frac{(k_g h)_{i+1/2,j} \left(\frac{P}{\mu Z}\right)^{(n+1)}_{i+1/2,j} [P^{(n+1)}_{i+1,j} - P^{(n+1)}_{i,j}]}{\overline{\Delta x}^2}$$

$$+ \frac{(k_g h)_{i,j-1/2} \left(\frac{P}{\mu Z}\right)^{(n+1/2)}_{i,j-1/2} [P^{(n+1/2)}_{i,j-1} - P^{(n+1/2)}_{i,j}]}{\overline{\Delta y}^2}$$

$$+ \frac{(k_g h)_{i,j+1/2} \left(\frac{P}{\mu Z}\right)^{(n+1)}_{i,j+1/2} [P^{(n+1)}_{i,j+1} - P^{(n+1)}_{i,j}]}{\overline{\Delta y}^2} - \left(\frac{RT}{M}\right) w^{(n+3/4)}_{i,j} \qquad \text{(V-11)}$$

where $\left(\frac{P}{\mu Z}\right)^{(n)}_{i-1/2,j} = \dfrac{\left(\frac{P}{\mu Z}\right)^{(n)}_{i-1,j} + \left(\frac{P}{\mu Z}\right)^{(n)}_{i,j}}{2}$

From the initial condition, all the quantities in (V-10) are known at some time $t = n\Delta t$. Then the quantities to be evaluated at $(n+1/2)\Delta t$ are: $P^{(n+1/2)}_{i-1,j}$, $P^{(n+1/2)}_{i,j-1}$

$P_{i,j}^{(n+1/2)}$, $(\frac{P}{Z})_{i,j}^{(n+1/2)}$, $(\frac{P}{\mu Z})_{i-1,j}^{(n+1/2)}$, $(\frac{P}{\mu Z})_{i,j}^{(n+1/2)}$, and $(\frac{P}{\mu Z})_{i,j-1}^{(n+1/2)}$

The last three quantities arise from evaluating $(\frac{P}{\mu Z})_{i-1/2,j}^{(n+1/2)}$ and $(\frac{P}{\mu Z})_{i,j-1/2}^{(n+1/2)}$. As usual, advantage is taken of the boundary conditions by choosing the starting point at a corner of the system network. For points adjacent to the boundary, $P_{i-1,j}^{(n+1/2)}$, $P_{i,j-1}^{(n+1/2)}$, $(\frac{P}{\mu Z})_{i-1,j}^{(n+1/2)}$, $(\frac{P}{\mu Z})_{i,j-1}^{(n+1/2)}$ either disappear or are known by virtue of the boundary conditions. For an interior point, these are known from previous computation. Therefore, in general, $P_{i,j}^{(n+1/2)}$, $(\frac{P}{Z})_{i,j}^{(n+1/2)}$, and $(\frac{P}{\mu Z})_{i,j}^{(n+1/2)}$ are the only unknowns to be determined. For an isothermal reservoir, μ, and Z depend only on pressure. Therefore, the evaluation of these quantities involves solving, normally, a relatively low degree polynomial in one variable in each equation. The basis of calculational procedure in the reverse sweep, using (V-11), is similar except that the starting point now is the corner point where the

forward sweep ended.

Numerical Example:

A hypothetical gas field containing 37 arbitrarily located gas wells was analyzed [35]. The geometrical configuration of the field is shown in Figure 7. The grid spacing was 0.5 mile giving a total of 179 gridpoints. The initial pressure distribution in the reservoir was assumed to be uniform at 1000 psia. The spatial variation of the reservoir properties is shown in the form of two matrices in Appendix B (Tables (B-1) and B-2)).

In the example chosen, both P and $(P/\mu Z)$ could be expressed as linear function of (P/Z) (Appendix B). These reduce each of equations (V-10) and (V-11) to a quadratic, which can be solved explicitly for the unknown $(P/Z)_{i,j}$. If higher degree polynomials are needed for these correlations, then presumably some

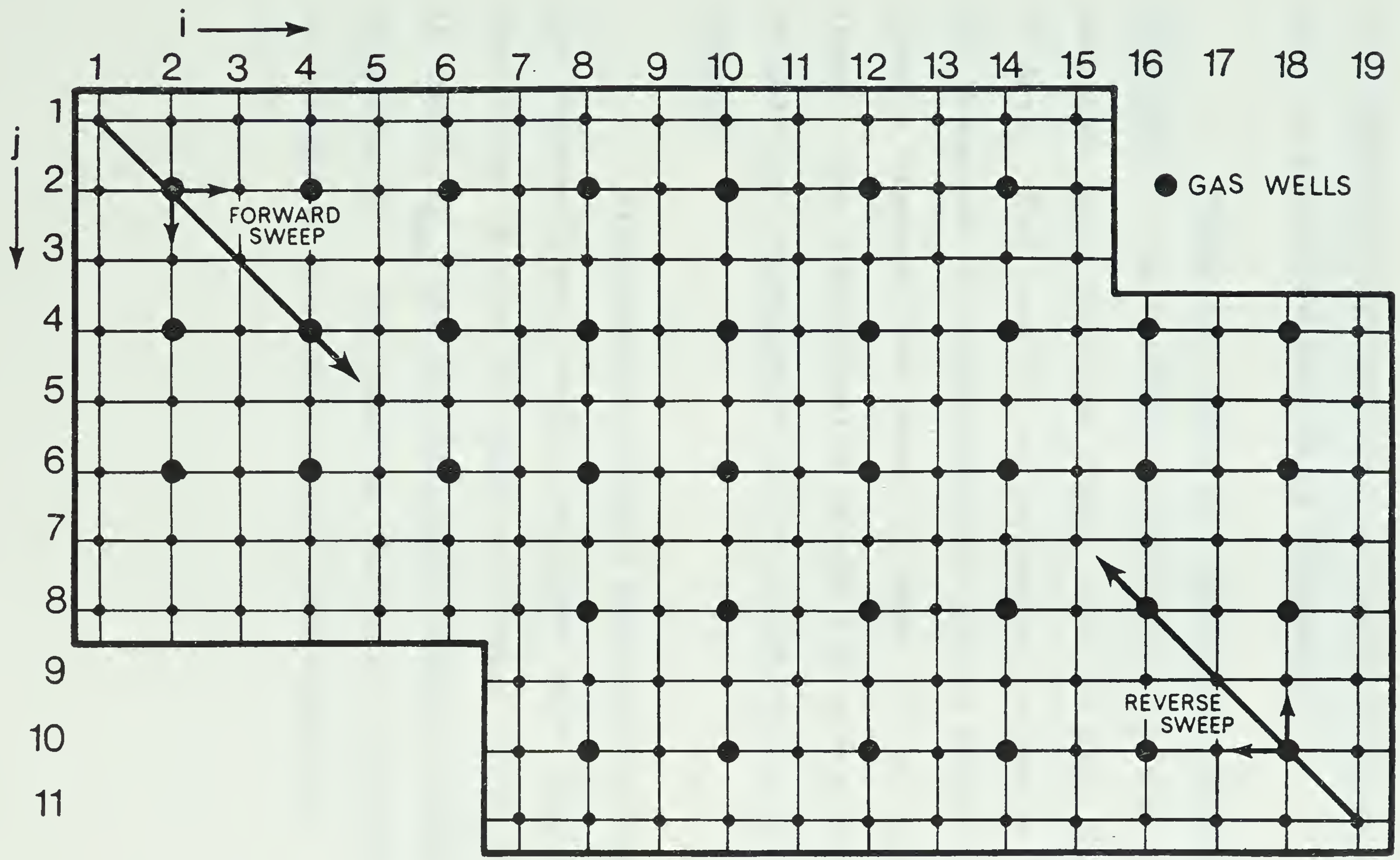

FIGURE 7: MATHEMATICAL MODEL OF GAS RESERVOIR

root-seeking procedure such as Newton-Raphson could be used to solve the resulting polynomial.

Table B-3 shows the production pattern in the reservoir. The pressure distribution in the field was computed using two time steps, $\Delta t = 30$ days and $\Delta t =$ 60 days. The results are presented in Tables B-4 and B-5. The pressures at representative points approximately along a diagonal are shown in Table 10 for three time steps. The corresponding values are sufficiently close so that the convergence of the solution may be reasonably assumed.

Although the results for only two time steps are reported here, stable solutions were obtained for time steps of over 240 days per cycle. This is about 35 times Δt_{max} for the classical explicit method (Appendix B). The limiting factor in the choice of Δt is, therefore, accuracy rather than stability.

TABLE 10

EFFECT OF TIME STEP SIZE ON PREDICTED PRESSURE DISTRIBUTION IN GAS RESERVOIR

Grid point Location (i,j)	Pressure at end of 960 days (Psia)		
	Δt, days		
	30	60	120
1,1	789	789	792
* 2,2	727	727	730
3,3	732	733	735
* 4,4	674	674	677
6,5	677	678	680
* 8,6	626	627	629
10,7	630	630	632
* 12,8	597	597	599
14,9	630	630	631
* 16,10	611	611	611
18,11	699	699	699

* Gas wells

Material Balance

The solutions were checked for material balance according to the following relations:

$$\text{Estimated cumulative production (moles)} = \frac{\overline{\Delta x}^2}{RT} \sum_i \sum_j (\phi h)_{i,j} \left(\frac{P}{Z}\right)_{i,j} = E$$

$$\text{Actual cumulative production (moles)} = \frac{\overline{\Delta x}^2}{M} \sum_i \sum_j w_{i,j} = A$$

The maximum deviation of the ratio, A/E, from the correct value of unity was less than 0.9 percent. To examine the effect of locating the wells at corners, the calculations were repeated using seven wells at seven corners of the gas field (Table B-6). The predicted pressure distribution and the material balance are presented in Tables B-7 and B-8 respectively. The material balance remained substantially the same as in the previous case. The maximum deviation of the ratio A/E from the correct value was about 1.0 percent, which is acceptable for many reservoir problems.

In summary, the ADEP has been applied to an inherently nonlinear system to which it is most suitable. The predicted behavior of the gas reservoir is consistent with that encountered in practice.

VI. SIMULATION OF TWO-PHASE FLOW IN OIL RESERVOIRS

Displacement of oil by injecting water into a reservoir is an important method in secondary recovery of oil. Physically, this system is more complex than either of the previous two, as it involves simultaneous flow of two immiscible fluids, both of which are slightly compressible. Mathematically, the system is governed by two simultaneous nonlinear parabolic equations. The objective in this study is to demonstrate the application of the alternating direction explicit procedure (ADEP) in the study of two phase flow in reservoirs.

Water is injected at a known rate at selected locations in a reservoir, and oil is withdrawn from other suitable locations. It is required to determine the pressure distribution and the advance of the water-front in the reservoir as a function of time. Mathematically, the formulation of this problem is based on treating each phase separately for the purpose of mass conservation and flow, and interrelating the two through their interaction to capillary forces. Accounting for the variation of saturation with time, the continuity

equation given by (IV-1) is rewritten for each phase as:

$$\phi h \frac{\partial}{\partial t}(\rho s) = -\frac{\partial}{\partial x}[h\ m_x] - \frac{\partial}{\partial y}[h\ m_y] \pm w(x,y,t) \qquad \text{(VI-1)}$$

Incorporating the Darcy's empirical flow relation for each phase,

$$m_x = -\frac{k_r k}{\mu}\rho\frac{\partial P}{\partial x} \qquad \text{(VI-2)}$$

and approximating the space derivatives as in (IV-4), the basic equations governing the flow of water and oil in the reservoir are given by:

$$\phi h\left[C_w S_w \frac{\partial P_w}{\partial t} + \frac{\partial S_w}{\partial t}\right] = \frac{\partial}{\partial x}\left[\frac{k_{rw}\ kh}{\mu_w}\frac{\partial P_w}{\partial x}\right]$$

$$+ \frac{\partial}{\partial y}\left[\frac{k_{rw}\ kh}{\mu_w}\frac{\partial P_w}{\partial y}\right] + q_w(x,y,t) \qquad \text{(VI-3)}$$

and

$$\phi h\left[C_o S_o \frac{\partial P_o}{\partial t} + \frac{\partial S_o}{\partial t}\right] = \frac{\partial}{\partial x}\left[\frac{k_{ro}\ kh}{\mu_o}\frac{\partial P_o}{\partial x}\right]$$

$$+ \frac{\partial}{\partial y}\left[\frac{k_{ro}\ kh}{\mu_o}\ \frac{\partial P_o}{\partial y}\right] - q_o(x,y,t) \qquad \text{(VI-4)}$$

where $k_r \equiv k_r(Sw)$, is the relative permeability to a phase.

Subscripts w and o indicate water and oil phases respectively. The water and oil pressures are interrelated through capillary pressure, P_c.

$$P_c = P_o - P_w = f\ (S_w) \qquad \text{(VI-5)}$$

where $f(S_w)$ is a known function of S_w.

Further, $S_o = 1 - S_w$

The boundaries of the system are closed so that,

$$\frac{\partial P_w}{\partial \gamma} = \frac{\partial P_o}{\partial \gamma} = 0 \qquad \text{(VI-6a)}$$

where γ is the normal to the boundary. The pressure throughout the reservoir is defined at a certain time t_o by:

$$P_w(x,y,t_o) = F_w(x,y)$$
$$P_o(x,y,t_o) = F_o(x,y) \qquad \text{(VI-6b)}$$

The three nonlinear equations, (VI-3) to (VI-5), subject to equations (VI-6), have to be solved for the three unknowns P_w, P_o, and S_w, S_o being known from S_w.

An implicit numerical solution of this problem was provided by Douglas, Peaceman, and Rachford [17] with certain approximations. Firstly, the fluids were considered incompressible. Secondly, the nonlinearity arising from k_r being a function of S_w, was linearized by using k_r always one time step behind. Then, the resulting equations are implicit only with respect to water and oil pressures. The equations were solved by the alternating direction iterative procedure. Each iteration step involved four implicit sets of equations: each set further leads to a bi-tridiagonal system. Notwithstanding the simplifying assumptions made, the procedure is involved and time consuming.

The application of ADEP, on the other hand, does not require further simplification of equations (VI-3) and VI-4). When applied to these equations, ADEP leads to two nonlinear algebraic equations in three unknowns - P_w, P_o, and S_w - all of which occur, significantly, at one gridpoint only. Together with the third equation (VI-5), these can be solved by an iterative procedure such as Newton-Raphson. The compressibility C and viscosity μ of the fluid normally do not vary significantly under isothermal reservoir conditions and are therefore assumed constant. On multiplying equations (VI-3) and (VI-4) by $\mu dx\, dy$, the finite difference approximation by ADEP is given by the following equations.

<u>Forward Sweep</u>

1. <u>Water Phase</u>

$$\frac{\Delta x \Delta y}{\Delta t/2} \mu_w (\phi h)_{i,j} \Big[C_w \, S_{w_{i,j}}^{(n+1/4)} (P_{w_{i,j}}^{(n+1/2)} - P_{w_{i,j}}^{(n)}) + (S_{w_{i,j}}^{(n+1/2)} - S_{w_{i,j}}^{(n)}) \Big]$$

$$= \frac{\Delta y}{\Delta x}\Bigg[(kh)_{i-1/2,j}k_{rw}(S^{(n+1/2)}_{w_{i-1/2,j}})\ [P^{(n+1/2)}_{w_{i-1,j}} - P^{(n+1/2)}_{w_{i,j}}]$$

$$+ (kh)_{i+1/2,j}\ k_{rw}(S^{(n)}_{w_{i+1/2,j}})[P^{(n)}_{w_{i+1,j}} - P^{(n)}_{w_{i,j}}]\Bigg]$$

$$+ \frac{\Delta x}{\Delta y}\Bigg[(kh)_{i,j-1/2}\ k_{rw}(S^{(n+1/2)}_{w_{i,j-1/2}})[P^{(n+1/2)}_{w_{i,j-1}} - P^{(n+1/2)}_{w_{i,j}}]$$

$$+ (kh)_{i,j+1/2}\ k_{rw}(S^{(n)}_{w_{i,j+1/2}})[P^{(n)}_{w_{i,j+1}} - P^{(n)}_{w_{i,j}}]\Bigg]$$

$$+ \mu_w\ \tilde{Q}^{(n+1/4)}_{w_{i,j}} \qquad \text{(VI-7)}$$

2. <u>Oil Phase</u>

$$\frac{\Delta x\Delta y}{\Delta t/2}\ \mu_o(\phi h)_{i,j}\Bigg[C_o(1-S^{(n+1/4)}_{w_{i,j}})(P^{(n+1/2)}_{o_{i,j}} - P^{(n)}_{o_{i,j}})$$

$$- (S^{(n+1/2)}_{w_{i,j}} - S^{(n)}_{w_{i,j}})\Bigg]$$

$$= \frac{\Delta y}{\Delta x}\left[(kh)_{i-1/2,j}\ k_{ro}(S_{w_{i-1/2,j}}^{(n+1/2)})[P_{o_{i-1,j}}^{(n+1/2)} - P_{o_{i,j}}^{(n+1/2)}]\right.$$

$$\left. + \ (kh)_{i+1/2,j}\ k_{ro}(S_{w_{i+1/2,j}}^{(n)})[P_{o_{i+1,j}}^{(n)} - P_{o_{i,j}}^{(n)}]\right]$$

$$+ \frac{\Delta x}{\Delta y}\left[(kh)_{i,j-1/2}\ k_{ro}(S_{w_{i,j-1/2}}^{(n+1/2)})[P_{o_{i,j-1}}^{(n+1/2)} - P_{o_{i,j}}^{(n+1/2)}]\right.$$

$$\left. + \ (kh)_{i,j+1/2}\ k_{ro}(S_{w_{i,j+1/2}}^{(n)})[P_{o_{i,j+1}}^{(n)} - P_{o_{i,j}}^{(n)}]\right]$$

$$- \ \mu_o\ Q_{o_{i,j}}^{(n+1/4)} \qquad \text{(VI-8)}$$

where $-\frac{\partial S_w}{\partial t}$ is substituted for $\frac{\partial S_o}{\partial t}$. Q and q are related by (IV-11).

The new pressures, $P_{w_{i,j}}^{(n+1/2)}$ and $P_{o_{i,j}}^{(n+1/2)}$ and the new saturation $S_{w_{i,j}}^{(n+1/2)}$ must satisfy, in addition to (VI-7) and (VI-8), the capillary pressure relation:

$$P_{o_{i,j}}^{(n+1/2)} - P_{w_{i,j}}^{(n+1/2)} = f(S_{w_{i,j}}^{(n+1/2)}) \qquad \text{(VI-9)}$$

<u>Reverse Sweep</u>

1. <u>Water Phase</u>

$$\frac{\Delta x \Delta y}{\Delta t/2} \mu_w (\phi h)_{i,j} \left[C_w \, S_{w_{i,j}}^{(n+3/4)} \, [P_{w_{i,j}}^{(n+1)} - P_{w_{i,j}}^{(n+1/2)}] \right.$$

$$\left. + \; [S_{w_{i,j}}^{(n+1)} - S_{w_{i,j}}^{(n+1/2)}] \right]$$

$$= \frac{\Delta y}{\Delta x} \left[(kh)_{i-1/2,j} \, k_{rw}(S_{w_{i-1/2,j}}^{(n+1/2)}) [P_{w_{i-1,j}}^{(n+1/2)} - P_{w_{i,j}}^{(n+1/2)}] \right.$$

$$\left. + \; (kh)_{i+1/2,j} \, k_{rw}(S_{w_{i+1/2,j}}^{(n+1)}) [P_{w_{i+1,j}}^{(n+1)} - P_{w_{i,j}}^{(n+1)}] \right]$$

$$+ \frac{\Delta x}{\Delta y} \left[(kh)_{i,j-1/2} k_{rw}(S_{w_{i,j-1/2}}^{(n+1/2)}) [P_{w_{i,j-1}}^{(n+1/2)} - P_{w_{i,j}}^{(n+1/2)}] \right.$$

$$\left. + \; (kh)_{i,j+1/2} \, k_{rw}(S_{w_{i,j+1/2}}^{(n+1)}) [P_{w_{i,j+1}}^{(n+1)} - P_{w_{i,j}}^{(n+1)}] \right]$$

$$+ \; \mu_w \, Q_{w_{i,j}}^{(n+3/4)} \qquad \text{(VI-10)}$$

2. <u>Oil Phase</u>

$$\frac{\Delta x \Delta y}{\Delta t/2} \mu_o (\phi h)_{i,j} \Big[C_o (1 - S_{w_{i,j}}^{(n+3/4)}) [P_{o_{i,j}}^{(n+1)} - P_{o_{i,j}}^{(n+1/2)}]$$

$$- [S_{w_{i,j}}^{(n+1)} - S_{w_{i,j}}^{(n+1/2)}] \Big]$$

$$= \frac{\Delta y}{\Delta x} \Big[(kh)_{i-1/2,j} \, k_{ro} (S_{w_{i-1/2,j}}^{(n+1/2)}) [P_{o_{i-1,j}}^{(n+1/2)} - P_{o_{i,j}}^{(n+1/2)}]$$

$$+ (kh)_{i+1/2,j} \, k_{ro} (S_{w_{i+1/2,j}}^{(n+1)}) [P_{o_{i+1,j}}^{(n+1)} - P_{o_{i,j}}^{(n+1)}] \Big]$$

$$+ \frac{\Delta x}{\Delta y} \Big[(kh)_{i,j-1/2} \, k_{ro} (S_{w_{i,j-1/2}}^{(n+1/2)}) [P_{o_{i,j-1}}^{(n+1/2)} - P_{o_{i,j}}^{(n+1/2)}]$$

$$+ (kh)_{i,j+1/2} \, k_{ro} (S_{w_{i,j+1/2}}^{(n+1)}) [P_{o_{i,j+1}}^{(n+1)} - P_{o_{i,j}}^{(n+1)}] \Big]$$

$$- \mu_o \, Q_{o_{i,j}}^{(n+3/4)} \qquad \text{(VI-11)}$$

Again, the new pressures must satisfy the capillary

pressure relation:

$$P_{o_{i,j}}^{(n+1)} - P_{w_{i,j}}^{(n+1)} = f\ (S_{w_{i,j}}^{(n+1)}) \qquad \text{(VI-12)}$$

The points along the boundary are associated with cells each of which has an area equal to $\Delta x \Delta y/2$ and the corner points with cells, each of which has an area equal to $\Delta x \Delta y/4$. Equations (VI-7), (VI-8), (VI-10), and (VI-11) are modified accordingly.

Let $P_{w_{i,j}}^{(n+1/2)} = Z_1$

$$P_{o_{i,j}}^{(n+1/2)} = Z_2$$

$$S_{w_{i,j}}^{(n+1/2)} = Z_3$$

Then, on bringing all the terms to the left hand side, each of equations (VI-7), (VI-8), and (VI-9) may be written as:

$$G_i\ (Z_1, Z_2, Z_3) = 0 \qquad i = 1,2,3 \qquad \text{(VI-13)}$$

or in vector notation,

$$\underline{G}(\underline{Z}) = \underline{0}$$

where G_i are nonlinear functions of Z_j and $\underline{G}$ is the vector having G_i as its elements and $\underline{Z}$ is the vector having Z_j as the elements. The other variables at the time level $(n+1/2)\Delta t$ in (VI-7) and (VI-8) are in effect known as usual in ADEP.

The set of equations(VI-13) are solved iteratively using the Newton-Raphson procedure. Let $\underline{Z}^{(o)}$ be the initial guess of the solution. Then expanding $\underline{G}$ about $\underline{Z}^{(o)}$ and truncating the terms of second and higher order in $\Delta\underline{Z}$,

$$\underline{G}(\underline{Z}) \simeq \underline{G}(\underline{Z}^{(o)}) + \underline{J}(\underline{Z}^{(o)}) \; [\underline{Z} - \underline{Z}^{(o)}] \qquad \text{(VI-14)}$$

where $\underline{J}(\underline{Z})$ is the Jacobian matrix defined by:

$$\underline{J}(\underline{Z}) = \begin{bmatrix} \dfrac{\partial G_1}{\partial Z_1} & \dfrac{\partial G_1}{\partial Z_2} & \dfrac{\partial G_1}{\partial Z_3} \\ \dfrac{\partial G_2}{\partial Z_1} & \dfrac{\partial G_2}{\partial Z_2} & \dfrac{\partial G_2}{\partial Z_3} \\ \dfrac{\partial G_3}{\partial Z_1} & \dfrac{\partial G_3}{\partial Z_2} & \dfrac{\partial G_3}{\partial Z_3} \end{bmatrix} \qquad \text{(VI-15)}$$

The solution of (VI-14), which will be the next improved guess, is:

$$\underline{Z}^{(1)} = \underline{Z}^{(0)} - \underline{J}^{-1}(\underline{Z}^{(0)})\ \underline{G}\ (\underline{Z}^{(0)})$$

From this, the general recurrence relation for the (ℓ+1) iteration follows:

$$\underline{Z}^{(\ell+1)} = \underline{Z}^{(\ell)} - \underline{J}^{-1}\ (\underline{Z}^{(\ell)})\ \underline{G}\ (\underline{Z}^{(\ell)}) \qquad \text{(VI-16)}$$

where $\underline{J}^{-1}$ is assumed to exist. The criterion for terminating iteration is given by:

$$|Z_j^{(\ell+1)} - Z_j^{(\ell)}| < \varepsilon_j, \quad j=1,2,3 \qquad \text{(VI-17)}$$

ε_j being a prescribed small value.

While the method is not guaranteed to converge, no convergence problems are anticipated in this case since the number of variables is only three and the initial guess can be made sufficiently close to the true solution in the physical problem.

Numerical Example

A hypothetical reservoir shown in Figure 8 was studied. Superposition of a square network leads to a system of 289 grid points spaced at a distance of 132 ft. A five-spot model was used for water-injection, that is, four injecting wells near the four corners of the reservoir and a centrally located producing well.

The source listing of the computer program using ADEP is given in Appendix D. The properties of both reservoir formation and oil are believed to fall within the range of those found in Alberta and are described in Appendix C. These include the distribution of (ϕh) and (kh) of the formation (Tables C-1 and C-2) and the capillary pressure data in graphical form (Figure C-1). For better accuracy, the capillary pressure data were divided into two sections, and each section was curve-fitted separately using least squares criteria. The data at the ends of the saturation range were extrapolated. The relative permeability

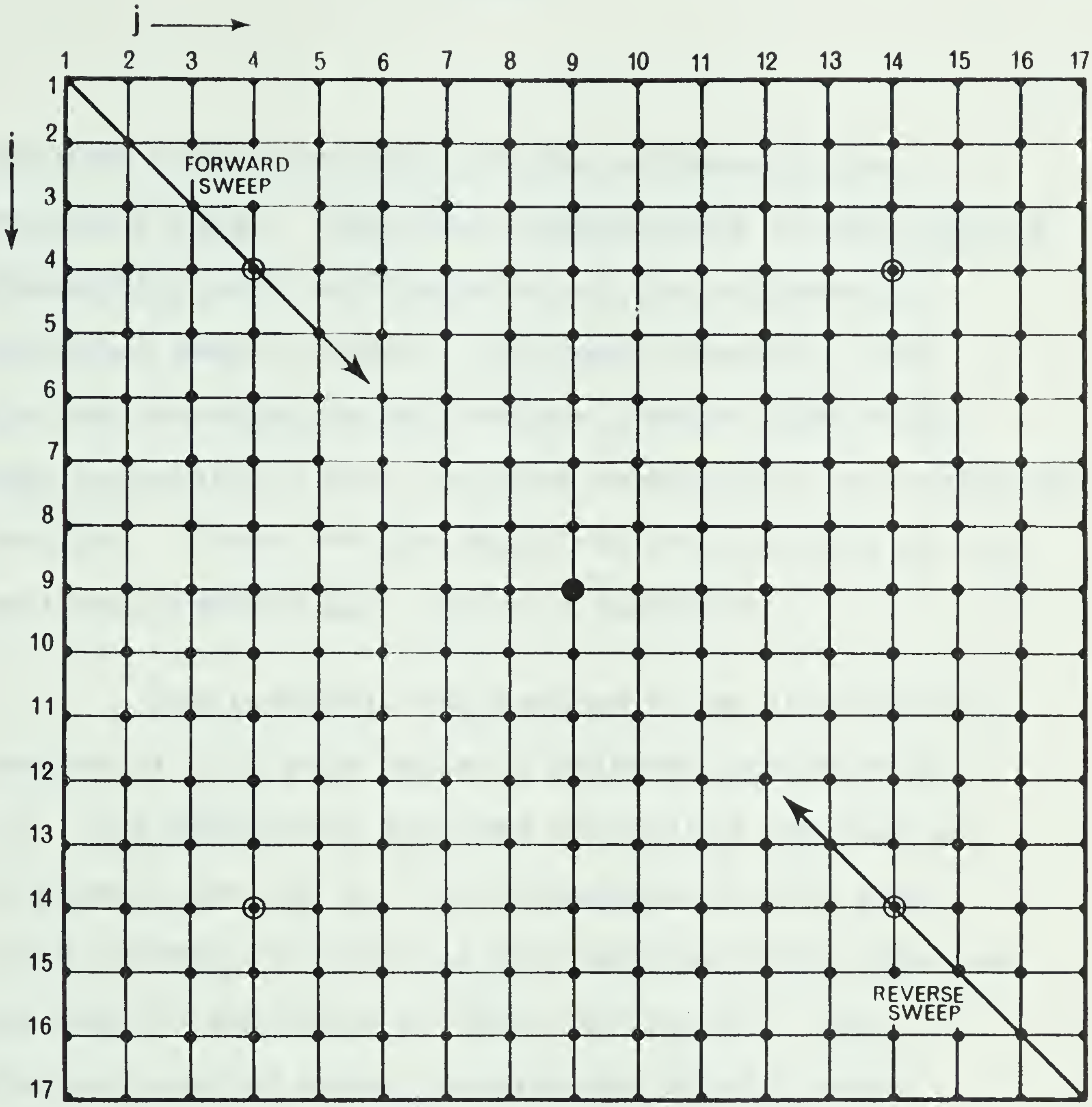

FIGURE 8: SIMULATION OF WATER-INJECTED RESERVOIR

data were also unavailable at the extremes of the saturation range. Empirical correlations of the relative permeability with extrapolation at the extremes of saturation range proved to be unsatisfactory. The solution converged to saturations greater than unity. These correlations were replaced subsequently by analytical functions. These and the empirical correlations for the capillary pressure are given in Appendix C.

The reservoir was assumed to be at a uniform pressure of 2700 psia and at a uniform saturation of 0.15. The production rate was maintained constant at 125 barrels per day of fluids measured at 2700 psia. It was assumed that the oil and water produced from the well were in the ratio of their mobilities. The injection rate of water, distributed equally among the four wells, was also constant during any run. This was, however, varied in a series of runs from 1.0 to 1.25 times the production rate. Computations were performed using comparatively large time steps of 10, 20, and 40 days per cycle. The initial guess

vector and the criteria for terminating iteration in equation (VI-17) are given in Appendix C. A typical matrix of iterations at all the points in the network is given in Table C-3 in Appendix C. The solution converges in approximately 3 to 4 iterations on the average.

A production rate of 125 barrels per day, and an equal injection rate, caused rapid pressure decline in the producing well. Over a period of 640 days, the pressure dropped from 2700 psia to 260 psia. During the same period, the pressure at an injecting well rose to about 3180 psia. This large gradient in pressure between the injecting and producing wells was caused obviously by the low absolute permeability of the formation. A decrease in production rate was not considered desirable since it was already low compared to the total oil present in the reservoir (1.7 million barrels). Alternatively, the injection rate was increased to 1.10 times the production rate. Consequently, the producing well pressure could be

maintained at a reasonable level. With these rates, it took nearly 5,000 days of real time to advance the water-front close to the producing well. To save computing time, a large time step of 20 days per cycle was used. The computing time was approximately 3.50 hours on IBM 7040.

Table C-4 in Appendix C gives a complete description of the distribution of water pressure, oil pressure and the water saturation throughout the reservoir at different times. Assuming an arbitrary saturation of 0.20 to represent the water-front, contours of the advancing water-front are shown in Figure 9. The saturation profiles between the producing well and two injecting wells are shown in Figures 10 and 11. The pressure build-up at an injecting well and the pressure decline at the producing well are described in Figures 12 and 13 respectively.

The injecting well at (4,4) (Figure 9) is surrounded by a low permeability zone (Table C-2).

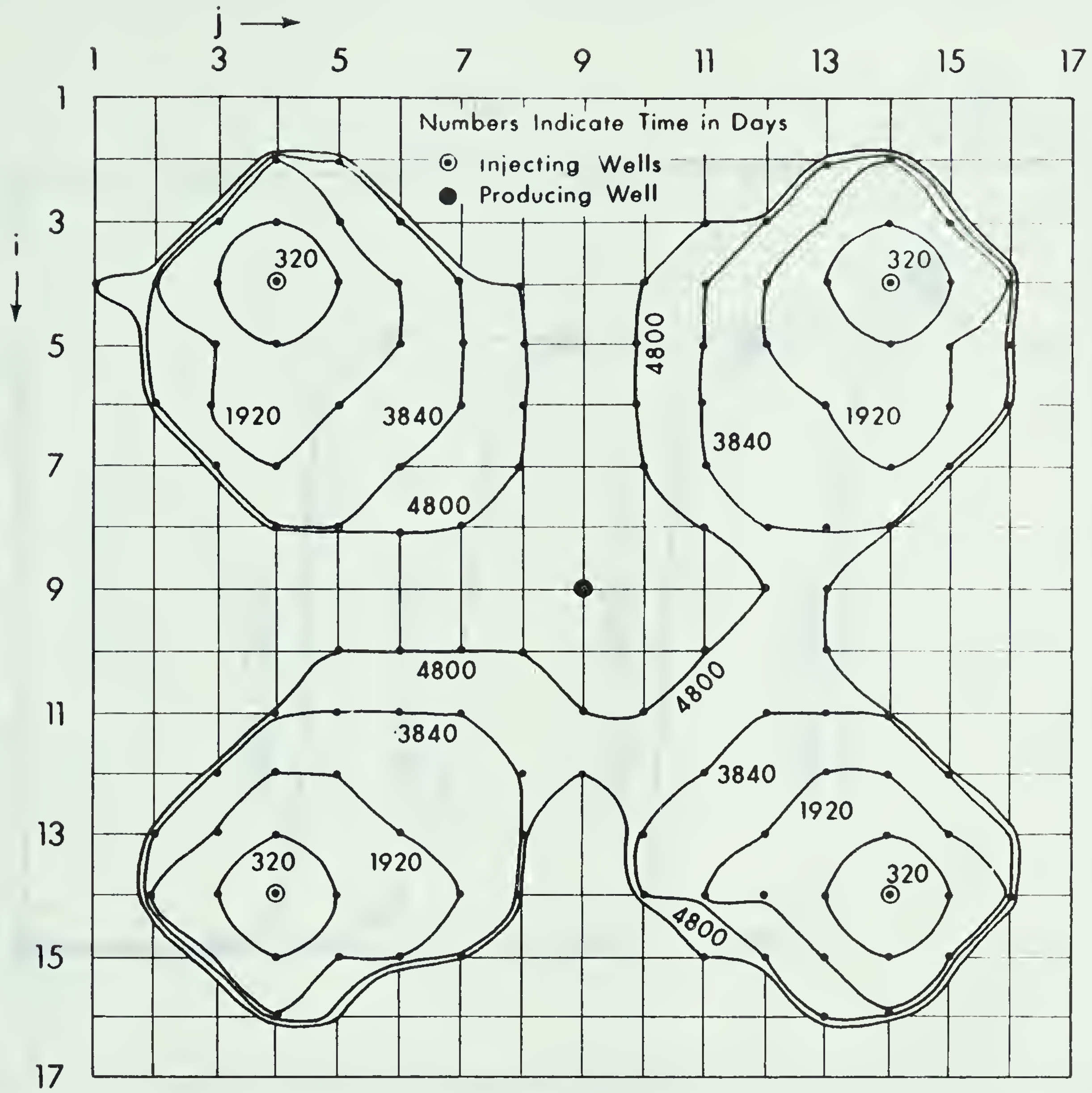

FIGURE 9 ADVANCE OF WATER-FRONT WITH TIME

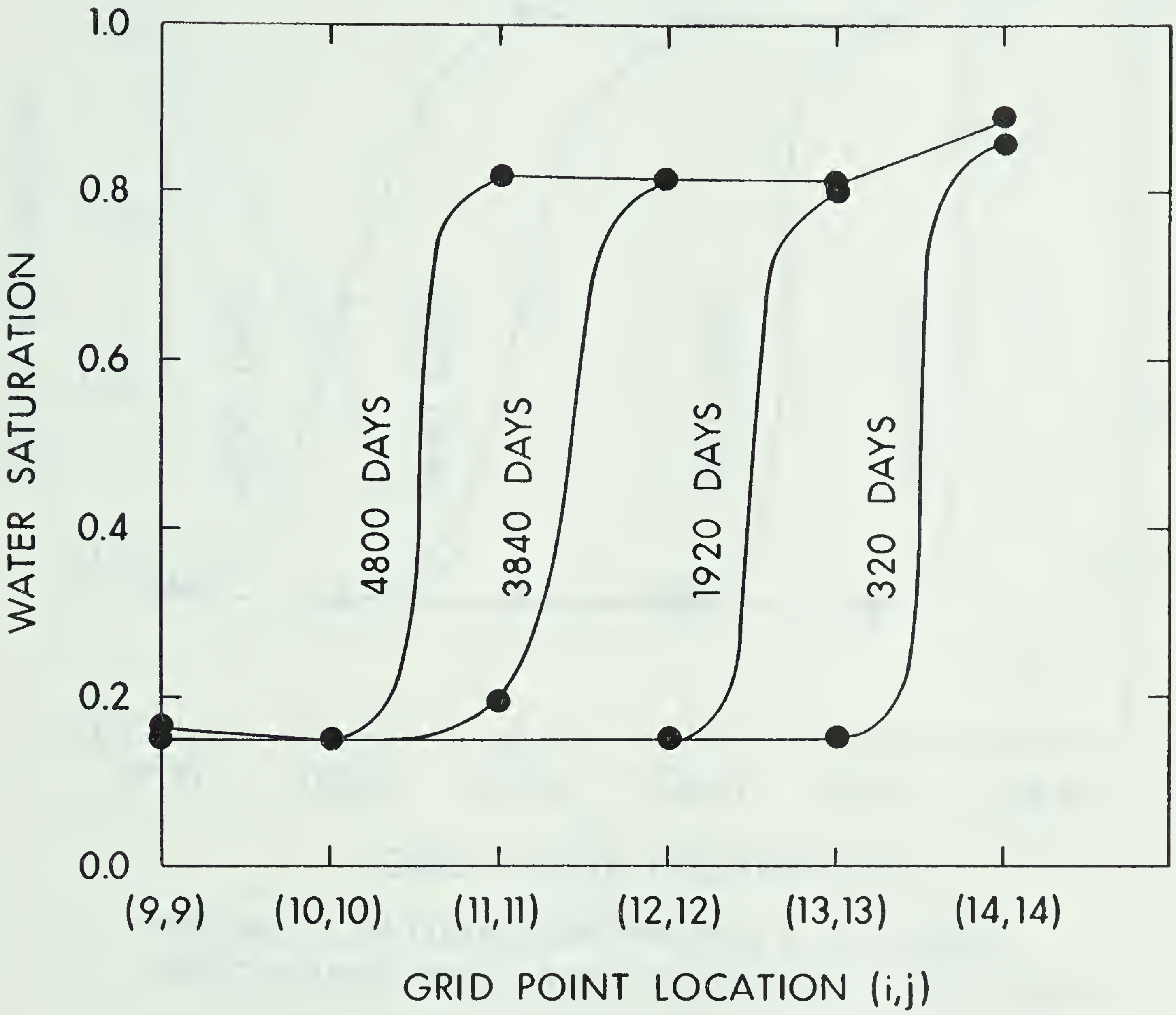

FIGURE 10 SATURATION PROFILES BETWEEN PRODUCING WELL AND INJECTING WELL (14,14)

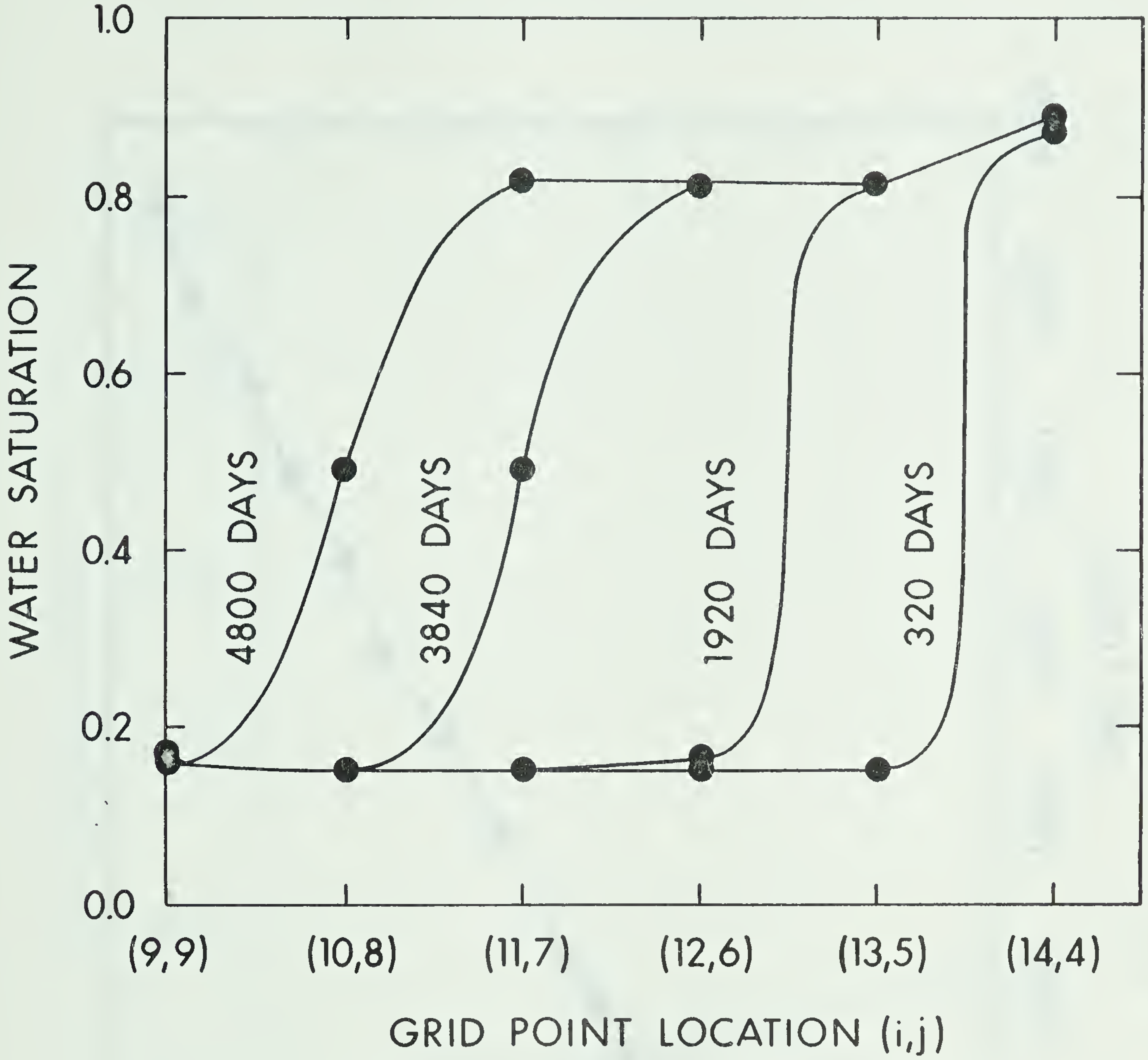

FIGURE 11 SATURATION PROFILES BETWEEN PRODUCING WELL AND INJECTING WELL (14,4)

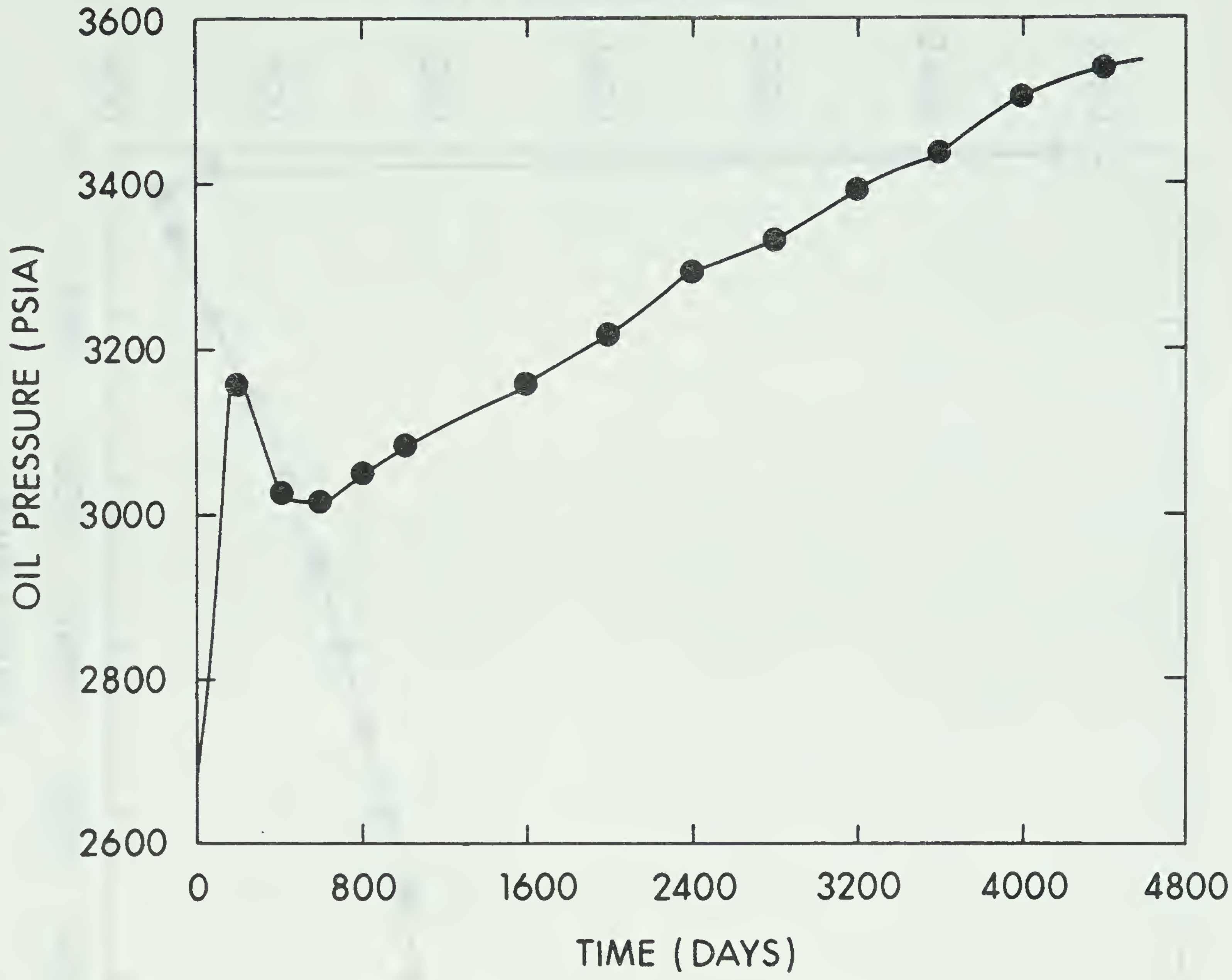

FIGURE 12 INJECTING WELL PRESSURE : LOCATION (15,15)

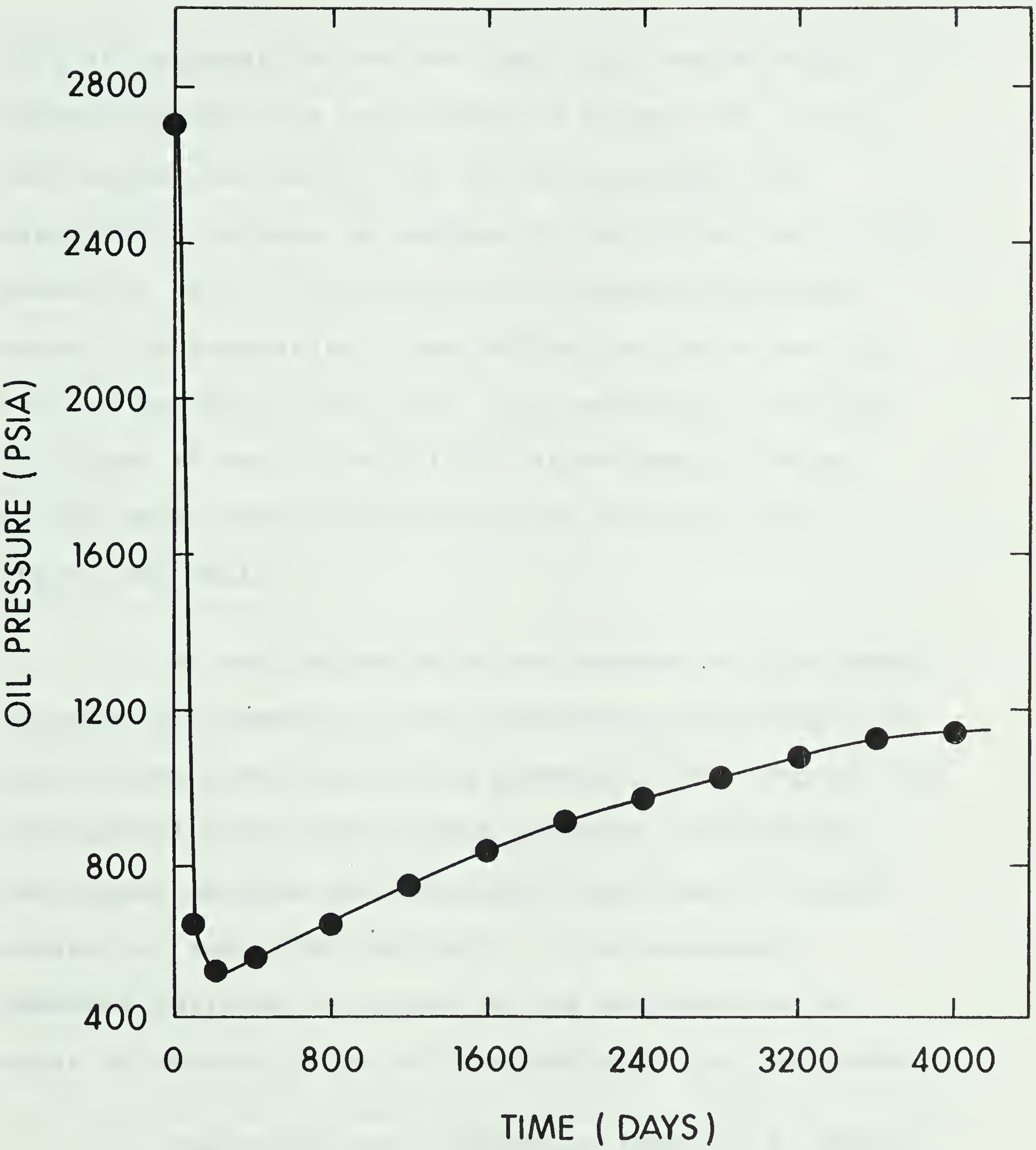

FIGURE 13 PRODUCING WELL PRESSURE

This is responsible for the fact that the 4800-day contour around this well does not merge with those of the neighboring wells. As can be expected, the water-front advance is maximum in the direction of the producing well. The contours are almost coincident toward the boundaries, thus reflecting the effect of no-flux boundary condition. The saturation profiles of Figure 10 and 11 exhibit a rather sharp gradient at the water-front indicating that the capillary effect was small.

At the beginning of the operation, the sharp increase in pressure at the injecting well (Figure 12) and the sharp decline at the producing well (Figure 13) are natural since the overall pressure distribution was rather uniform and there was less flow of fluids towards or away from the wells. The subsequent pressure build-up is caused by the accumulation of water in excess of the oil removed from the reservoir.

The fractional saturation (Table C-4) beyond the water-front exhibited a small decrease. This is

contrary to the anticipated increase in water saturation throughout the reservoir. This effect is greater for greater excess of water injected. The decrease in saturation was caused probably by a combination of factors. There is an accumulation of mass throughout the reservoir as a result of excess water injected. In the low saturation region, the relative permeability to oil is large while the relative permeability to water is negligible. This causes an influx of oil and little or no influx of water. The influx of oil could be accommodated by the compression of fluids, largely by that of water because of its greater compressibility. This results in an increase in oil saturation or decrease in water saturation. In this case the value of compressibility of water used happened to be greater than that of oil, although the reverse is true in practice.

Stability

In the absence of a proper method of stability analysis of simultaneous nonlinear equations, it is not possible to make a definite statement regarding the stability criterion. The maximum time step, Δt_{max}, for the classical explicit method, given below, is only a

crude estimate. Considering the linear form of equations (VI-3) and (VI-4),

$$\Delta t_{max} = \frac{1}{4} \mu c \left[\frac{S_w}{k_r}\right]_{min} \left[\frac{\phi h}{kh}\right]_{min} \frac{\overline{\Delta x}^2}{6.32} \qquad \text{(VI-18)}$$

It is observed that the value of Δt_{max} will be considerably different for water and oil, becuase their k_r values are different. Further, since k_r varies, so will the value of Δt_{max}. Based on the conditions at the beginning of the reservoir operation, the maximum time step for water will be obviously a large value. On the other hand, Δt_{max} for oil is only 0.116 day. In solving equations (VI-7) to (VI-12), time steps of up to 40 days per cycle were used and the solutions obtained were apparently stable.

Material Balance

The conservation of mass in each phase was checked at regular intervals and the results are shown

in Table C-5. Two points are significant. The material balance deviation for water is consistently small while it is significantly large for oil. The latter, however, decays with time. It appears that the errors inherent in using large time steps probably caused the deviation in material balance. It may be noted that a time step of 20 days is about 172 times Δt_{max} for oil at the beginning of the operation when the errors are large. The value of this factor decreases with time since Δt_{max} value increases (equation VI-18) as a result of decreasing relative permeability to oil. Significantly, the errors also decrease with time. This is apparently because the factor $\Delta t/\Delta t_{max}$ decreases with time. For a large part of the reservoir operation, Δt_{max} for water is large and hence $\Delta t/\Delta t_{max}$ is comparatively small. Probably for this reason the material balance deviation is small for water.

In this study, an unduly large time step was used in order to approach breakthrough within a reasonable computing time. To ensure accuracy in

material balance, it is imperative that a more reasonable time step of, say, about 20 times Δt_{max}, should be used at least in the early part of the reservoir operation.

In summary, the alternating direction explicit procedure could successfully solve simultaneous nonlinear parabolic equations. Iteration involved variables at one grid point only. The solution converged in less than four iterations. It is believed that in solving non-linear equations such as the above, ADEP is computationally faster than the alternating direction implicit procedures. To ensure accuracy in material balance, it is imperative that time steps of reasonable size should be used. The magnitude of such a time step is satisfactory for most practical problems.

CONCLUSIONS AND RECOMMENDATIONS

The second order parabolic partial differential equation resulting from a diffusion process can be converted into a first order matrix differential equation. The various numerical methods have been shown to result from different splittings of the coefficient matrix in this equation. The stability of the solution is determined by the properties of the integrating matrix. The matrix method of stability analysis is more general than the von Neumann technique. Unlike the latter, the matrix method can treat the finite difference approximations of linear parabolic equations with variable as well as constant coefficients. Techniques need to be developed for analyzing the stability of the numerical solution of nonlinear parabolic equations. The application of matrix theory is useful in the analysis of numerical procedures and also in providing an insight into new approaches to numerical solution of partial differential equations.

The alternating direction explicit procedure (ADEP) combines the computational simplicity of the

explicit methods with the unconditional stability of the implicit methods. The ADEP, extended to the three-dimensional case, compared favorably with the best alternating direction implicit procedure (B-D-R method) at average size time steps. Their accuracies were of the same order and ADEP was computationally faster. At larger size time steps, the greater computational speed of ADEP must be weighed against the somewhat better accuracy of the B-D-R method.

The concept of ADEP was extended to the solution of nonlinear parabolic partial differential equations. The computational speed of ADEP, compared to that of any ADIP, is a greater advantage in the nonlinear case than in the linear case.

While these methods represent considerable advance over the classical implicit and the classical explicit methods, for reasonable accuracy they still require large amounts of computing time in the solution of nonlinear and large linear systems. Two of the possible approaches for further study in reducing the computing time are:

1. To seek a more accurate approximation of the partial differential equation by a matrix differential equation by using a higher order correct discretization of the space derivative.
2. To find an accurate and efficient procedure for integrating the matrix differential equation.

No computing problems were encountered in solving, by ADEP, two-dimensional models of three types of petroleum reservoirs - a linear model of a liquid reservoir, a nonlinear model of a gas reservoir, and a rigorous mathematical model of two-phase flow in a water-injected oil reservoir. The solution of the linear model by ADEP checked well with that obtained by ADIP of Peaceman and Rachford. The ADEP solutions in all three cases were apparently stable. For accuracy of the solution and of the material balance, reasonable size time step should be used. Such a size is, however, large enough for most practical

purposes. The material balance deviation observed when using very large time steps needs to be investigated further.

Although only two-dimensional models were solved in this study, the author believes that ADEP can, as well, solve three-dimensional models, which more accurately represent many petroleum reservoirs.

NOMENCLATURE

SCALARS

B	-	Formation volume factor
c	-	Compressibility, 1/psia
g	-	Capacity term
h	-	Formation height, ft.
K	-	Rate coefficient
k	-	Absolute permeability, darcy
k_f	-	Effective permeability to a fluid, darcy
k_r	-	Relative permeability
m	-	Number of grid points along a column or row
m_x	-	Mass flux in x-direction
m_y	-	Mass flux in y-direction
M	-	Molecular weight
P	-	Pressure, psia
Q	-	Production or injection rate at reservoir conditions, bbl/day
Q'	-	Production or injection rate at surface conditions, STB/day
q	-	Production or injection rate per unit area at reservoir conditions, (bbl.)/ (ft^2)(day)
R	-	Gas constant, (psia)(ft^3)/(lb-mole)(°R)
r_i	-	Elemental region or cell at i^{th} grid point
S	-	Source term

S_w	-	Fractional saturation of water
T	-	Absolute temperature, °R
T	-	True solution
t	-	Time, days
u	-	Approximate solution to T
w	-	Gas production rate per unit area, (lb)/ft^2)(day)
x,y,z	-	Space coordinates (rectangular system)
Z	-	Compressibility factor

GREEK LETTERS

α	-	Real Gas psuedo-pressure
β	-	Dummy variable
γ	-	Normal to a system boundary
$\Delta t, \Delta x, \Delta y, \Delta z$	-	Increments in t,x,y, and z respectively
Δ_β	-	Forward difference with respect to β
ε	-	Criterion to terminate iteration
η	-	Dummy variable
θ	-	$\Delta t/\overline{\Delta x}^2$
λ	-	Eigenvalue of a matrix

μ - Viscosity, cp

$\rho(\underline{A})$ - Spectral radius of matrix $\underline{A}$

ρ - Density, lb/ft^3

τ - Truncation error

ϕ - Formation porosity (fraction)

ω - Relaxation factor in PSOR method

VECTORS, MATRICES

$\underline{A}$ - Matrix operator representing space derivatives

$\underline{B}$ - Coefficient matrix of a numerical method

$\underline{C}$ - Coefficient matrix of a numerical method

$\underline{D}$ - Diagonal matrix formed from $\underline{A}$

$\underline{D}_1, \underline{D}_2$ - Matrices resulting from splitting $\underline{D}$ according to ADEP

$\underline{E}$ - Strictly lower triangular matrix formed from $\underline{A}$

$\underline{F}$ - Strictly upper triangular matrix formed from $\underline{A}$

$\underline{G}$ - Diagonal matrix whose diagonal elements represent average capacities of elemental regions or cells

$\underline{H}$ - Tridiagonal matrix derived from splitting $\underline{A}$ according to ADIP

$\underline{J}$	-	Jacobian matrix defined by equation (VI-15)
$\underline{M},\underline{N}$	-	Matrices derived from general splitting of $\underline{A}$
$\underline{P}$	-	Matrix to be inverted in any numerical method
$\underline{p}^{(n)}$	-	Pressure vector at time $n\Delta t$
$\underline{Q}$	-	Vector of production or injection rates
$\underline{R}$	-	Coefficient matrix in the explicit part of a numerical method
$\underline{S}$	-	Vector of source terms
$\underline{T}$	-	True solution vector
$\underline{u}$	-	Approximate solution vector
$\underline{V}$	-	Tridiagonal matrix derived from splitting $\underline{A}$ according to ADIP
$\underline{Z}^{(\ell)}$	-	ℓth vector iterate of $\underline{Z}$
$\underline{\tau}$	-	Truncation error vector

SUBSCRIPTS

i	-	Position
i,j,k	-	Position in x,y, and z directions respectively
F	-	Forward Sweep in ADEP
f	-	Fluid
g	-	Gas

o	-	Oil
r	-	Relative
R	-	Reverse Sweep in ADEP
w	-	water

SUPERSCRIPTS

(n)	-	nth time step
(ℓ)	-	ℓth iteration
*	-	Intermediate estimate of approximate solution

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A P P E N D I X

APPENDIX A

OIL RESERVOIR

Reservoir

Initial Pressure	P_o	1065 psia
Saturation Pressure of oil	P_s	500 psia

Oil

If the production rate is specified at surface conditions, i.e., Q' in stock tank barrels, the rate at reservoir conditions, Q, is given by:

$$Q = Q' * B(P)$$

where P = reservoir pressure, psia

B = formation volume factor.

$$B(P) = 1.132 - 7.611 * 10^{-6}P$$

$$\mu(P) = 2.377 + 1.327 * 10^{-4}P$$

$$C = 6.7617 * 10^{-6} \text{ (psia)}^{-1}$$

$$S = 1.0$$

Network

Grid spacing	Δx $(=\Delta y)$	1320 ft.
No. of grid points		179
No. of producing wells		14

Δt_{max} for the FDE Scheme

Δt_{max} is the maximum time step consistent with stability of the Forward Difference Explicit Scheme. The calculation of Δt_{max} for the variable coefficient equation (IV-5) requires determination of the eigen-values of the coefficient matrix. A simpler approximate method is to estimate Δt_{max} on the basis of $(k/\phi\mu)_{max}$.

$$\frac{1}{CS} \left[\frac{K}{\phi\mu} \right]_{max} \left[\frac{\Delta t}{\Delta x^2} + \frac{\Delta t}{\Delta y^2} \right] \leq 1/2$$

Then, $\Delta t_{max} \doteq 0.3$ day

TABLE A- 1

PHI*H MATRIX (FT) - OIL RESERVOIR

J / I	1	2	3	4	5	6	7	8	9	10	11
1	14.3	9.7	8.0	7.3	7.5	5.8	5.1	4.3			
2	14.3	9.5	8.8	8.1	8.0	8.0	5.8	4.7			
3	18.2	11.0	8.7	9.0	8.3	7.3	6.2	5.2			
4	16.5	9.9	10.2	8.7	10.0	7.2	6.7	6.0			
5	13.0	10.9	9.6	8.3	7.7	7.4	7.4	7.1			
6	10.0	9.5	9.0	8.2	7.6	7.3	6.8	6.7			
7	9.8	9.3	8.8	8.4	10.3	7.5	6.7	6.4	6.0	4.2	2.8
8	14.3	11.0	9.7	8.2	12.0	8.4	8.3	7.3	7.2	4.6	3.2
9	16.5	14.2	14.2	10.1	12.5	10.2	7.7	6.6	6.7	5.3	3.0
10	16.6	14.1	8.0	10.9	11.6	10.6	6.8	6.8	6.2	5.0	3.0
11	13.3	15.1	10.3	10.2	11.0	8.0	8.0	6.0	4.5	3.7	2.7
12	16.5	12.8	11.0	8.9	10.2	8.4	8.4	5.8	3.7	3.0	2.6
13	15.7	13.2	12.5	12.4	8.8	7.7	7.7	6.0	4.0	4.0	4.1
14	16.2	12.5	12.2	11.5	10.5	7.9	7.3	6.3	5.2	5.5	5.2
15	7.3	9.0	9.7	14.0	10.0	10.2	7.9	7.1	6.1	6.8	5.8
16				11.0	9.1	10.3	12.0	9.2	7.6	6.7	6.1
17				7.6	8.8	11.2	12.7	11.8	12.0	7.1	6.4
18				7.7	12.0	13.5	13.1	12.7	12.2	8.1	6.8
19				11.0	14.7	13.7	12.7	12.3	11.2	10.0	6.4

TABLE A- 2

K*H MATRIX (DARCY-FT) - OIL RESERVOIR

I \ J	1	2	3	4	5	6	7	8	9	10	11
1	14.0	16.7	13.8	13.3	15.0	12.3	13.1	11.3			
2	14.9	17.0	15.2	14.5	15.1	16.3	13.2	12.1			
3	18.2	19.3	15.5	16.4	15.4	13.8	11.9	11.3			
4	16.2	17.7	18.2	15.8	18.2	13.3	12.6	11.8			
5	13.3	20.2	16.6	14.6	13.7	13.5	13.7	13.7			
6	16.7	17.6	16.1	14.6	13.8	13.0	12.4	12.4			
7	15.1	16.0	16.3	15.3	18.7	13.6	12.4	11.9	11.5	9.3	8.0
8	14.0	13.7	16.7	14.6	21.4	15.3	15.4	13.0	13.6	10.2	9.1
9	14.3	12.9	13.7	17.4	20.2	14.6	14.8	12.2	12.6	12.3	10.0
10	14.3	12.8	13.8	17.0	17.8	17.1	13.9	14.2	13.2	12.8	10.7
11	11.3	13.7	10.8	18.5	18.6	19.0	17.4	15.0	12.9	11.6	10.8
12	14.0	10.4	9.3	9.9	18.2	16.2	18.3	14.5	11.2	9.4	9.3
13	13.7	10.6	10.1	11.3	11.0	13.7	16.0	14.0	10.0	11.4	11.4
14	15.4	10.6	9.7	9.4	9.5	13.6	13.8	13.1	11.8	12.5	11.6
15	8.1	8.8	8.2	11.2	8.6	10.2	13.9	13.7	12.0	13.6	12.1
16				9.2	7.3	9.0	11.7	10.8	13.1	12.2	12.0
17				8.4	7.3	9.0	10.8	11.3	11.8	12.2	11.9
18				8.1	10.1	10.8	10.1	10.9	11.5	10.1	12.1
19				10.9	12.9	10.5	9.4	9.5	10.2	11.1	11.0

TABLE A-3

PRODUCTION PATTERN OF OIL RESERVOIR

LOCATION OF WELL I	J	PRODUCTION Q(I,J) S.T.B./DAY
3	3	75.
6	3	75.
9	3	75.
12	3	75.
3	6	125.
6	6	125.
9	6	125.
12	6	125.
15	6	125.
18	6	125.
9	9	75.
12	9	75.
15	9	75.
18	9	75.

TABLE A- 4

MATRIX OF PRESSURE IN OIL RESERVOIR,(PSIA). BY ADEP

(TIME STEP = 10.00 DAYS)

TIME = 100. DAYS

I \ J	1	2	3	4	5	6	7	8	9	10	11
1	1029.	1028.	1026.	1024.	1022.	1020.	1020.	1020.			
2	1028.	1027.	1024.	1023.	1020.	1018.	1019.	1019.			
3	1028.	1025.	1021.	1021.	1018.	1012.	1017.	1018.			
4	1027.	1025.	1023.	1021.	1019.	1016.	1017.	1018.			
5	1026.	1025.	1022.	1020.	1018.	1015.	1016.	1017.			
6	1026.	1024.	1019.	1019.	1016.	1009.	1014.	1015.			
7	1026.	1024.	1021.	1020.	1017.	1014.	1015.	1016.	1015.	1016.	1016.
8	1027.	1025.	1022.	1020.	1017.	1014.	1015.	1015.	1014.	1015.	1015.
9	1028.	1025.	1019.	1019.	1016.	1010.	1013.	1013.	1010.	1013.	1014.
10	1029.	1027.	1022.	1020.	1017.	1014.	1014.	1013.	1012.	1014.	1014.
11	1030.	1028.	1023.	1020.	1017.	1014.	1014.	1013.	1012.	1013.	1014.
12	1031.	1028.	1020.	1020.	1017.	1010.	1013.	1012.	1008.	1013.	1014.
13	1032.	1030.	1026.	1023.	1019.	1015.	1014.	1014.	1013.	1014.	1015.
14	1033.	1031.	1027.	1025.	1020.	1015.	1015.	1015.	1014.	1015.	1016.
15	1033.	1031.	1026.	1025.	1020.	1009.	1014.	1015.	1012.	1016.	1017.
16				1026.	1023.	1017.	1017.	1017.	1016.	1018.	1019.
17				1026.	1022.	1018.	1019.	1019.	1018.	1019.	1020.
18				1024.	1021.	1012.	1018.	1019.	1015.	1019.	1021.
19				1024.	1022.	1019.	1020.	1020.	1019.	1021.	1021.

CCNTC. TABLE A- 4

MATRIX OF PRESSURE IN OIL RESERVOIR,(PSIA). BY ADEP

(TIME STEP = 10.00 DAYS)

TIME = 300. DAYS

J / I	1	2	3	4	5	6	7	8	9	10	11
1	939.	938.	936.	934.	932.	930.	930.	930.			
2	939.	937.	934.	933.	930.	928.	928.	929.			
3	938.	936.	931.	931.	928.	922.	926.	928.			
4	937.	935.	933.	931.	929.	926.	926.	927.			
5	937.	935.	932.	930.	928.	925.	926.	926.			
6	936.	934.	929.	929.	926.	919.	924.	925.			
7	936.	934.	931.	929.	927.	924.	924.	925.	925.	925.	925.
8	937.	935.	931.	929.	927.	924.	924.	924.	923.	924.	924.
9	938.	935.	929.	929.	926.	919.	923.	922.	919.	922.	924.
10	939.	937.	932.	930.	927.	924.	923.	923.	922.	923.	923.
11	940.	937.	933.	930.	927.	924.	923.	922.	921.	922.	923.
12	941.	938.	930.	930.	926.	919.	922.	921.	917.	922.	923.
13	942.	940.	935.	932.	928.	924.	923.	923.	922.	923.	924.
14	943.	941.	937.	934.	929.	924.	924.	923.	923.	924.	925.
15	942.	940.	935.	935.	929.	918.	923.	923.	920.	924.	926.
16				935.	932.	926.	926.	926.	925.	926.	927.
17				935.	931.	927.	927.	927.	926.	927.	928.
18				933.	929.	920.	926.	927.	923.	927.	929.
19				933.	931.	928.	928.	929.	928.	929.	929.

CONTD. TABLE A- 4

MATRIX OF PRESSURE IN OIL RESERVOIR,(PSIA). BY ADEP

(TIME STEP = 10.00 DAYS)

TIME = 500. DAYS

I \ J	1	2	3	4	5	6	7	8	9	10	11
1	849.	847.	845.	843.	841.	840.	839.	839.			
2	848.	846.	844.	842.	840.	837.	838.	838.			
3	847.	845.	840.	840.	838.	831.	836.	837.			
4	847.	845.	842.	840.	838.	835.	836.	837.			
5	846.	844.	842.	840.	837.	834.	835.	836.			
6	845.	843.	838.	838.	836.	828.	833.	834.			
7	846.	844.	841.	839.	836.	834.	834.	834.	834.	835.	835.
8	847.	844.	841.	839.	836.	834.	834.	833.	833.	834.	834.
9	847.	845.	838.	838.	835.	829.	832.	832.	828.	832.	833.
10	848.	846.	842.	839.	836.	833.	833.	832.	831.	832.	833.
11	849.	847.	842.	839.	836.	833.	832.	832.	831.	832.	832.
12	850.	847.	839.	839.	835.	829.	831.	831.	827.	831.	833.
13	851.	849.	845.	842.	837.	833.	833.	832.	831.	832.	833.
14	852.	850.	846.	843.	838.	833.	833.	833.	832.	833.	834.
15	852.	849.	844.	844.	838.	828.	833.	833.	830.	834.	835.
16				844.	841.	835.	835.	835.	834.	835.	836.
17				844.	840.	836.	837.	836.	836.	837.	837.
18				842.	839.	830.	836.	836.	833.	837.	838.
19				842.	840.	837.	838.	838.	837.	838.	839.

CCNTD. TABLE A- 4

MATRIX OF PRESSURE IN OIL RESERVOIR,(PSIA). BY ADEP

(TIME STEP = 10.00 DAYS)

TIME = 700. DAYS

I \ J	1	2	3	4	5	6	7	8	9	10	11
1	758.	756.	755.	753.	751.	749.	749.	749.			
2	757.	756.	753.	751.	749.	747.	747.	748.			
3	757.	754.	750.	750.	747.	741.	745.	747.			
4	756.	754.	751.	750.	747.	745.	745.	746.			
5	755.	754.	751.	749.	747.	744.	745.	745.			
6	755.	753.	748.	748.	745.	738.	743.	744.			
7	755.	753.	750.	748.	746.	743.	743.	744.	744.	744.	744.
8	756.	754.	750.	748.	746.	743.	743.	743.	742.	743.	743.
9	757.	754.	748.	748.	745.	738.	741.	741.	738.	741.	742.
10	758.	755.	751.	748.	746.	743.	742.	742.	740.	741.	742.
11	759.	756.	751.	749.	746.	742.	742.	741.	740.	741.	742.
12	760.	756.	748.	748.	745.	738.	741.	740.	736.	740.	742.
13	761.	758.	754.	751.	747.	743.	742.	742.	741.	742.	743.
14	761.	759.	755.	752.	748.	743.	743.	742.	741.	743.	744.
15	761.	759.	753.	753.	748.	737.	742.	742.	739.	743.	744.
16				753.	750.	745.	745.	744.	744.	745.	746.
17				753.	750.	745.	746.	746.	745.	746.	747.
18				751.	748.	739.	745.	746.	742.	746.	747.
19				751.	749.	746.	747.	747.	746.	747.	748.

CCNTC. TABLE A- 4

MATRIX OF PRESSURE IN OIL RESERVOIR,(PSIA). BY ADEP

(TIME STEP = 10.00 DAYS)

TIME = 1000. DAYS

I \ J	1	2	3	4	5	6	7	8	9	10	11
1	622.	620.	618.	616.	615.	613.	612.	613.			
2	621.	619.	617.	615.	613.	611.	611.	612.			
3	620.	618.	613.	614.	611.	605.	609.	611.			
4	620.	618.	615.	614.	611.	609.	609.	610.			
5	619.	617.	615.	613.	611.	608.	608.	609.			
6	619.	616.	612.	612.	609.	602.	607.	608.			
7	619.	617.	614.	612.	610.	607.	607.	608.	608.	608.	608.
8	620.	617.	614.	612.	610.	607.	607.	607.	606.	607.	607.
9	620.	618.	612.	611.	609.	602.	605.	605.	602.	605.	606.
10	621.	619.	615.	612.	610.	607.	606.	606.	604.	605.	606.
11	622.	620.	615.	613.	610.	606.	606.	605.	604.	605.	606.
12	623.	620.	612.	612.	609.	602.	605.	604.	600.	604.	606.
13	624.	622.	618.	615.	611.	606.	606.	606.	605.	606.	607.
14	625.	623.	619.	616.	612.	607.	607.	606.	605.	607.	608.
15	625.	622.	617.	617.	612.	601.	606.	606.	603.	607.	608.
16				617.	614.	609.	609.	608.	608.	609.	610.
17				617.	614.	609.	610.	610.	609.	610.	611.
18				615.	612.	603.	609.	610.	606.	610.	611.
19				615.	613.	610.	611.	611.	610.	611.	612.

CONTD. TABLE A- 4

MATRIX OF PRESSURE IN OIL RESERVOIR,(PSIA). BY ADEP

(TIME STEP = 10.00 DAYS)

TIME = 1200. DAYS

I \ J	1	2	3	4	5	6	7	8	9	10	11
1	531.	529.	528.	526.	524.	522.	522.	522.			
2	530.	529.	526.	524.	522.	520.	520.	521.			
3	530.	527.	523.	523.	520.	514.	518.	520.			
4	529.	527.	525.	523.	521.	518.	519.	519.			
5	528.	527.	524.	522.	520.	517.	518.	518.			
6	528.	526.	521.	521.	518.	511.	516.	517.			
7	528.	526.	523.	521.	519.	516.	517.	517.	517.	517.	517.
8	529.	527.	523.	521.	519.	516.	516.	516.	515.	516.	517.
9	530.	527.	521.	521.	518.	512.	515.	514.	511.	515.	516.
10	531.	528.	524.	522.	519.	516.	515.	515.	514.	515.	515.
11	531.	529.	524.	522.	519.	516.	515.	514.	513.	514.	515.
12	532.	529.	521.	521.	518.	511.	514.	514.	510.	514.	515.
13	533.	531.	527.	524.	520.	516.	515.	515.	514.	515.	516.
14	534.	532.	528.	525.	521.	516.	516.	515.	515.	516.	517.
15	534.	531.	526.	526.	521.	510.	515.	515.	512.	516.	518.
16				526.	523.	518.	518.	518.	517.	518.	519.
17				526.	523.	518.	519.	519.	518.	519.	520.
18				524.	521.	512.	518.	519.	515.	519.	521.
19				524.	523.	519.	520.	520.	519.	521.	521.

TABLE A- 5

MATRIX OF PRESSURE IN OIL RESERVOIR,(PSIA). BY ADIP

(TIME STEP = 10.00 DAYS)

TIME = 100. DAYS

I \ J	1	2	3	4	5	6	7	8	9	10	11
1	1029.	1027.	1025.	1023.	1021.	1020.	1019.	1019.			
2	1028.	1026.	1024.	1022.	1020.	1018.	1018.	1019.			
3	1027.	1025.	1020.	1020.	1018.	1011.	1016.	1018.			
4	1027.	1025.	1022.	1021.	1018.	1016.	1016.	1017.			
5	1026.	1024.	1022.	1020.	1018.	1015.	1016.	1017.			
6	1026.	1024.	1019.	1019.	1016.	1009.	1014.	1016.			
7	1026.	1024.	1021.	1019.	1017.	1014.	1015.	1016.	1016.	1016.	1016.
8	1027.	1025.	1021.	1019.	1017.	1014.	1014.	1014.	1014.	1014.	1015.
9	1028.	1025.	1019.	1019.	1016.	1010.	1013.	1013.	1009.	1013.	1014.
10	1029.	1027.	1023.	1020.	1017.	1014.	1014.	1013.	1012.	1013.	1014.
11	1031.	1028.	1023.	1021.	1018.	1014.	1014.	1013.	1012.	1013.	1014.
12	1032.	1029.	1021.	1021.	1017.	1010.	1013.	1012.	1008.	1013.	1014.
13	1034.	1032.	1028.	1024.	1019.	1015.	1015.	1014.	1013.	1014.	1015.
14	1035.	1034.	1031.	1027.	1021.	1016.	1015.	1015.	1014.	1015.	1016.
15	1036.	1035.	1032.	1029.	1022.	1010.	1015.	1015.	1012.	1016.	1017.
16				1032.	1024.	1018.	1018.	1017.	1016.	1018.	1018.
17				1028.	1023.	1019.	1019.	1019.	1018.	1019.	1020.
18				1025.	1021.	1012.	1018.	1019.	1015.	1019.	1020.
19				1025.	1023.	1019.	1020.	1020.	1019.	1020.	1021.

CCNTD. TABLE A- 5

MATRIX OF PRESSURE IN OIL RESERVOIR,(PSIA). BY ADIP

(TIME STEP = 10.00 DAYS)

TIME = 300. DAYS

I \ J	1	2	3	4	5	6	7	8	9	10	11
1	941.	940.	938.	936.	934.	932.	932.	932.			
2	941.	939.	937.	935.	933.	930.	931.	932.			
3	940.	938.	933.	933.	931.	924.	929.	931.			
4	940.	938.	935.	933.	931.	928.	929.	930.			
5	939.	937.	935.	933.	930.	928.	929.	930.			
6	939.	936.	932.	932.	929.	922.	927.	929.			
7	939.	937.	934.	932.	930.	927.	928.	928.	929.	928.	928.
8	940.	938.	935.	932.	930.	927.	927.	927.	927.	927.	928.
9	941.	939.	932.	932.	929.	923.	926.	926.	922.	926.	927.
10	943.	940.	936.	933.	931.	927.	927.	926.	925.	926.	927.
11	944.	942.	937.	934.	931.	927.	927.	926.	925.	926.	927.
12	946.	943.	934.	934.	930.	923.	926.	925.	921.	925.	927.
13	948.	946.	941.	938.	933.	928.	927.	927.	925.	927.	928.
14	949.	948.	944.	940.	934.	929.	928.	927.	926.	928.	929.
15	950.	948.	946.	942.	935.	923.	928.	928.	924.	928.	929.
16				946.	938.	931.	930.	930.	929.	930.	931.
17				941.	937.	931.	931.	931.	930.	931.	932.
18				938.	934.	925.	931.	931.	927.	931.	932.
19				938.	936.	932.	933.	932.	931.	932.	933.

CONTD. TABLE A- 5

MATRIX OF PRESSURE IN OIL RESERVOIR,(PSIA). BY ADIP

(TIME STEP = 10.00 DAYS)

TIME = 500. DAYS

J / I	1	2	3	4	5	6	7	8	9	10	11
1	854.	853.	851.	849.	847.	845.	845.	845.			
2	854.	852.	849.	848.	846.	843.	844.	844.			
3	853.	851.	846.	846.	844.	837.	842.	844.			
4	853.	851.	848.	846.	844.	841.	842.	843.			
5	852.	850.	848.	846.	843.	841.	842.	843.			
6	851.	849.	845.	845.	842.	835.	840.	842.			
7	852.	850.	847.	845.	843.	840.	841.	841.	842.	841.	841.
8	853.	851.	847.	845.	843.	840.	840.	840.	840.	840.	841.
9	854.	851.	845.	845.	842.	836.	839.	839.	835.	839.	840.
10	856.	853.	849.	846.	844.	840.	840.	839.	838.	839.	840.
11	857.	855.	850.	847.	844.	840.	840.	839.	838.	839.	840.
12	859.	856.	847.	847.	843.	836.	839.	838.	834.	838.	840.
13	861.	859.	854.	851.	846.	841.	840.	840.	839.	840.	841.
14	862.	861.	857.	853.	847.	842.	841.	840.	839.	841.	842.
15	862.	861.	859.	855.	848.	836.	841.	841.	837.	841.	842.
16				859.	851.	844.	843.	843.	842.	843.	844.
17				854.	849.	844.	844.	844.	843.	844.	845.
18				851.	847.	838.	844.	844.	840.	844.	845.
19				851.	849.	845.	846.	845.	844.	845.	846.

CONTD. TABLE A- 5

MATRIX OF PRESSURE IN OIL RESERVOIR,(PSIA). BY ADIP

(TIME STEP = 10.00 DAYS)

TIME = 700. DAYS

I \ J	1	2	3	4	5	6	7	8	9	10	11
1	767.	766.	764.	762.	760.	758.	758.	758.			
2	766.	765.	762.	761.	759.	756.	757.	757.			
3	766.	764.	759.	759.	757.	750.	755.	756.			
4	765.	763.	761.	759.	757.	754.	755.	756.			
5	765.	763.	760.	759.	756.	754.	755.	755.			
6	764.	762.	757.	758.	755.	748.	753.	755.			
7	765.	763.	760.	758.	756.	753.	754.	754.	755.	754.	754.
8	766.	764.	760.	758.	756.	753.	753.	753.	753.	753.	754.
9	767.	764.	758.	758.	755.	749.	752.	752.	748.	752.	753.
10	768.	766.	762.	759.	756.	753.	753.	752.	751.	752.	753.
11	770.	767.	763.	760.	757.	753.	753.	752.	751.	752.	753.
12	772.	769.	760.	760.	756.	749.	752.	751.	747.	751.	753.
13	774.	771.	767.	763.	759.	754.	753.	753.	751.	753.	754.
14	775.	773.	770.	766.	760.	755.	754.	753.	752.	754.	755.
15	775.	774.	771.	768.	761.	749.	754.	753.	750.	754.	755.
16				771.	764.	757.	756.	756.	755.	756.	757.
17				767.	762.	757.	757.	757.	756.	757.	758.
18				764.	760.	751.	756.	757.	753.	757.	758.
19				764.	761.	758.	758.	758.	757.	758.	759.

CONTD. TABLE A- 5

MATRIX OF PRESSURE IN OIL RESERVOIR,(PSIA). BY ADIP

(TIME STEP = 10.00 DAYS)

TIME = 1000. DAYS

I \ J	1	2	3	4	5	6	7	8	9	10	11
1	636.	635.	633.	631.	629.	628.	627.	627.			
2	636.	634.	632.	630.	628.	626.	626.	627.			
3	635.	633.	628.	628.	626.	619.	624.	626.			
4	635.	633.	630.	628.	626.	624.	625.	625.			
5	634.	632.	630.	628.	626.	623.	624.	625.			
6	634.	631.	627.	627.	624.	617.	622.	624.			
7	634.	632.	629.	627.	625.	622.	623.	624.	624.	624.	624.
8	635.	633.	630.	628.	625.	623.	623.	623.	622.	623.	623.
9	636.	633.	627.	627.	625.	618.	621.	621.	618.	621.	622.
10	638.	635.	631.	629.	626.	623.	622.	621.	620.	621.	622.
11	639.	637.	632.	629.	626.	623.	622.	621.	620.	621.	622.
12	641.	638.	629.	629.	626.	619.	621.	621.	617.	621.	622.
13	643.	640.	636.	633.	628.	623.	623.	622.	621.	622.	623.
14	644.	642.	639.	635.	630.	624.	623.	623.	622.	623.	624.
15	644.	643.	641.	637.	630.	619.	623.	623.	619.	623.	625.
16				641.	633.	626.	626.	625.	624.	625.	626.
17				636.	632.	627.	627.	626.	625.	626.	627.
18				633.	629.	620.	626.	626.	622.	626.	628.
19				633.	631.	627.	628.	628.	627.	628.	628.

CONTD. TABLE A- 5

MATRIX OF PRESSURE IN OIL RESERVOIR,(PSIA). BY ADIP

(TIME STEP = 10.00 DAYS)

TIME = 1200. DAYS

I \ J	1	2	3	4	5	6	7	8	9	10	11
1	549.	547.	546.	544.	542.	540.	540.	540.			
2	548.	547.	544.	543.	541.	538.	539.	539.			
3	548.	546.	541.	541.	539.	532.	537.	539.			
4	547.	545.	543.	541.	539.	536.	537.	538.			
5	547.	545.	542.	541.	538.	536.	537.	538.			
6	546.	544.	540.	540.	537.	530.	535.	537.			
7	547.	545.	542.	540.	538.	535.	536.	536.	537.	537.	537.
8	548.	546.	542.	540.	538.	535.	535.	535.	535.	535.	536.
9	549.	546.	540.	540.	537.	531.	534.	534.	531.	534.	535.
10	550.	548.	544.	541.	539.	535.	535.	534.	533.	534.	535.
11	552.	549.	545.	542.	539.	535.	535.	534.	533.	534.	535.
12	553.	550.	542.	542.	538.	532.	534.	533.	529.	533.	535.
13	555.	553.	549.	545.	541.	536.	536.	535.	534.	535.	536.
14	557.	555.	552.	548.	542.	537.	536.	536.	535.	536.	537.
15	557.	556.	553.	550.	543.	532.	536.	536.	532.	536.	537.
16				553.	545.	539.	538.	538.	537.	538.	539.
17				549.	544.	539.	539.	539.	538.	539.	540.
18				546.	542.	533.	539.	539.	535.	539.	540.
19				546.	543.	540.	541.	540.	539.	540.	541.

TABLE A- 6

MATRIX OF PRESSURE IN OIL RESERVOIR,(PSIA). BY ADEP

(TIME STEP = 20.00 DAYS)

TIME = 100. DAYS

J / I	1	2	3	4	5	6	7	8	9	10	11
1	1030.	1028.	1027.	1025.	1024.	1023.	1023.	1023.			
2	1029.	1028.	1025.	1024.	1022.	1020.	1021.	1022.			
3	1029.	1026.	1022.	1022.	1020.	1015.	1019.	1021.			
4	1028.	1026.	1024.	1023.	1021.	1018.	1019.	1020.			
5	1028.	1026.	1023.	1022.	1020.	1017.	1018.	1019.			
6	1027.	1025.	1021.	1021.	1018.	1012.	1016.	1018.			
7	1027.	1026.	1023.	1021.	1019.	1016.	1017.	1018.	1018.	1018.	1019.
8	1028.	1026.	1023.	1021.	1019.	1016.	1017.	1017.	1016.	1018.	1018.
9	1029.	1026.	1020.	1021.	1018.	1012.	1015.	1015.	1013.	1016.	1017.
10	1030.	1028.	1024.	1022.	1019.	1016.	1016.	1016.	1015.	1016.	1017.
11	1031.	1029.	1024.	1022.	1019.	1016.	1016.	1016.	1015.	1016.	1017.
12	1032.	1029.	1021.	1022.	1019.	1013.	1015.	1015.	1012.	1015.	1017.
13	1033.	1031.	1027.	1024.	1021.	1017.	1017.	1016.	1015.	1017.	1017.
14	1034.	1032.	1029.	1026.	1022.	1017.	1017.	1017.	1016.	1017.	1018.
15	1033.	1031.	1027.	1027.	1022.	1012.	1017.	1017.	1014.	1017.	1019.
16				1027.	1024.	1019.	1019.	1019.	1018.	1019.	1020.
17				1028.	1024.	1020.	1021.	1020.	1019.	1020.	1021.
18				1026.	1023.	1014.	1020.	1020.	1016.	1021.	1022.
19				1026.	1024.	1021.	1022.	1022.	1021.	1022.	1022.

CCNTC. TABLE A- 6

MATRIX OF PRESSURE IN OIL RESERVOIR,(PSIA). BY ADEP

(TIME STEP = 20.00 DAYS)

TIME = 300. CAYS

J / I	1	2	3	4	5	6	7	8	9	10	11
1	941.	940.	938.	936.	934.	932.	932.	932.			
2	941.	939.	936.	935.	932.	930.	930.	931.			
3	940.	938.	933.	933.	930.	924.	928.	930.			
4	939.	937.	935.	933.	931.	928.	928.	929.			
5	938.	937.	934.	932.	930.	927.	928.	928.			
6	938.	936.	931.	931.	928.	921.	926.	927.			
7	938.	936.	933.	931.	929.	926.	926.	927.	927.	927.	927.
8	939.	937.	933.	931.	929.	926.	926.	926.	925.	926.	927.
9	940.	937.	931.	931.	928.	921.	924.	924.	921.	924.	926.
10	941.	938.	934.	932.	929.	926.	925.	925.	924.	925.	925.
11	942.	939.	935.	932.	929.	925.	925.	924.	923.	924.	925.
12	943.	940.	931.	932.	928.	921.	924.	924.	920.	924.	925.
13	944.	942.	937.	934.	930.	926.	925.	925.	924.	925.	926.
14	944.	942.	939.	936.	931.	926.	926.	926.	925.	926.	927.
15	944.	942.	936.	937.	931.	920.	925.	926.	922.	926.	928.
16				936.	934.	928.	928.	928.	927.	928.	929.
17				937.	933.	929.	929.	929.	928.	929.	930.
18				935.	932.	923.	929.	929.	925.	930.	931.
19				935.	933.	930.	931.	931.	930.	931.	932.

CCNTD. TABLE A- 6

MATRIX OF PRESSURE IN OIL RESERVOIR,(PSIA). BY ADEP

(TIME STEP = 20.00 DAYS)

TIME = 500. DAYS

J / I	1	2	3	4	5	6	7	8	9	10	11
1	851.	849.	847.	845.	843.	841.	841.	841.			
2	850.	848.	846.	844.	842.	839.	840.	840.			
3	849.	847.	842.	842.	840.	833.	838.	839.			
4	849.	847.	844.	842.	840.	837.	838.	839.			
5	848.	846.	843.	842.	839.	836.	837.	838.			
6	847.	845.	840.	840.	837.	830.	835.	836.			
7	848.	846.	843.	841.	838.	835.	836.	836.	836.	837.	837.
8	848.	846.	843.	841.	838.	835.	835.	835.	835.	835.	836.
9	849.	846.	840.	840.	837.	831.	834.	834.	830.	834.	835.
10	850.	848.	843.	841.	838.	835.	835.	834.	833.	834.	835.
11	851.	849.	844.	841.	838.	835.	834.	834.	833.	834.	834.
12	852.	849.	841.	841.	837.	831.	833.	833.	829.	833.	834.
13	853.	851.	846.	843.	839.	835.	835.	834.	833.	834.	835.
14	853.	852.	848.	845.	840.	835.	835.	835.	834.	835.	836.
15	853.	851.	845.	846.	840.	830.	835.	835.	832.	836.	837.
16				845.	843.	837.	837.	837.	836.	838.	838.
17				846.	843.	838.	839.	838.	838.	839.	839.
18				844.	841.	832.	838.	838.	835.	839.	840.
19				844.	842.	839.	840.	840.	839.	840.	841.

CCNTC. TABLE A- 6

MATRIX CF PRESSURE IN OIL RESERVOIR,(PSIA). BY ADEP

(TIME STEP = 20.00 DAYS)

TIME = 700. DAYS

J / I	1	2	3	4	5	6	7	8	9	10	11
1	760.	758.	756.	754.	752.	751.	750.	750.			
2	759.	757.	755.	753.	751.	749.	749.	750.			
3	758.	756.	751.	752.	749.	742.	747.	748.			
4	758.	756.	753.	752.	749.	746.	747.	748.			
5	757.	755.	753.	751.	748.	746.	746.	747.			
6	757.	754.	750.	750.	747.	740.	744.	745.			
7	757.	755.	752.	750.	748.	745.	745.	746.	745.	746.	746.
8	758.	755.	752.	750.	748.	745.	745.	745.	744.	745.	745.
9	758.	756.	749.	749.	747.	740.	743.	743.	740.	743.	744.
10	759.	757.	753.	750.	748.	744.	744.	743.	742.	743.	744.
11	760.	758.	753.	750.	747.	744.	744.	743.	742.	743.	744.
12	761.	758.	750.	750.	747.	740.	743.	742.	738.	742.	744.
13	762.	760.	756.	753.	748.	744.	744.	743.	742.	744.	745.
14	763.	761.	757.	754.	749.	745.	744.	744.	743.	745.	746.
15	762.	760.	754.	755.	749.	739.	744.	744.	741.	745.	746.
16				754.	752.	747.	747.	746.	746.	747.	748.
17				755.	752.	747.	748.	748.	747.	748.	749.
18				754.	750.	741.	747.	748.	744.	748.	749.
19				754.	752.	748.	749.	749.	748.	749.	750.

CCNTD. TABLE A- 6

MATRIX OF PRESSURE IN OIL RESERVOIR,(PSIA). BY ADEP

(TIME STEP = 20.00 DAYS)

TIME = 1000. DAYS

J / I	1	2	3	4	5	6	7	8	9	10	11
1	623.	622.	620.	618.	616.	615.	614.	614.			
2	623.	621.	619.	617.	615.	612.	613.	613.			
3	622.	620.	615.	615.	613.	606.	611.	612.			
4	622.	620.	617.	615.	613.	610.	611.	612.			
5	621.	619.	616.	615.	612.	609.	610.	611.			
6	620.	618.	613.	613.	611.	603.	608.	609.			
7	621.	619.	616.	614.	611.	609.	609.	609.	609.	610.	610.
8	621.	619.	616.	614.	611.	609.	609.	609.	608.	609.	609.
9	622.	619.	613.	613.	610.	604.	607.	607.	604.	607.	608.
10	623.	621.	616.	614.	611.	608.	608.	607.	606.	607.	608.
11	624.	621.	617.	614.	611.	608.	608.	607.	606.	607.	608.
12	625.	622.	614.	614.	610.	604.	607.	606.	602.	606.	608.
13	626.	624.	619.	616.	612.	608.	608.	607.	606.	608.	609.
14	626.	624.	621.	618.	613.	608.	608.	608.	607.	608.	609.
15	626.	624.	618.	619.	613.	603.	608.	608.	605.	609.	610.
16				618.	616.	611.	611.	610.	609.	611.	611.
17				619.	616.	611.	612.	612.	611.	612.	612.
18				617.	614.	605.	611.	611.	608.	612.	613.
19				617.	615.	612.	613.	613.	612.	613.	614.

CCNTC. TABLE A- 6

MATRIX OF PRESSURE IN OIL RESERVOIR,(PSIA). BY ADEP

(TIME STEP = 20.00 DAYS)

TIME = 1200. DAYS

I \ J	1	2	3	4	5	6	7	8	9	10	11
1	533.	531.	529.	527.	525.	524.	523.	523.			
2	532.	530.	528.	526.	524.	522.	522.	523.			
3	531.	529.	524.	525.	522.	515.	520.	521.			
4	531.	529.	526.	524.	522.	519.	520.	521.			
5	530.	528.	526.	524.	521.	519.	519.	520.			
6	529.	527.	523.	523.	520.	513.	517.	518.			
7	530.	528.	525.	523.	521.	518.	518.	519.	518.	519.	519.
8	530.	528.	525.	523.	521.	518.	518.	518.	517.	518.	518.
9	531.	528.	522.	522.	520.	513.	516.	516.	513.	516.	517.
10	532.	530.	526.	523.	521.	517.	517.	516.	515.	516.	517.
11	533.	531.	526.	523.	520.	517.	517.	516.	515.	516.	517.
12	534.	531.	523.	523.	520.	513.	516.	515.	511.	515.	517.
13	535.	533.	528.	526.	521.	517.	517.	516.	515.	517.	518.
14	535.	533.	530.	527.	522.	518.	517.	517.	516.	518.	519.
15	535.	533.	527.	528.	522.	512.	517.	517.	514.	518.	519.
16				527.	525.	520.	520.	519.	519.	520.	521.
17				528.	525.	520.	521.	521.	520.	521.	522.
18				526.	523.	514.	520.	521.	517.	521.	522.
19				526.	524.	521.	522.	522.	521.	522.	523.

TABLE A- 7

MATRIX OF PRESSURE IN OIL RESERVOIR,(PSIA). BY ADIP

(TIME STEP = 20.00 DAYS)

TIME = 100. DAYS

J / I	1	2	3	4	5	6	7	8	9	10	11
1	1029.	1028.	1026.	1024.	1022.	1020.	1020.	1020.			
2	1027.	1026.	1023.	1021.	1019.	1016.	1017.	1018.			
3	1027.	1025.	1020.	1021.	1018.	1011.	1016.	1018.			
4	1027.	1025.	1022.	1021.	1019.	1016.	1017.	1018.			
5	1026.	1025.	1022.	1020.	1018.	1014.	1016.	1017.			
6	1025.	1023.	1018.	1018.	1015.	1008.	1014.	1015.			
7	1026.	1024.	1021.	1019.	1017.	1013.	1015.	1015.	1015.	1015.	1015.
8	1028.	1026.	1022.	1021.	1019.	1015.	1016.	1016.	1015.	1016.	1016.
9	1027.	1024.	1018.	1018.	1015.	1008.	1012.	1012.	1008.	1012.	1013.
10	1030.	1027.	1022.	1020.	1017.	1013.	1013.	1013.	1011.	1013.	1013.
11	1031.	1029.	1024.	1022.	1019.	1015.	1015.	1014.	1013.	1014.	1015.
12	1032.	1029.	1020.	1020.	1017.	1010.	1013.	1012.	1008.	1012.	1013.
13	1034.	1032.	1027.	1024.	1020.	1015.	1015.	1014.	1013.	1014.	1015.
14	1036.	1034.	1031.	1027.	1022.	1016.	1016.	1015.	1014.	1016.	1017.
15	1035.	1034.	1031.	1028.	1021.	1009.	1014.	1014.	1011.	1015.	1016.
16				1031.	1025.	1017.	1018.	1018.	1016.	1018.	1019.
17				1027.	1024.	1018.	1019.	1019.	1017.	1018.	1019.
18				1025.	1021.	1011.	1018.	1018.	1014.	1018.	1020.
19				1025.	1023.	1019.	1021.	1021.	1019.	1021.	1021.

CCNTD. TABLE A- 7

MATRIX OF PRESSURE IN OIL RESERVOIR,(PSIA). BY ADIP

(TIME STEP = 20.00 DAYS)

TIME = 300. DAYS

I \ J	1	2	3	4	5	6	7	8	9	10	11
1	941.	940.	938.	936.	934.	933.	932.	932.			
2	940.	939.	936.	934.	932.	930.	931.	931.			
3	940.	938.	933.	933.	931.	924.	929.	931.			
4	940.	938.	935.	933.	931.	928.	929.	930.			
5	939.	937.	935.	933.	930.	928.	929.	929.			
6	938.	936.	931.	932.	929.	922.	927.	929.			
7	939.	937.	934.	932.	930.	927.	928.	928.	929.	928.	928.
8	940.	938.	935.	933.	930.	927.	928.	928.	927.	928.	928.
9	941.	938.	932.	932.	929.	922.	926.	925.	922.	926.	927.
10	943.	940.	936.	933.	930.	927.	927.	926.	925.	926.	926.
11	944.	942.	937.	934.	931.	927.	927.	926.	925.	926.	927.
12	946.	943.	934.	934.	930.	923.	926.	925.	921.	925.	926.
13	948.	946.	941.	938.	933.	928.	927.	927.	925.	927.	928.
14	949.	948.	944.	940.	934.	929.	928.	927.	926.	928.	929.
15	949.	948.	946.	942.	935.	923.	928.	927.	924.	928.	929.
16				946.	938.	931.	930.	930.	929.	930.	931.
17				941.	936.	931.	931.	931.	930.	931.	931.
18				938.	934.	925.	931.	931.	927.	931.	932.
19				938.	936.	932.	933.	932.	931.	932.	933.

CONTD. TABLE A- 7

MATRIX OF PRESSURE IN OIL RESERVOIR,(PSIA). BY ADIP

(TIME STEP = 20.00 DAYS)

TIME = 500. DAYS

I \ J	1	2	3	4	5	6	7	8	9	10	11
1	854.	853.	851.	849.	847.	845.	845.	845.			
2	853.	852.	849.	848.	846.	843.	844.	844.			
3	853.	851.	846.	846.	844.	837.	842.	844.			
4	852.	851.	848.	846.	844.	841.	842.	843.			
5	852.	850.	847.	846.	843.	841.	842.	842.			
6	851.	849.	844.	845.	842.	835.	840.	842.			
7	852.	850.	847.	845.	843.	840.	841.	841.	842.	841.	841.
8	853.	851.	847.	845.	843.	840.	840.	840.	840.	840.	840.
9	854.	851.	845.	845.	842.	836.	839.	839.	835.	839.	840.
10	856.	853.	849.	846.	843.	840.	840.	839.	838.	839.	840.
11	857.	855.	850.	847.	844.	840.	840.	839.	838.	839.	840.
12	859.	856.	847.	847.	843.	836.	839.	838.	834.	838.	840.
13	861.	858.	854.	851.	846.	841.	840.	840.	838.	840.	841.
14	862.	860.	857.	853.	847.	842.	841.	840.	839.	841.	842.
15	862.	861.	858.	855.	848.	836.	841.	840.	837.	841.	842.
16				858.	851.	844.	843.	843.	842.	843.	844.
17				854.	849.	844.	844.	844.	843.	844.	844.
18				851.	847.	838.	843.	844.	840.	844.	845.
19				851.	848.	845.	845.	845.	844.	845.	846.

CONTD. TABLE A- 7

MATRIX OF PRESSURE IN OIL RESERVOIR,(PSIA). BY ADIP

(TIME STEP = 20.00 DAYS)

TIME = 700. DAYS

J / I	1	2	3	4	5	6	7	8	9	10	11
1	767.	766.	764.	762.	760.	758.	758.	758.			
2	766.	765.	762.	761.	759.	756.	757.	757.			
3	766.	764.	759.	759.	757.	750.	755.	756.			
4	765.	763.	761.	759.	757.	754.	755.	756.			
5	765.	763.	760.	759.	756.	754.	755.	755.			
6	764.	762.	757.	758.	755.	748.	753.	755.			
7	765.	763.	760.	758.	756.	753.	754.	754.	755.	754.	754.
8	766.	764.	760.	758.	756.	753.	753.	753.	753.	753.	753.
9	767.	764.	758.	758.	755.	749.	752.	752.	748.	752.	753.
10	768.	766.	762.	759.	756.	753.	753.	752.	751.	752.	753.
11	770.	767.	763.	760.	757.	753.	753.	752.	751.	752.	752.
12	772.	768.	760.	760.	756.	749.	752.	751.	747.	751.	752.
13	773.	771.	767.	763.	758.	754.	753.	753.	751.	753.	754.
14	775.	773.	770.	766.	760.	755.	754.	753.	752.	754.	754.
15	775.	774.	771.	768.	761.	749.	754.	753.	750.	754.	755.
16				771.	763.	757.	756.	756.	755.	756.	756.
17				767.	762.	757.	757.	757.	756.	757.	757.
18				764.	760.	751.	756.	757.	753.	757.	758.
19				764.	761.	758.	758.	758.	757.	758.	759.

CCNTC. TABLE A- 7

MATRIX OF PRESSURE IN OIL RESERVOIR,(PSIA). BY ADIP

(TIME STEP = 20.00 DAYS)

TIME = 1000. DAYS

I \ J	1	2	3	4	5	6	7	8	9	10	11
1	636.	635.	633.	631.	629.	627.	627.	627.			
2	636.	634.	631.	630.	628.	625.	626.	627.			
3	635.	633.	628.	628.	626.	619.	624.	626.			
4	634.	633.	630.	628.	626.	624.	624.	625.			
5	634.	632.	630.	628.	626.	623.	624.	625.			
6	633.	631.	627.	627.	624.	617.	622.	624.			
7	634.	632.	629.	627.	625.	622.	623.	624.	624.	624.	624.
8	635.	633.	629.	627.	625.	622.	623.	622.	622.	622.	623.
9	636.	633.	627.	627.	624.	618.	621.	621.	618.	621.	622.
10	638.	635.	631.	628.	626.	622.	622.	621.	620.	621.	622.
11	639.	636.	632.	629.	626.	623.	622.	621.	620.	621.	622.
12	641.	638.	629.	629.	625.	619.	621.	621.	616.	620.	622.
13	642.	640.	636.	633.	628.	623.	623.	622.	621.	622.	623.
14	644.	642.	639.	635.	629.	624.	623.	623.	622.	623.	624.
15	644.	643.	640.	637.	630.	619.	623.	623.	619.	623.	624.
16				640.	633.	626.	626.	625.	624.	625.	626.
17				636.	631.	626.	627.	626.	625.	626.	627.
18				633.	629.	620.	626.	626.	622.	626.	627.
19				633.	631.	627.	628.	627.	626.	628.	628.

CCNTC. TABLE A- 7

MATRIX OF PRESSURE IN OIL RESERVOIR,(PSIA). BY ADIP

(TIME STEP = 20.00 DAYS)

TIME = 1200. DAYS

J / I	1	2	3	4	5	6	7	8	9	10	11
1	549.	547.	546.	544.	542.	540.	540.	540.			
2	548.	547.	544.	543.	541.	538.	539.	539.			
3	548.	545.	541.	541.	539.	532.	537.	538.			
4	547.	545.	543.	541.	539.	536.	537.	538.			
5	547.	545.	542.	541.	538.	536.	537.	537.			
6	546.	544.	539.	540.	537.	530.	535.	537.			
7	547.	545.	542.	540.	538.	535.	536.	536.	537.	536.	536.
8	548.	546.	542.	540.	538.	535.	535.	535.	535.	535.	535.
9	549.	546.	540.	540.	537.	531.	534.	534.	530.	534.	535.
10	550.	548.	544.	541.	538.	535.	535.	534.	533.	534.	535.
11	552.	549.	545.	542.	539.	535.	535.	534.	533.	534.	535.
12	553.	550.	542.	542.	538.	531.	534.	533.	529.	533.	535.
13	555.	553.	549.	545.	541.	536.	535.	535.	534.	535.	536.
14	556.	555.	552.	548.	542.	537.	536.	535.	534.	536.	537.
15	557.	556.	553.	550.	543.	531.	536.	536.	532.	536.	537.
16				553.	545.	539.	538.	538.	537.	538.	539.
17				548.	544.	539.	539.	539.	538.	539.	539.
18				546.	542.	533.	538.	539.	535.	539.	540.
19				545.	543.	540.	540.	540.	539.	540.	541.

TABLE A-8

PRODUCTION PATTERN OF OIL RESERVOIR

LOCATION OF WELL I	J	PRODUCTION Q(I,J) S.T.B./DAY
1	1	50.
15	1	50.
3	3	75.
6	3	75.
9	3	75.
12	3	75.
15	4	50.
19	4	50.
3	6	125.
6	6	125.
9	6	125.
12	6	125.
15	6	125.
18	6	125.
1	8	50.
7	8	50.
9	9	75.
12	9	75.
15	9	75.
18	9	75.
7	11	50.
19	11	50.

TABLE A-9

MATRIX OF PRESSURE IN OIL RESERVOIR,(PSIA). BY ADEP

WITH EIGHT WELLS AT CORNERS

TIME = 100. DAYS

I \ J	1	2	3	4	5	6	7	8	9	10	11
1	1008.	1010.	1010.	1009.	1007.	1005.	1003.	998.			
2	1011.	1011.	1009.	1009.	1007.	1004.	1003.	1002.			
3	1013.	1011.	1007.	1008.	1005.	999.	1002.	1003.			
4	1015.	1013.	1010.	1009.	1006.	1003.	1003.	1003.			
5	1015.	1013.	1010.	1009.	1006.	1003.	1003.	1002.			
6	1015.	1013.	1009.	1008.	1005.	998.	1001.	999.			
7	1017.	1015.	1011.	1009.	1007.	1003.	1002.	999.	999.	999.	993.
8	1018.	1016.	1012.	1010.	1007.	1004.	1003.	1002.	1001.	1001.	999.
9	1019.	1016.	1010.	1010.	1007.	1001.	1003.	1002.	999.	1001.	1001.
10	1020.	1018.	1013.	1011.	1008.	1005.	1004.	1003.	1002.	1003.	1003.
11	1020.	1018.	1014.	1011.	1008.	1005.	1005.	1004.	1002.	1003.	1004.
12	1020.	1017.	1010.	1010.	1007.	1001.	1004.	1003.	1000.	1003.	1004.
13	1019.	1017.	1014.	1012.	1009.	1005.	1005.	1004.	1003.	1004.	1005.
14	1016.	1016.	1013.	1011.	1008.	1004.	1005.	1005.	1003.	1005.	1005.
15	1009.	1012.	1009.	1007.	1006.	998.	1004.	1004.	1001.	1004.	1005.
16				1009.	1009.	1005.	1006.	1006.	1004.	1005.	1005.
17				1009.	1008.	1005.	1006.	1006.	1004.	1004.	1004.
18				1005.	1004.	997.	1005.	1005.	1000.	1002.	1001.
19				1000.	1004.	1004.	1006.	1006.	1003.	1001.	996.

CONTD. TABLE A-9

MATRIX OF PRESSURE IN OIL RESERVOIR,(PSIA). BY ADEP

WITH EIGHT WELLS AT CORNERS

TIME = 300. DAYS

J / I	1	2	3	4	5	6	7	8	9	10	11
1	894.	896.	896.	894.	891.	888.	885.	879.			
2	898.	897.	895.	894.	891.	887.	886.	884.			
3	900.	898.	893.	893.	889.	882.	885.	885.			
4	901.	899.	896.	893.	890.	887.	886.	886.			
5	901.	899.	896.	894.	890.	886.	885.	885.			
6	901.	899.	893.	893.	889.	880.	883.	881.			
7	902.	900.	896.	894.	891.	886.	885.	881.	881.	880.	873.
8	903.	901.	897.	894.	891.	887.	886.	884.	883.	882.	879.
9	905.	901.	895.	894.	891.	883.	885.	884.	880.	882.	882.
10	905.	903.	898.	895.	892.	888.	887.	885.	883.	884.	884.
11	905.	903.	898.	895.	891.	888.	887.	885.	884.	884.	885.
12	904.	902.	893.	894.	890.	883.	886.	885.	880.	884.	885.
13	903.	901.	897.	895.	891.	887.	887.	886.	885.	886.	887.
14	900.	899.	897.	894.	891.	887.	887.	886.	885.	887.	887.
15	893.	896.	891.	890.	889.	880.	886.	886.	883.	886.	888.
16				891.	891.	887.	888.	888.	887.	887.	888.
17				892.	890.	887.	888.	888.	887.	887.	887.
18				887.	886.	879.	887.	887.	883.	885.	884.
19				881.	885.	885.	888.	888.	886.	884.	880.

CONTD. TABLE A-9

MATRIX OF PRESSURE IN OIL RESERVOIR,(PSIA). BY ADEP

WITH EIGHT WELLS AT CORNERS

TIME = 500. DAYS

J / I	1	2	3	4	5	6	7	8	9	10	11
1	777.	779.	778.	777.	774.	771.	767.	762.			
2	781.	780.	778.	776.	773.	769.	768.	766.			
3	782.	780.	775.	775.	772.	764.	767.	768.			
4	783.	781.	778.	776.	773.	769.	769.	768.			
5	784.	781.	778.	776.	773.	769.	768.	767.			
6	784.	781.	776.	775.	772.	763.	766.	764.			
7	785.	782.	779.	776.	773.	769.	767.	764.	764.	763.	755.
8	786.	783.	779.	777.	773.	770.	768.	766.	765.	764.	762.
9	787.	784.	777.	776.	773.	766.	768.	766.	762.	764.	764.
10	788.	785.	780.	777.	774.	770.	769.	768.	766.	766.	766.
11	787.	785.	780.	777.	774.	770.	769.	768.	766.	766.	767.
12	787.	784.	776.	776.	772.	765.	768.	767.	763.	766.	768.
13	785.	784.	780.	777.	773.	769.	769.	768.	767.	768.	769.
14	782.	782.	779.	776.	773.	769.	769.	769.	768.	769.	769.
15	775.	778.	773.	772.	771.	762.	768.	768.	765.	768.	770.
16				773.	773.	769.	770.	770.	769.	769.	770.
17				774.	772.	769.	770.	770.	769.	769.	769.
18				769.	768.	761.	769.	769.	765.	767.	767.
19				764.	768.	767.	770.	770.	768.	767.	762.

CONTD. TABLE A-9

MATRIX OF PRESSURE IN OIL RESERVOIR,(PSIA). BY ADEP

WITH EIGHT WELLS AT CORNERS

TIME = 700. DAYS

J / I	1	2	3	4	5	6	7	8	9	10	11
1	659.	661.	661.	659.	656.	653.	650.	644.			
2	663.	662.	660.	658.	655.	652.	650.	649.			
3	664.	662.	658.	657.	654.	647.	650.	650.			
4	665.	663.	660.	658.	655.	651.	651.	651.			
5	666.	664.	661.	658.	655.	651.	650.	649.			
6	666.	663.	658.	658.	654.	645.	648.	646.			
7	667.	665.	661.	659.	655.	651.	649.	646.	646.	645.	638.
8	668.	665.	662.	659.	656.	652.	650.	649.	647.	646.	644.
9	669.	666.	659.	658.	655.	648.	650.	649.	644.	647.	647.
10	670.	667.	662.	659.	656.	652.	651.	650.	648.	648.	648.
11	669.	667.	662.	659.	656.	652.	651.	650.	648.	649.	649.
12	669.	666.	658.	658.	655.	648.	650.	649.	645.	649.	650.
13	667.	666.	662.	659.	655.	652.	651.	650.	649.	650.	651.
14	664.	664.	661.	658.	655.	651.	651.	651.	650.	651.	652.
15	657.	660.	656.	655.	653.	645.	650.	650.	647.	651.	652.
16				656.	655.	651.	652.	652.	651.	652.	652.
17				656.	654.	651.	653.	653.	651.	651.	651.
18				651.	650.	644.	651.	652.	647.	650.	649.
19				646.	650.	650.	652.	653.	650.	649.	644.

CCNTD. TABLE A-9

MATRIX OF PRESSURE IN OIL RESERVOIR,(PSIA). BY ADEP

WITH EIGHT WELLS AT CORNERS

TIME = 900. DAYS

J / I	1	2	3	4	5	6	7	8	9	10	11
1	541.	543.	543.	541.	538.	535.	532.	527.			
2	545.	544.	542.	540.	538.	534.	533.	531.			
3	546.	544.	540.	540.	536.	529.	532.	532.			
4	547.	545.	542.	540.	537.	534.	533.	533.			
5	548.	546.	543.	540.	537.	533.	532.	532.			
6	548.	545.	540.	540.	536.	528.	530.	528.			
7	549.	547.	543.	541.	537.	533.	532.	528.	528.	527.	520.
8	550.	547.	544.	541.	538.	534.	532.	531.	530.	528.	526.
9	551.	548.	541.	541.	537.	530.	532.	531.	526.	529.	529.
10	552.	549.	544.	541.	538.	535.	533.	532.	530.	530.	530.
11	551.	549.	544.	541.	538.	534.	533.	532.	530.	531.	531.
12	551.	548.	540.	540.	537.	530.	532.	531.	527.	531.	532.
13	549.	548.	544.	541.	538.	534.	533.	533.	531.	532.	533.
14	546.	546.	543.	540.	537.	533.	533.	533.	532.	533.	534.
15	539.	542.	538.	537.	535.	527.	532.	532.	529.	533.	534.
16				538.	538.	534.	534.	534.	533.	534.	534.
17				538.	536.	533.	535.	535.	533.	534.	533.
18				533.	532.	526.	533.	534.	529.	532.	531.
19				528.	532.	532.	534.	535.	533.	531.	526.

TABLE A-10

MATERIAL BALANCE OF ADEP SOLUTION:

OIL RESERVOIR WITH EIGHT CORNER WELLS

$\Delta t = 33 * \Delta t_{max}$.

Time Days	Actual Cumulative Production / Estimated Cumulative Production
10	1.0008
30	1.0181
100	1.0082
200	1.0025
300	1.0008
400	1.0001
500	0.9998
600	0.9996
800	0.9996
900	0.9997
950	0.9997

APPENDIX B

GAS RESERVOIR

P, μ, Z Interrelations

The following curve-fitted emperical relations satisfactorily represent the gas properties.

$$P = 0.800 \left(\frac{P}{Z}\right) + 80.00; \quad P \text{ is in psia}$$

$$\frac{P}{\mu Z} = 74.74 \left(\frac{P}{Z}\right) + 6815; \quad \mu \text{ is in cp}$$

The reservoir temperature was assumed constant at 80°F and the gas saturation was assumed to be unity throughout the reservoir.

Δt_{max} for the FDE Method

If the Forward Difference Explicit Method were used to solve equation (V-4), the maximum time step consistent with stability is given [5], for the case of uniform properties, by:

$$\Delta t_{max} = \frac{1}{4} \frac{\overline{\Delta x}^2 \phi h \quad S_g \mu}{K_g h \, P}$$

For a heterogeneous medium, Δt_{max} is estimated by employing the most severe field conditions, that is, $(\mu/P)_{min}$ and $(\phi h/K_g h)_{min}$. This occurs, in this case, at i = 11, j = 11.

Then, Δt_{max} = 6.8 days.

TABLE B- 1

PHI*H MATRIX (FT) - GAS RESERVOIR

J / I	1	2	3	4	5	6	7	8	9	10	11
1	1.43	0.97	0.80	0.73	0.75	0.58	0.51	0.43			
2	1.43	0.95	0.88	0.81	0.80	0.80	0.58	0.47			
3	1.82	1.10	0.87	0.90	0.83	0.73	0.62	0.52			
4	1.65	0.99	1.02	0.87	1.00	0.72	0.67	0.60			
5	1.30	1.09	0.96	0.83	0.77	0.74	0.74	0.71			
6	1.00	0.95	0.90	0.82	0.76	0.73	0.68	0.67			
7	0.98	0.93	0.88	0.84	1.03	0.75	0.67	0.64	0.60	0.42	0.28
8	1.43	1.10	0.97	0.82	1.20	0.84	0.83	0.73	0.72	0.46	0.32
9	1.65	1.42	1.42	1.01	1.25	1.02	0.77	0.66	0.67	0.53	0.30
10	1.66	1.41	0.80	1.09	1.16	1.06	0.68	0.68	0.62	0.50	0.30
11	1.33	1.51	1.03	1.02	1.10	0.80	0.80	0.60	0.45	0.37	0.27
12	1.65	1.28	1.10	0.89	1.02	0.84	0.84	0.58	0.37	0.30	0.26
13	1.57	1.32	1.25	1.24	0.88	0.77	0.77	0.60	0.40	0.40	0.41
14	1.62	1.25	1.22	1.15	1.05	0.79	0.73	0.63	0.52	0.55	0.52
15	0.73	0.90	0.97	1.40	1.00	1.02	0.79	0.71	0.61	0.68	0.58
16				1.10	0.91	1.03	1.20	0.92	0.76	0.67	0.61
17				0.76	0.88	1.12	1.27	1.18	1.20	0.71	0.64
18				0.77	1.20	1.35	1.31	1.27	1.22	0.81	0.68
19				1.10	1.47	1.37	1.27	1.23	1.12	1.00	0.64

TABLE B- 2

K*H MATRIX (DARCY-FT) - GAS RESERVOIR

I \ J	1	2	3	4	5	6	7	8	9	10	11
1	0.17	0.21	0.17	0.16	0.19	0.15	0.16	0.14			
2	0.18	0.21	0.19	0.18	0.19	0.20	0.16	0.15			
3	0.23	0.24	0.19	0.20	0.19	0.17	0.15	0.14			
4	0.20	0.22	0.23	0.20	0.23	0.17	0.16	0.15			
5	0.16	0.25	0.21	0.18	0.17	0.17	0.17	0.17			
6	0.21	0.22	0.20	0.18	0.17	0.16	0.15	0.15			
7	0.19	0.20	0.20	0.19	0.23	0.17	0.15	0.15	0.14	0.12	0.10
8	0.17	0.17	0.21	0.18	0.27	0.19	0.19	0.16	0.17	0.13	0.11
9	0.18	0.16	0.17	0.22	0.25	0.18	0.18	0.15	0.16	0.15	0.13
10	0.18	0.16	0.17	0.21	0.22	0.21	0.17	0.18	0.16	0.16	0.13
11	0.14	0.17	0.13	0.23	0.23	0.24	0.22	0.19	0.16	0.14	0.13
12	0.17	0.13	0.12	0.12	0.23	0.20	0.23	0.18	0.14	0.12	0.11
13	0.17	0.13	0.13	0.14	0.14	0.17	0.20	0.17	0.13	0.14	0.14
14	0.19	0.13	0.12	0.12	0.12	0.17	0.17	0.16	0.15	0.15	0.14
15	0.10	0.11	0.10	0.14	0.11	0.13	0.17	0.17	0.15	0.17	0.15
16				0.11	0.09	0.11	0.14	0.13	0.16	0.15	0.15
17				0.10	0.09	0.11	0.13	0.14	0.15	0.15	0.15
18				0.10	0.13	0.13	0.13	0.14	0.14	0.13	0.15
19				0.13	0.16	0.13	0.12	0.12	0.13	0.14	0.14

TABLE B-3

PRODUCTION PATTERN OF GAS RESERVOIR

Location of Well (i,j)	Vol. Rate $(V_g)_{i,j}$ mmscf/Day	Location of Well (i,j)	Vol. Rate $(V_g)_{i,j}$ mmscf/Day
2,2	0.8	8,6	0.8
4,2	0.9	10,6	1.0
6,2	0.8	12,6	0.8
8,2	1.0	14,6	0.8
10,2	1.2	16,6	1.0
12,2	1.1	18,6	1.2
14,2	1.0	8,8	0.7
2,4	0.6	10,8	0.7
4,4	0.8	12,8	0.6
6,4	0.7	14,8	0.6
8,4	0.8	16,8	1.0
10,4	1.0	18,8	1.0
12,4	0.9	8,10	0.5
14,4	1.0	10,10	0.5
16,4	0.5	12,10	0.4
18,4	0.5	14,10	0.5
2,6	0.8	16,10	0.8
4,6	0.9	18,10	0.8
6,6	0.8		

TABLE B- 4

MATRIX OF PRESSURE IN GAS RESERVOIR,(PSIA). BY ADEP

(TIME STEP = 30. DAYS)

TIME = 120. DAYS

J / I	1	2	3	4	5	6	7	8	9	10	11
1	989.	977.	978.	973.	974.	969.	980.	987.			
2	982.	951.	967.	949.	962.	941.	973.	985.			
3	984.	967.	969.	961.	965.	958.	975.	985.			
4	979.	945.	961.	938.	956.	930.	970.	984.			
5	981.	964.	967.	958.	962.	956.	974.	984.			
6	974.	945.	960.	939.	956.	932.	969.	980.			
7	978.	962.	966.	958.	964.	956.	968.	969.	971.	967.	971.
8	977.	935.	959.	937.	959.	935.	956.	936.	959.	939.	962.
9	984.	965.	970.	958.	965.	956.	961.	953.	961.	955.	962.
10	978.	928.	955.	932.	955.	929.	950.	930.	951.	936.	956.
11	984.	964.	968.	955.	962.	951.	957.	949.	954.	950.	958.
12	979.	927.	962.	926.	956.	933.	952.	933.	948.	934.	957.
13	986.	968.	976.	962.	966.	953.	959.	951.	956.	953.	963.
14	980.	934.	966.	923.	960.	929.	953.	933.	952.	939.	963.
15	987.	976.	982.	969.	974.	957.	962.	951.	958.	954.	967.
16				945.	964.	915.	957.	918.	951.	927.	962.
17				966.	975.	960.	973.	960.	969.	957.	970.
18				942.	968.	919.	965.	929.	965.	934.	969.
19				980.	986.	977.	985.	978.	983.	976.	981.

CONTD. TABLE B- 4

MATRIX OF PRESSURE IN GAS RESERVOIR,(PSIA). BY ADEP

(TIME STEP = 30. DAYS)

TIME = 360. DAYS

I \ J	1	2	3	4	5	6	7	8	9	10	11
1	940.	922.	917.	907.	907.	903.	917.	926.			
2	929.	892.	902.	878.	892.	871.	908.	923.			
3	929.	905.	901.	888.	891.	886.	909.	922.			
4	919.	878.	888.	860.	878.	852.	901.	919.			
5	916.	895.	892.	879.	883.	878.	903.	917.			
6	906.	871.	883.	855.	873.	848.	892.	907.			
7	909.	888.	887.	875.	879.	871.	885.	888.	888.	882.	886.
8	909.	858.	878.	850.	871.	845.	868.	847.	871.	849.	874.
9	919.	891.	888.	871.	875.	864.	869.	861.	870.	863.	871.
10	912.	849.	871.	840.	863.	831.	854.	831.	856.	840.	863.
11	919.	889.	885.	865.	869.	855.	860.	851.	858.	854.	864.
12	916.	851.	880.	834.	862.	835.	855.	833.	851.	836.	862.
13	927.	899.	900.	876.	875.	858.	862.	853.	859.	857.	870.
14	922.	864.	891.	835.	870.	832.	856.	834.	856.	843.	871.
15	930.	913.	912.	890.	889.	864.	868.	855.	865.	862.	878.
16				862.	880.	819.	864.	821.	860.	836.	876.
17				889.	897.	875.	888.	874.	886.	875.	890.
18				870.	896.	838.	887.	848.	889.	857.	896.
19				919.	924.	911.	920.	911.	916.	908.	913.

CCNTD. TABLE B- 4

MATRIX OF PRESSURE IN GAS RESERVOIR,(PSIA). BY ADEP

(TIME STEP = 30. DAYS)

TIME = 600. DAYS

I \ J	1	2	3	4	5	6	7	8	9	10	11
1	881.	861.	854.	841.	840.	835.	850.	860.			
2	869.	828.	837.	809.	823.	799.	839.	856.			
3	867.	841.	834.	819.	820.	814.	839.	854.			
4	854.	808.	819.	786.	805.	776.	828.	848.			
5	849.	824.	820.	804.	807.	801.	828.	843.			
6	836.	797.	808.	776.	794.	766.	814.	829.			
7	838.	813.	810.	795.	798.	788.	803.	805.	803.	795.	799.
8	837.	780.	799.	765.	787.	757.	782.	757.	783.	758.	785.
9	848.	815.	809.	787.	790.	776.	781.	771.	780.	772.	781.
10	840.	769.	790.	752.	775.	738.	763.	736.	764.	745.	771.
11	849.	813.	805.	779.	781.	764.	769.	758.	765.	761.	772.
12	847.	773.	801.	745.	773.	741.	762.	737.	757.	740.	770.
13	860.	828.	824.	792.	788.	767.	771.	760.	767.	764.	779.
14	856.	791.	815.	748.	783.	738.	765.	739.	764.	749.	781.
15	865.	844.	838.	810.	806.	776.	779.	764.	775.	773.	790.
16				779.	797.	727.	777.	728.	773.	746.	791.
17				812.	820.	793.	807.	791.	804.	792.	809.
18				795.	822.	757.	810.	766.	811.	775.	818.
19				850.	855.	840.	848.	837.	843.	833.	838.

CONTD. TABLE B- 4

MATRIX OF PRESSURE IN GAS RESERVOIR,(PSIA). BY ADEP

(TIME STEP = 30. DAYS)

TIME = 960. DAYS

J / I	1	2	3	4	5	6	7	8	9	10	11
1	789.	765.	756.	741.	738.	731.	747.	758.			
2	774.	727.	736.	704.	718.	690.	735.	753.			
3	771.	740.	732.	713.	714.	706.	733.	749.			
4	755.	702.	713.	674.	694.	659.	719.	742.			
5	747.	718.	713.	693.	695.	687.	716.	733.			
6	730.	685.	696.	657.	677.	642.	697.	715.			
7	730.	702.	697.	677.	679.	666.	682.	683.	677.	666.	670.
8	728.	661.	682.	640.	664.	626.	655.	623.	653.	622.	654.
9	740.	701.	693.	664.	667.	648.	652.	639.	649.	639.	649.
10	731.	647.	670.	621.	648.	601.	630.	596.	628.	606.	636.
11	741.	699.	687.	654.	654.	632.	636.	623.	630.	624.	638.
12	741.	653.	683.	612.	644.	604.	628.	597.	620.	599.	635.
13	756.	718.	710.	670.	662.	635.	638.	625.	632.	629.	647.
14	752.	676.	701.	618.	657.	601.	632.	600.	630.	612.	650.
15	762.	737.	728.	692.	684.	647.	650.	632.	645.	642.	662.
16				656.	676.	591.	650.	590.	644.	611.	665.
17				696.	704.	673.	688.	668.	682.	668.	688.
18				678.	709.	632.	693.	640.	692.	649.	699.
19				743.	748.	730.	738.	724.	730.	719.	724.

CONTD. TABLE B- 4

MATRIX OF PRESSURE IN GAS RESERVOIR,(PSIA). BY ADEP

(TIME STEP = 30. DAYS)

TIME = 1200. DAYS

I \ J	1	2	3	4	5	6	7	8	9	10	11
1	726.	700.	690.	673.	669.	661.	679.	691.			
2	710.	658.	668.	632.	647.	615.	665.	685.			
3	706.	672.	663.	642.	643.	633.	663.	680.			
4	688.	629.	641.	597.	620.	580.	647.	671.			
5	678.	647.	641.	618.	620.	611.	643.	661.			
6	659.	609.	621.	577.	599.	559.	621.	640.			
7	659.	627.	621.	599.	601.	586.	603.	603.	596.	582.	587.
8	656.	580.	604.	556.	583.	539.	572.	534.	567.	531.	568.
9	668.	625.	616.	583.	585.	564.	568.	552.	563.	551.	563.
10	658.	564.	589.	534.	563.	508.	542.	502.	540.	512.	548.
11	669.	623.	609.	571.	571.	545.	550.	533.	542.	535.	550.
12	669.	571.	604.	523.	559.	512.	540.	503.	530.	505.	547.
13	686.	643.	634.	589.	580.	548.	552.	536.	544.	541.	561.
14	681.	597.	625.	530.	574.	508.	545.	507.	541.	520.	565.
15	693.	665.	654.	614.	605.	563.	565.	544.	559.	555.	578.
16				573.	596.	497.	565.	496.	558.	519.	582.
17				618.	627.	592.	609.	586.	602.	585.	608.
18				598.	634.	546.	614.	554.	612.	564.	620.
19				671.	676.	656.	665.	649.	655.	642.	648.

CONTD. TABLE B- 4

MATRIX OF PRESSURE IN GAS RESERVOIR,(PSIA). BY ADEP

(TIME STEP = 30. DAYS)

TIME = 1440. DAYS

I \ J	1	2	3	4	5	6	7	8	9	10	11
1	663.	635.	624.	605.	601.	592.	611.	624.			
2	646.	588.	600.	559.	576.	540.	595.	617.			
3	641.	604.	594.	570.	571.	560.	593.	612.			
4	622.	556.	569.	519.	545.	499.	575.	601.			
5	610.	575.	569.	543.	546.	534.	570.	590.			
6	589.	532.	547.	496.	521.	474.	545.	566.			
7	588.	552.	546.	521.	523.	505.	525.	524.	516.	500.	505.
8	584.	499.	527.	471.	502.	451.	489.	444.	483.	440.	484.
9	597.	550.	539.	503.	505.	480.	485.	466.	478.	464.	478.
10	586.	479.	509.	444.	480.	414.	455.	406.	451.	418.	461.
11	599.	547.	532.	488.	488.	458.	464.	444.	454.	445.	463.
12	598.	487.	526.	431.	475.	418.	452.	407.	440.	409.	460.
13	617.	570.	559.	509.	498.	462.	466.	447.	457.	452.	475.
14	612.	517.	549.	439.	491.	414.	458.	412.	453.	427.	480.
15	624.	594.	581.	536.	527.	478.	481.	457.	474.	469.	496.
16				490.	516.	400.	481.	398.	472.	425.	499.
17				541.	552.	512.	530.	504.	522.	502.	528.
18				517.	558.	457.	536.	466.	534.	477.	542.
19				599.	606.	583.	592.	575.	581.	567.	573.

TABLE B- 5

MATRIX OF PRESSURE IN GAS RESERVOIR,(PSIA). BY ADEP

(TIME STEP = 60. DAYS)

TIME = 120. DAYS

J / I	1	2	3	4	5	6	7	8	9	10	11
1	990.	978.	979.	974.	976.	971.	981.	987.			
2	982.	952.	967.	950.	964.	943.	974.	985.			
3	985.	968.	969.	962.	966.	959.	976.	985.			
4	980.	946.	961.	939.	956.	930.	970.	984.			
5	981.	965.	967.	958.	963.	956.	974.	983.			
6	975.	946.	961.	939.	956.	932.	968.	979.			
7	979.	963.	966.	959.	964.	956.	967.	968.	972.	968.	972.
8	978.	936.	959.	937.	959.	935.	956.	937.	959.	940.	963.
9	985.	966.	970.	958.	965.	956.	961.	954.	961.	956.	963.
10	979.	928.	956.	932.	955.	929.	950.	930.	952.	937.	957.
11	985.	965.	968.	955.	962.	951.	957.	950.	955.	951.	959.
12	980.	927.	963.	926.	956.	934.	953.	934.	949.	935.	957.
13	987.	968.	977.	962.	966.	954.	959.	951.	956.	953.	963.
14	981.	934.	966.	923.	960.	929.	953.	933.	952.	939.	963.
15	988.	977.	982.	969.	974.	957.	962.	951.	958.	954.	967.
16				944.	964.	915.	957.	918.	951.	927.	962.
17				966.	975.	960.	973.	960.	969.	957.	970.
18				941.	967.	919.	965.	929.	965.	934.	969.
19				979.	985.	977.	985.	978.	983.	975.	980.

CONTD. TABLE B- 5

MATRIX OF PRESSURE IN GAS RESERVOIR,(PSIA). BY ADEP

(TIME STEP = 60. DAYS)

TIME = 360. DAYS

J I	1	2	3	4	5	6	7	8	9	10	11
1	940.	923.	918.	908.	908.	904.	918.	927.			
2	930.	893.	903.	879.	893.	872.	909.	924.			
3	929.	906.	902.	889.	892.	886.	909.	923.			
4	920.	878.	889.	861.	879.	853.	901.	920.			
5	917.	895.	893.	879.	883.	878.	903.	917.			
6	907.	872.	883.	856.	873.	848.	892.	907.			
7	910.	888.	887.	875.	880.	871.	886.	888.	888.	882.	886.
8	910.	859.	878.	850.	871.	845.	869.	847.	871.	850.	875.
9	920.	891.	889.	871.	876.	864.	869.	861.	870.	864.	872.
10	912.	849.	872.	840.	863.	831.	855.	832.	856.	840.	863.
11	919.	889.	885.	865.	869.	856.	861.	852.	858.	854.	864.
12	917.	851.	881.	834.	862.	835.	855.	833.	851.	836.	862.
13	927.	899.	900.	876.	875.	858.	863.	853.	859.	857.	870.
14	922.	865.	891.	835.	870.	832.	857.	834.	856.	843.	871.
15	931.	913.	912.	890.	889.	864.	868.	855.	865.	862.	878.
16				862.	880.	819.	864.	821.	860.	836.	876.
17				889.	897.	874.	889.	874.	886.	874.	890.
18				870.	896.	837.	887.	848.	888.	856.	895.
19				919.	924.	911.	919.	910.	916.	908.	912.

CONTD. TABLE B- 5

MATRIX OF PRESSURE IN GAS RESERVOIR,(PSIA). BY ADEP

(TIME STEP = 60. DAYS)

TIME = 600. DAYS

J / I	1	2	3	4	5	6	7	8	9	10	11
1	882.	862.	854.	842.	840.	835.	850.	860.			
2	870.	828.	838.	810.	823.	800.	840.	856.			
3	867.	841.	835.	819.	821.	815.	839.	854.			
4	855.	809.	819.	786.	805.	776.	829.	849.			
5	850.	825.	821.	805.	808.	802.	828.	843.			
6	836.	798.	808.	776.	795.	766.	814.	830.			
7	838.	814.	811.	795.	799.	789.	804.	805.	803.	795.	799.
8	838.	780.	800.	766.	787.	757.	782.	757.	783.	758.	786.
9	848.	815.	810.	787.	791.	776.	781.	771.	780.	773.	782.
10	841.	769.	791.	752.	776.	738.	763.	737.	764.	746.	771.
11	849.	814.	806.	780.	782.	765.	769.	759.	766.	761.	772.
12	848.	774.	802.	745.	774.	742.	763.	738.	757.	740.	770.
13	860.	828.	824.	793.	788.	767.	771.	760.	767.	765.	779.
14	856.	792.	816.	749.	783.	739.	765.	739.	764.	750.	782.
15	865.	845.	839.	810.	806.	776.	779.	764.	776.	773.	790.
16				779.	797.	728.	777.	728.	773.	746.	791.
17				812.	819.	793.	807.	791.	804.	792.	809.
18				794.	822.	757.	810.	766.	810.	775.	818.
19				850.	855.	840.	848.	837.	842.	833.	838.

CONTD. TABLE B- 5

MATRIX OF PRESSURE IN GAS RESERVOIR,(PSIA). BY ADEP

(TIME STEP = 60. DAYS)

TIME = 960. DAYS

I \ J	1	2	3	4	5	6	7	8	9	10	11
1	789.	766.	757.	741.	738.	732.	748.	759.			
2	775.	727.	737.	704.	718.	690.	735.	754.			
3	771.	741.	733.	714.	715.	706.	734.	750.			
4	756.	702.	713.	674.	695.	660.	720.	742.			
5	747.	719.	713.	693.	696.	687.	717.	734.			
6	730.	685.	697.	658.	678.	643.	698.	715.			
7	731.	702.	697.	678.	680.	667.	683.	683.	678.	667.	671.
8	729.	661.	683.	641.	665.	627.	656.	624.	653.	622.	655.
9	740.	701.	693.	665.	667.	649.	653.	640.	649.	639.	650.
10	731.	647.	670.	622.	648.	601.	630.	597.	629.	606.	637.
11	741.	700.	688.	654.	654.	633.	637.	623.	631.	625.	638.
12	741.	654.	683.	613.	645.	604.	629.	597.	621.	600.	636.
13	756.	718.	710.	670.	662.	635.	639.	625.	633.	630.	647.
14	752.	676.	701.	619.	657.	601.	632.	600.	630.	612.	651.
15	762.	738.	728.	692.	685.	647.	650.	632.	645.	642.	662.
16				656.	676.	591.	650.	590.	644.	611.	665.
17				696.	704.	673.	688.	668.	682.	668.	688.
18				678.	709.	632.	692.	640.	692.	649.	699.
19				743.	748.	730.	738.	724.	730.	718.	723.

CONTD. TABLE B- 5

MATRIX OF PRESSURE IN GAS RESERVOIR,(PSIA). BY ADEP

(TIME STEP = 60. DAYS)

TIME = 1200. DAYS

I \ J	1	2	3	4	5	6	7	8	9	10	11
1	727.	701.	691.	673.	670.	662.	679.	691.			
2	711.	658.	669.	632.	648.	616.	666.	685.			
3	706.	673.	664.	643.	643.	633.	663.	681.			
4	689.	630.	642.	598.	620.	580.	647.	672.			
5	679.	647.	641.	619.	621.	611.	643.	661.			
6	660.	609.	622.	578.	600.	560.	621.	640.			
7	659.	628.	622.	600.	602.	586.	604.	603.	596.	583.	587.
8	657.	581.	605.	557.	584.	540.	572.	535.	568.	532.	569.
9	668.	626.	616.	584.	586.	564.	568.	553.	563.	552.	563.
10	659.	564.	590.	534.	564.	509.	543.	503.	540.	513.	549.
11	670.	623.	610.	572.	571.	546.	550.	534.	542.	535.	550.
12	670.	572.	605.	523.	560.	512.	540.	503.	531.	505.	547.
13	686.	644.	634.	589.	580.	549.	552.	536.	545.	541.	561.
14	682.	597.	625.	530.	574.	509.	545.	507.	542.	521.	565.
15	693.	666.	654.	614.	605.	563.	565.	545.	559.	555.	579.
16				573.	596.	498.	565.	496.	558.	519.	582.
17				618.	627.	592.	609.	586.	602.	585.	608.
18				598.	634.	546.	614.	554.	612.	564.	620.
19				671.	676.	656.	664.	649.	655.	642.	647.

CONTD. TABLE B- 5

MATRIX OF PRESSURE IN GAS RESERVOIR,(PSIA). BY ADEP

(TIME STEP = 60. DAYS)

TIME = 1440. DAYS

I \ J	1	2	3	4	5	6	7	8	9	10	11
1	664.	636.	624.	605.	601.	592.	611.	624.			
2	646.	589.	600.	559.	576.	540.	596.	618.			
3	641.	605.	594.	571.	572.	560.	593.	613.			
4	622.	556.	570.	520.	546.	500.	575.	602.			
5	611.	576.	569.	544.	546.	535.	571.	590.			
6	589.	533.	547.	497.	522.	475.	545.	567.			
7	589.	553.	547.	521.	524.	506.	525.	525.	516.	500.	506.
8	585.	499.	527.	472.	503.	451.	489.	445.	483.	440.	484.
9	598.	550.	540.	503.	505.	481.	485.	467.	479.	465.	478.
10	587.	479.	510.	445.	480.	415.	455.	407.	452.	418.	461.
11	599.	547.	532.	489.	488.	459.	464.	445.	454.	446.	463.
12	599.	488.	527.	431.	475.	419.	453.	407.	441.	410.	460.
13	617.	570.	560.	509.	498.	462.	466.	447.	457.	452.	476.
14	612.	517.	549.	440.	491.	414.	458.	412.	454.	428.	480.
15	624.	594.	582.	536.	527.	479.	482.	457.	474.	469.	496.
16				490.	516.	401.	481.	398.	472.	426.	499.
17				541.	552.	512.	530.	504.	522.	502.	528.
18				517.	558.	457.	536.	466.	534.	477.	542.
19				599.	605.	583.	592.	575.	581.	566.	573.

TABLE B-6

PRODUCTION PATTERN OF GAS RESERVOIR

WITH SEVEN CORNER WELLS

Location of Wells (i,j)	Vol. Rate, $(V_g)_{i,j}$ mmscf/Day	Location of Wells (i,j)	Vol. Rate, $(V_g)_{i,j}$ mmscf/Day
* 1,1	0.5	8,6	0.8
*15,1	0.5	10,6	1.0
4,2	0.9	12,6	0.8
6,2	0.8	14,6	0.8
8,2	1.0	16,6	1.0
10,2	1.2	18.6	1.2
12,2	1.1	* 1,8	0.5
2,4	0.6	* 7,8	0.5
4,4	0.8	10,8	0.7
6,4	0.7	12,8	0.6
8,4	0.8	14,8	0.6
10,4	1.0	16,8	1.0
12,4	0.9	18,8	1.0
14,4	1.0	10,10	0.5
16,4	0.5	12,10	0.4
*19,4	0.5	14,10	0.5
2,6	0.8	16,10	0.8
4,6	0.9	* 7,11	0.5
6,6	0.8	*19,11	0.5

* Corner Wells

TABLE B-7

MATRIX OF PRESSURE IN GAS RESERVOIR,(PSIA). BY ADEP

WITH SEVEN WELLS AT CORNERS

TIME = 120. DAYS

J / I	1	2	3	4	5	6	7	8	9	10	11
1	958.	979.	982.	975.	973.	962.	956.	921.			
2	983.	982.	977.	952.	963.	938.	961.	959.			
3	987.	977.	975.	964.	966.	957.	970.	975.			
4	981.	949.	963.	940.	957.	930.	968.	979.			
5	982.	966.	968.	959.	963.	955.	972.	980.			
6	975.	946.	961.	939.	956.	931.	965.	971.			
7	979.	963.	966.	959.	965.	956.	965.	951.	966.	953.	896.
8	978.	936.	960.	938.	959.	938.	965.	969.	974.	966.	950.
9	985.	966.	970.	958.	965.	958.	966.	965.	971.	965.	963.
10	979.	928.	956.	932.	956.	929.	953.	934.	956.	941.	959.
11	985.	965.	969.	955.	962.	952.	958.	951.	957.	953.	961.
12	981.	930.	964.	927.	956.	934.	953.	934.	950.	936.	958.
13	990.	981.	981.	963.	966.	954.	959.	951.	957.	953.	964.
14	984.	988.	979.	926.	960.	929.	953.	933.	952.	939.	963.
15	940.	981.	986.	971.	974.	957.	962.	951.	959.	956.	968.
16				947.	965.	915.	957.	919.	954.	931.	964.
17				977.	979.	961.	974.	962.	975.	969.	973.
18				975.	974.	920.	965.	932.	976.	977.	968.
19				947.	979.	976.	985.	978.	985.	974.	934.

CONTD. TABLE B-7

MATRIX OF PRESSURE IN GAS RESERVOIR,(PSIA). BY ADEP

WITH SEVEN WELLS AT CORNERS

TIME = 360. DAYS

I \ J	1	2	3	4	5	6	7	8	9	10	11
1	906.	926.	923.	907.	898.	879.	868.	829.			
2	933.	928.	916.	882.	886.	853.	875.	873.			
3	936.	920.	910.	891.	888.	875.	889.	895.			
4	925.	885.	894.	862.	876.	845.	889.	904.			
5	920.	899.	896.	880.	882.	874.	895.	906.			
6	909.	874.	885.	857.	873.	846.	886.	896.			
7	911.	889.	888.	876.	881.	872.	883.	869.	884.	865.	799.
8	911.	859.	879.	852.	873.	850.	881.	887.	891.	879.	859.
9	920.	892.	890.	873.	878.	869.	879.	879.	885.	877.	873.
10	914.	851.	873.	842.	866.	835.	861.	842.	866.	849.	868.
11	923.	893.	889.	868.	872.	859.	865.	858.	865.	860.	869.
12	924.	861.	888.	838.	865.	838.	858.	837.	856.	841.	867.
13	939.	924.	915.	883.	879.	861.	865.	856.	863.	861.	874.
14	934.	935.	918.	846.	875.	835.	859.	837.	860.	847.	876.
15	886.	927.	926.	898.	894.	867.	871.	859.	870.	868.	884.
16				871.	885.	822.	867.	826.	868.	847.	885.
17				905.	904.	878.	892.	881.	898.	895.	900.
18				905.	902.	840.	890.	856.	908.	910.	899.
19				874.	911.	908.	921.	915.	924.	911.	864.

CONTD. TABLE B-7

MATRIX OF PRESSURE IN GAS RESERVOIR,(PSIA). BY ADEP

WITH SEVEN WELLS AT CORNERS

TIME = 600. DAYS

J / I	1	2	3	4	5	6	7	8	9	10	11
1	846.	865.	859.	839.	825.	802.	788.	743.			
2	875.	867.	851.	810.	811.	773.	796.	793.			
3	877.	858.	844.	820.	813.	796.	811.	817.			
4	863.	818.	825.	787.	800.	764.	811.	826.			
5	855.	830.	825.	805.	805.	794.	816.	828.			
6	840.	801.	811.	778.	794.	762.	806.	815.			
7	841.	816.	813.	797.	800.	789.	801.	785.	799.	777.	703.
8	840.	783.	802.	769.	791.	764.	797.	803.	806.	793.	770.
9	850.	818.	813.	791.	795.	784.	794.	793.	799.	790.	785.
10	844.	774.	795.	757.	781.	745.	773.	750.	777.	758.	779.
11	856.	821.	812.	786.	787.	771.	777.	768.	775.	771.	780.
12	860.	790.	814.	754.	780.	748.	769.	745.	765.	749.	778.
13	880.	861.	846.	806.	796.	774.	777.	767.	774.	772.	787.
14	876.	875.	851.	767.	793.	745.	771.	746.	772.	758.	790.
15	825.	867.	860.	824.	815.	783.	785.	772.	785.	784.	801.
16				793.	807.	734.	784.	738.	786.	762.	805.
17				831.	829.	800.	814.	802.	822.	819.	825.
18				832.	829.	760.	816.	779.	835.	838.	827.
19				800.	840.	836.	851.	846.	855.	841.	790.

CONTD. TABLE B-7

MATRIX OF PRESSURE IN GAS RESERVOIR,(PSIA). BY ADEP

WITH SEVEN WELLS AT CORNERS

TIME = 960. DAYS

I \ J	1	2	3	4	5	6	7	8	9	10	11
1	750.	770.	761.	735.	717.	689.	671.	617.			
2	781.	771.	751.	702.	701.	654.	680.	675.			
3	782.	759.	742.	712.	702.	681.	696.	703.			
4	765.	713.	719.	673.	686.	642.	695.	712.			
5	754.	725.	717.	693.	690.	676.	700.	713.			
6	735.	690.	700.	659.	676.	637.	687.	697.			
7	735.	706.	701.	681.	682.	668.	680.	658.	674.	647.	555.
8	733.	666.	687.	646.	670.	636.	674.	679.	682.	666.	638.
9	745.	707.	699.	671.	674.	659.	670.	667.	673.	661.	656.
10	739.	656.	678.	631.	657.	613.	645.	616.	647.	623.	649.
11	754.	713.	700.	665.	665.	644.	649.	637.	645.	639.	651.
12	761.	678.	703.	628.	657.	616.	640.	610.	634.	613.	648.
13	785.	762.	741.	691.	677.	647.	650.	637.	645.	643.	660.
14	782.	779.	748.	646.	673.	614.	644.	613.	643.	627.	664.
15	725.	770.	759.	714.	700.	660.	662.	645.	660.	659.	679.
16				676.	690.	603.	661.	606.	663.	635.	685.
17				721.	718.	683.	699.	685.	708.	704.	711.
18				722.	719.	638.	702.	659.	725.	728.	714.
19				686.	732.	727.	744.	738.	748.	732.	673.

CCNTC. TABLE B-7

MATRIX OF PRESSURE IN GAS RESERVOIR,(PSIA). BY ADEP

WITH SEVEN WELLS AT CORNERS

TIME = 1200. DAYS

I \ J	1	2	3	4	5	6	7	8	9	10	11
1	683.	705.	695.	666.	646.	614.	594.	532.			
2	717.	706.	684.	629.	627.	574.	603.	598.			
3	718.	693.	674.	640.	629.	604.	621.	628.			
4	699.	641.	648.	596.	610.	559.	619.	638.			
5	686.	654.	646.	618.	615.	598.	624.	638.			
6	665.	615.	626.	579.	598.	553.	608.	620.			
7	665.	633.	626.	603.	604.	587.	600.	575.	593.	562.	453.
8	662.	587.	611.	563.	590.	551.	594.	598.	601.	582.	551.
9	676.	633.	624.	592.	595.	577.	589.	585.	591.	578.	572.
10	669.	576.	601.	546.	575.	524.	560.	526.	562.	534.	563.
11	686.	640.	625.	586.	584.	560.	565.	551.	560.	553.	566.
12	694.	602.	629.	543.	575.	528.	555.	520.	547.	523.	563.
13	721.	695.	671.	615.	598.	564.	567.	551.	560.	557.	577.
14	718.	714.	679.	564.	594.	526.	560.	524.	559.	539.	582.
15	656.	704.	691.	640.	624.	578.	580.	561.	578.	576.	599.
16				598.	614.	514.	580.	517.	582.	549.	606.
17				647.	644.	605.	623.	606.	632.	627.	635.
18				649.	645.	555.	627.	577.	651.	654.	639.
19				609.	660.	655.	672.	666.	677.	659.	593.

CCNTC. TABLE B-7

MATRIX CF PRESSURE IN GAS RESERVOIR,(PSIA). BY ADEP

WITH SEVEN WELLS AT CORNERS

TIME = 1440. DAYS

I \ J	1	2	3	4	5	6	7	8	9	10	11
1	616.	639.	629.	597.	574.	539.	516.	443.			
2	653.	641.	617.	555.	554.	493.	526.	520.			
3	654.	626.	605.	568.	555.	528.	546.	554.			
4	633.	569.	576.	518.	534.	475.	543.	565.			
5	619.	584.	574.	543.	539.	520.	549.	564.			
6	596.	539.	552.	498.	519.	467.	531.	543.			
7	595.	559.	552.	526.	527.	507.	521.	492.	512.	477.	344.
8	592.	507.	534.	479.	511.	464.	514.	519.	521.	501.	464.
9	607.	560.	549.	513.	517.	496.	508.	504.	510.	495.	489.
10	599.	495.	523.	459.	494.	433.	476.	435.	477.	444.	479.
11	618.	568.	550.	506.	504.	476.	482.	465.	475.	466.	482.
12	628.	525.	555.	457.	494.	438.	470.	428.	460.	431.	479.
13	656.	628.	602.	539.	520.	481.	484.	466.	476.	472.	495.
14	653.	649.	611.	481.	515.	436.	476.	433.	474.	450.	501.
15	584.	638.	624.	567.	549.	498.	500.	477.	497.	494.	520.
16				519.	537.	422.	500.	424.	500.	462.	528.
17				575.	571.	527.	547.	528.	557.	551.	560.
18				577.	572.	469.	551.	495.	578.	582.	565.
19				531.	588.	583.	602.	594.	607.	587.	512.

TABLE B-8

MATERIAL BALANCE

GAS RESERVOIR WITH SEVEN CORNER WELLS

$\Delta t = 60$ days

Time Days	Actual Production / Estimated Production
120	1.007
240	1.004
480	1.002
840	1.001
1560	1.000

APPENDIX C

WATER-INJECTED OIL-RESERVOIR

1. Emperical correlations of capillary pressure data:

$$P_c = \frac{0.01299}{(S_w-0.14)^2} - \frac{0.05031}{(S_w-0.14)} + 7.0702 \qquad 0.14 \leq S_w \leq 0.20$$

$$P_c = \frac{0.3742}{S_w^{\ 2}} - \frac{0.8208}{S_w} + 3.9248 \qquad 0.20 < S_w \leq 0.50$$

$$P_c = 5.132 - 2.704\ S_w \qquad 0.50 < S_w < 0.712$$

$$P_c = -9.1247 + 37.348\ S_w - 28.13\ S_w^{\ 2} \qquad 0.712 \leq S_w \leq 1.0$$

P_c in psia; S_w is fractional water saturation.

2. Relative permeability relations:

$$k_{rw} = \left(\frac{S_w - 0.14}{0.86}\right)^4 \qquad 0.14 \leq S_w \leq 1.0$$

$$k_{ro} = \left(\frac{0.86 - S_w}{0.86}\right)^4 \qquad 0.14 \leq S_w \leq 0.86$$

For the purpose of iterative procedure, the following conditions are also imposed:

$k_{rw} = 0$ for $S_w < 0.14$

$k_{ro} = 0$ for $S_w > 0.86$

3. <u>Fluid Properties</u>

Compressibility of water	$c_w = 0.4248 \times 10^{-4} (\text{psia})^{-1}$
Compressibility of oil	$c_o = 0.7968 \times 10^{-5} (\text{psia})^{-1}$
Viscosity of water	$\mu_w = 0.4732$ cp
Viscosity of oil	$\mu_o = 1.170$ cp

4. <u>Initial guess vector</u>

$$Z_1^{(o)} = P_{w_{i,j}}^{(n)} - 5$$

$$Z_2^{(o)} = P_{o_{i,j}}^{(n)} - 5$$

$$Z_3^{(o)} = S_{w_{i,j}}^{(n)} + 5 \times 10^{-4}$$

5. <u>Criteria to terminate iteration</u>

$$\left| Z_1^{(\ell+1)} - Z_1^{(\ell)} \right| \leq \varepsilon_1 = 10^{-4} \text{ psia}$$

$$\left| Z_2^{(\ell+1)} - Z_2^{(\ell)} \right| \leq \varepsilon_2 = 10^{-4} \text{ psia}$$

$$\left| Z_3^{(\ell+1)} - Z_3^{(\ell)} \right| \leq \varepsilon_3 = 10^{-7}$$

TABLE C-1

MATRIX OF PHI*H (POROSITY*FORMATION HEIGHT) (FT)

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	2.430	2.430	2.447	2.448	2.417	2.386	2.340	2.309	2.308	2.363	2.417	1.509	1.432	2.622	2.658	2.673	2.673
2	2.430	2.430	2.447	2.448	2.417	2.386	2.340	2.309	2.308	2.363	2.417	1.509	1.432	2.622	2.658	2.673	2.673
3	2.494	2.494	2.511	2.512	2.480	2.418	2.356	2.309	2.308	1.493	2.417	1.495	1.432	2.596	2.611	2.632	2.632
4	2.541	2.541	2.558	2.560	2.512	2.418	2.348	2.309	2.308	2.308	2.417	2.482	2.544	2.608	2.630	2.638	2.638
5	2.605	2.605	2.623	2.608	2.552	2.433	2.356	2.309	2.308	2.342	2.419	2.457	2.520	2.572	2.601	2.650	2.650
6	2.652	2.652	2.638	2.607	2.560	2.433	2.356	2.294	2.292	2.320	2.405	2.445	2.513	2.552	2.590	2.613	2.613
7	2.649	2.649	2.627	2.582	2.526	2.433	2.340	2.294	2.293	2.329	2.392	2.448	2.506	2.563	2.574	2.631	2.631
8	2.642	2.642	2.616	2.588	2.542	2.433	2.332	2.278	2.255	2.305	2.363	2.453	2.511	2.572	2.617	2.789	2.789
9	2.655	2.655	2.630	2.603	2.526	2.417	2.340	2.294	2.263	2.307	2.396	2.457	2.540	2.625	2.655	2.715	2.715
10	2.685	2.685	2.645	2.586	2.517	2.432	2.371	2.340	2.339	2.377	2.468	2.552	2.630	2.693	2.739	2.782	2.782
11	2.718	2.718	2.652	2.586	2.541	2.487	2.464	2.433	2.432	2.462	2.555	2.633	2.712	2.785	2.839	2.898	2.898
12	2.769	2.769	2.695	2.593	2.540	2.493	2.478	2.494	2.518	2.573	2.635	2.732	2.812	2.878	2.949	3.010	3.010
13	2.769	2.769	2.675	2.550	2.520	2.490	2.475	2.475	2.507	2.586	2.716	2.832	2.898	3.013	3.027	3.021	3.021
14	2.759	2.759	2.670	2.563	2.516	2.453	2.438	2.470	2.501	2.565	2.712	2.872	2.999	3.102	3.062	2.778	2.778
15	2.718	2.718	2.667	2.555	2.462	2.414	2.400	2.414	2.445	2.540	2.661	2.769	2.876	2.910	2.839	2.766	2.766
16	2.640	2.640	2.587	2.506	2.442	2.408	2.391	2.405	2.436	2.517	2.628	2.694	2.745	2.730	2.712	2.661	2.661
17	2.640	2.640	2.587	2.506	2.442	2.408	2.391	2.405	2.436	2.517	2.628	2.694	2.745	2.730	2.712	2.661	2.661

TABLE C - 2

MATRIX OF K*H (PERMEABILITY*FORMATION HEIGHT) (DARCY-FT)

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0.073	0.073	0.071	0.068	0.064	0.061	0.059	0.057	0.058	0.065	0.074	0.050	0.051	0.093	0.091	0.090	0.090
2	0.073	0.073	0.071	0.068	0.064	0.061	0.059	0.057	0.058	0.065	0.074	0.050	0.051	0.093	0.091	0.090	0.090
3	0.070	0.070	0.068	0.067	0.066	0.063	0.059	0.057	0.058	0.041	0.075	0.053	0.054	0.098	0.096	0.094	0.094
4	0.082	0.082	0.077	0.068	0.066	0.064	0.060	0.057	0.058	0.058	0.074	0.089	0.100	0.104	0.101	0.098	0.098
5	0.093	0.093	0.086	0.081	0.077	0.070	0.062	0.057	0.058	0.063	0.074	0.087	0.103	0.107	0.106	0.105	0.105
6	0.104	0.104	0.097	0.089	0.081	0.071	0.062	0.059	0.058	0.062	0.073	0.087	0.106	0.110	0.111	0.111	0.111
7	0.119	0.119	0.106	0.096	0.087	0.079	0.067	0.059	0.058	0.061	0.074	0.095	0.111	0.120	0.117	0.094	0.094
8	0.135	0.135	0.120	0.108	0.097	0.084	0.073	0.060	0.058	0.061	0.077	0.102	0.121	0.128	0.125	0.126	0.126
9	0.152	0.152	0.138	0.142	0.109	0.095	0.082	0.070	0.059	0.063	0.090	0.115	0.133	0.140	0.136	0.132	0.132
10	0.166	0.166	0.157	0.140	0.125	0.110	0.096	0.083	0.072	0.081	0.105	0.132	0.142	0.148	0.147	0.143	0.143
11	0.180	0.180	0.171	0.157	0.145	0.129	0.117	0.108	0.102	0.112	0.130	0.144	0.155	0.159	0.157	0.153	0.153
12	0.185	0.185	0.178	0.170	0.166	0.149	0.139	0.130	0.130	0.133	0.145	0.157	0.169	0.173	0.171	0.164	0.164
13	0.188	0.188	0.187	0.182	0.180	0.174	0.165	0.158	0.153	0.155	0.168	0.177	0.184	0.192	0.180	0.154	0.154
14	0.196	0.196	0.196	0.193	0.187	0.181	0.180	0.181	0.183	0.185	0.189	0.194	0.199	0.203	0.189	0.174	0.174
15	0.189	0.189	0.190	0.185	0.171	0.160	0.152	0.152	0.153	0.158	0.169	0.180	0.186	0.188	0.178	0.166	0.166
16	0.181	0.181	0.180	0.174	0.158	0.146	0.155	0.135	0.134	0.139	0.148	0.158	0.148	0.147	0.144	0.136	0.136
17	0.181	0.181	0.180	0.174	0.158	0.146	0.155	0.135	0.134	0.139	0.148	0.158	0.148	0.147	0.144	0.136	0.136

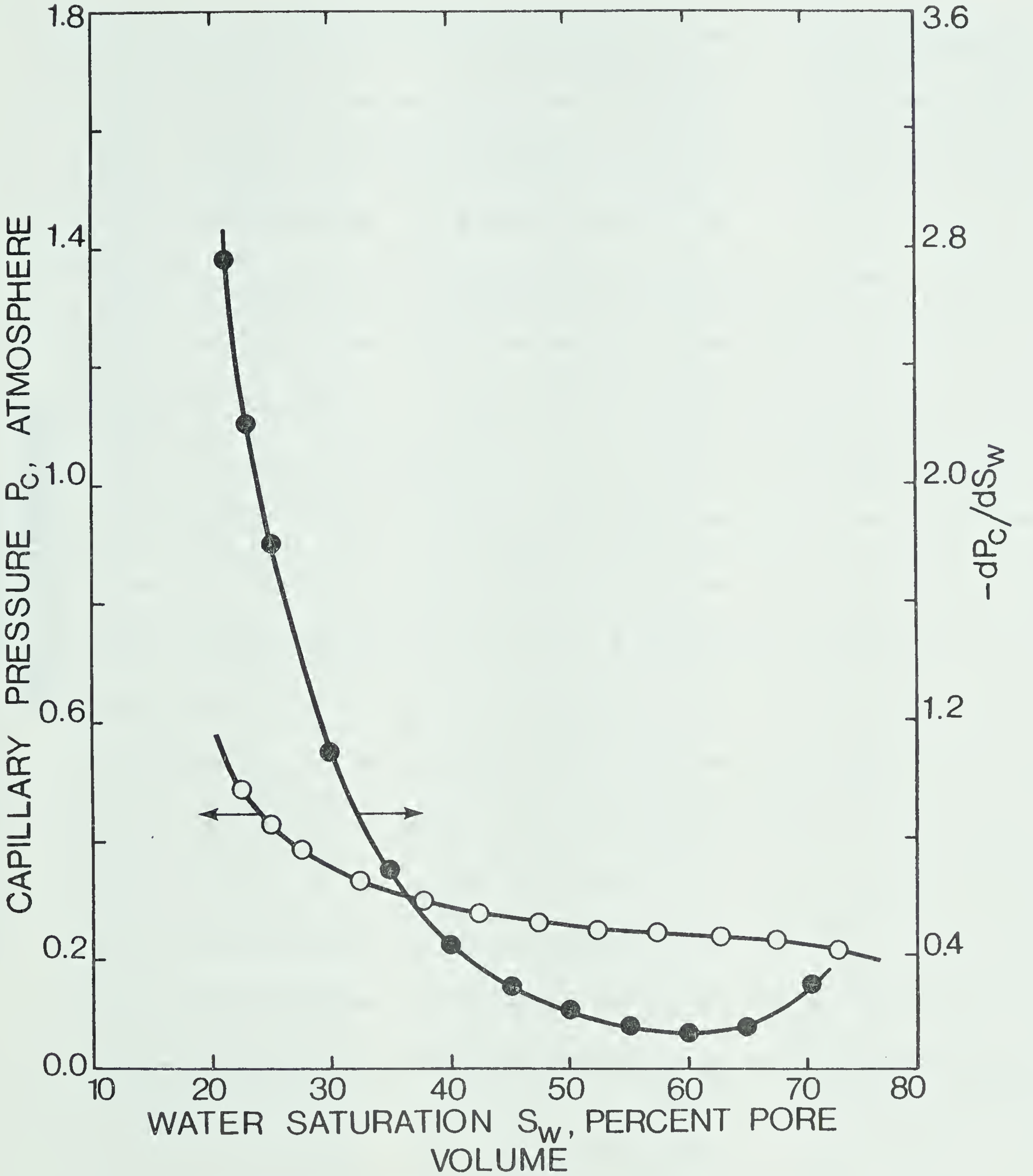

FIGURE C-1: CAPILLARY PRESSURE DATA

TABLE C-3

MATRIX OF ITERATIONS

J / I	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	3	3	3	3	3	3	3	3	3	3	3	3	4	3	3	3	3
2	3	3	3	4	3	3	3	3	3	3	3	4	3	4	3	3	3
3	3	3	4	3	4	3	4	3	3	3	3	4	3	3	4	3	3
4	3	4	3	4	3	4	3	3	3	3	4	3	3	4	3	4	4
5	3	4	4	3	4	4	3	3	3	3	3	4	3	3	4	3	4
6	3	4	4	4	5	3	3	3	3	3	3	4	5	4	4	3	4
7	3	3	4	4	3	3	4	4	4	3	3	3	3	3	3	3	3
8	3	3	3	3	3	3	4	4	4	3	4	4	4	3	3	3	3
9	3	3	3	3	3	3	4	4	5	4	4	4	4	4	3	3	3
10	3	3	3	3	3	3	3	3	4	4	4	4	4	4	3	3	3
11	3	3	3	4	3	3	3	4	4	4	4	4	4	4	4	3	3
12	3	3	4	3	4	4	3	4	4	4	4	4	5	4	3	3	4
13	3	4	3	3	3	4	3	3	4	4	4	4	3	3	4	3	3
14	3	4	3	3	3	4	3	3	3	4	4	4	3	3	3	3	3
15	3	3	4	3	4	3	3	3	3	4	4	4	4	3	3	3	3
16	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
17	3	3	3	3	3	3	3	3	3	3	3	4	3	3	3	3	3

TABLE C-4

Time = 320 Days

WATER PRESSURE MATRIX (PSIA)

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	2993.	2995.	2997.	2982.	2933.	2868.	2809.	2767.	2747.	2752.	2774.	2819.	2883.	2921.	2928.	2923.	2920.
2	2991.	2996.	3006.	2998.	2940.	2864.	2800.	2755.	2736.	2742.	2766.	2815.	2888.	2933.	2935.	2923.	2918.
3	2980.	2993.	3035.	3153.	2959.	2848.	2771.	2720.	2698.	2708.	2741.	2798.	2902.	3053.	2953.	2918.	2908.
4	2946.	2965.	3133.	3748.	3049.	2801.	2714.	2656.	2631.	2644.	2692.	2759.	2968.	3449.	3027.	2893.	2879.
5	2883.	2891.	2917.	3036.	2815.	2714.	2625.	2556.	2529.	2552.	2612.	2687.	2766.	2944.	2846.	2831.	2826.
6	2811.	2810.	2803.	2773.	2704.	2610.	2508.	2416.	2374.	2420.	2506.	2593.	2668.	2724.	2753.	2763.	2766.
7	2752.	2746.	2728.	2686.	2613.	2509.	2375.	2225.	2127.	2235.	2383.	2501.	2587.	2646.	2682.	2701.	2707.
8	2709.	2702.	2678.	2631.	2553.	2431.	2245.	1970.	1665.	1989.	2261.	2427.	2529.	2594.	2635.	2657.	2664.
9	2686.	2678.	2652.	2606.	2528.	2398.	2175.	1724.	532.	1745.	2193.	2392.	2501.	2569.	2611.	2635.	2642.
10	2679.	2672.	2649.	2607.	2537.	2430.	2270.	2038.	1775.	2049.	2274.	2416.	2509.	2570.	2609.	2630.	2637.
11	2687.	2682.	2663.	2627.	2570.	2489.	2387.	2274.	2202.	2274.	2377.	2467.	2539.	2591.	2625.	2641.	2646.
12	2705.	2703.	2692.	2663.	2614.	2550.	2481.	2418.	2386.	2411.	2465.	2526.	2583.	2629.	2655.	2664.	2666.
13	2729.	2731.	2737.	2795.	2660.	2601.	2547.	2505.	2486.	2498.	2532.	2579.	2633.	2746.	2697.	2695.	2693.
14	2747.	2753.	2834.	3046.	2784.	2640.	2591.	2557.	2543.	2551.	2577.	2618.	2741.	3068.	2800.	2723.	2716.
15	2754.	2757.	2769.	2834.	2717.	2662.	2618.	2589.	2576.	2582.	2604.	2640.	2687.	2797.	2740.	2729.	2724.
16	2752.	2752.	2752.	2743.	2711.	2670.	2634.	2609.	2597.	2601.	2619.	2647.	2683.	2717.	2727.	2726.	2725.
17	2751.	2751.	2748.	2736.	2708.	2672.	2639.	2615.	2604.	2607.	2624.	2649.	2681.	2709.	2721.	2724.	2724.

OIL PRESSURE MATRIX (PSIA)

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	3052.	3054.	3056.	3040.	2989.	2922.	2861.	2817.	2797.	2801.	2825.	2871.	2938.	2977.	2985.	2979.	2976.
2	3050.	3055.	3066.	3057.	2996.	2918.	2852.	2805.	2785.	2792.	2816.	2867.	2943.	2990.	2991.	2979.	2974.
3	3038.	3052.	3096.	3159.	3017.	2901.	2822.	2769.	2746.	2756.	2790.	2850.	2957.	3059.	3010.	2974.	2963.
4	3003.	3022.	3139.	3750.	3055.	2852.	2762.	2702.	2677.	2690.	2740.	2809.	2974.	3451.	3033.	2948.	2933.
5	2938.	2946.	2972.	3041.	2867.	2763.	2670.	2600.	2572.	2596.	2658.	2734.	2816.	2949.	2899.	2884.	2878.
6	2863.	2862.	2854.	2823.	2752.	2656.	2550.	2457.	2413.	2460.	2548.	2638.	2715.	2773.	2803.	2813.	2816.
7	2801.	2796.	2777.	2733.	2659.	2552.	2414.	2261.	2162.	2272.	2423.	2544.	2632.	2692.	2730.	2749.	2756.
8	2758.	2750.	2725.	2677.	2597.	2471.	2282.	2002.	1693.	2021.	2298.	2468.	2572.	2639.	2681.	2704.	2711.
9	2733.	2725.	2699.	2651.	2571.	2437.	2210.	1753.	551.	1774.	2229.	2432.	2544.	2613.	2657.	2661.	2689.
10	2727.	2719.	2696.	2652.	2581.	2471.	2307.	2071.	1804.	2083.	2312.	2456.	2550.	2614.	2654.	2676.	2683.
11	2735.	2729.	2710.	2673.	2615.	2532.	2427.	2312.	2238.	2311.	2416.	2509.	2582.	2636.	2670.	2688.	2693.
12	2754.	2751.	2740.	2710.	2659.	2594.	2523.	2458.	2426.	2451.	2507.	2569.	2628.	2675.	2701.	2711.	2713.
13	2778.	2780.	2786.	2800.	2706.	2646.	2591.	2548.	2529.	2540.	2575.	2623.	2679.	2752.	2745.	2742.	2741.
14	2797.	2803.	2839.	3048.	2789.	2686.	2635.	2601.	2587.	2594.	2621.	2664.	2747.	3070.	2807.	2772.	2765.
15	2803.	2807.	2819.	2839.	2766.	2709.	2664.	2634.	2621.	2627.	2650.	2686.	2735.	2804.	2789.	2778.	2773.
16	2802.	2802.	2802.	2793.	2759.	2717.	2680.	2654.	2642.	2646.	2664.	2693.	2730.	2766.	2776.	2775.	2774.
17	2801.	2800.	2797.	2785.	2756.	2719.	2685.	2661.	2649.	2652.	2669.	2696.	2728.	2757.	2770.	2773.	2773.

MATRIX OF FRACTIONAL WATER SATURATION

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0.148	0.148	0.148	0.148	0.148	0.149	0.149	0.149	0.149	0.149	0.149	0.149	0.149	0.148	0.148	0.148	0.148
2	0.148	0.148	0.148	0.148	0.148	0.149	0.149	0.149	0.149	0.149	0.149	0.149	0.149	0.148	0.148	0.148	0.148
3	0.148	0.148	0.148	0.271	0.148	0.149	0.149	0.150	0.150	0.150	0.149	0.149	0.148	0.268	0.148	0.148	0.148
4	0.148	0.148	0.287	0.876	0.296	0.149	0.150	0.150	0.150	0.150	0.150	0.149	0.306	0.874	0.279	0.148	0.149
5	0.149	0.148	0.148	0.318	0.149	0.150	0.150	0.151	0.151	0.151	0.150	0.150	0.149	0.323	0.149	0.149	0.149
6	0.149	0.149	0.149	0.149	0.150	0.150	0.151	0.152	0.152	0.151	0.151	0.150	0.150	0.150	0.149	0.149	0.149
7	0.149	0.149	0.150	0.150	0.150	0.150	0.152	0.153	0.153	0.153	0.152	0.151	0.150	0.150	0.150	0.150	0.150
8	0.150	0.150	0.150	0.150	0.151	0.151	0.153	0.154	0.156	0.154	0.153	0.151	0.151	0.150	0.150	0.150	0.150
9	0.150	0.150	0.150	0.150	0.151	0.152	0.153	0.156	0.164	0.156	0.153	0.152	0.151	0.151	0.150	0.150	0.150
10	0.150	0.150	0.150	0.150	0.151	0.151	0.152	0.154	0.156	0.154	0.152	0.152	0.151	0.151	0.150	0.150	0.150
11	0.150	0.150	0.150	0.150	0.151	0.151	0.152	0.152	0.153	0.152	0.152	0.151	0.151	0.150	0.150	0.150	0.150
12	0.150	0.150	0.150	0.150	0.150	0.151	0.151	0.152	0.152	0.152	0.151	0.151	0.150	0.150	0.150	0.150	0.150
13	0.150	0.150	0.149	0.314	0.150	0.150	0.151	0.151	0.151	0.151	0.151	0.150	0.150	0.264	0.150	0.150	0.150
14	0.149	0.149	0.281	0.877	0.332	0.150	0.150	0.151	0.151	0.151	0.150	0.150	0.270	0.858	0.240	0.150	0.150
15	0.149	0.149	0.149	0.290	0.150	0.150	0.150	0.150	0.150	0.150	0.150	0.150	0.150	0.247	0.149	0.150	0.150
16	0.149	0.149	0.149	0.149	0.150	0.150	0.150	0.150	0.150	0.150	0.150	0.150	0.150	0.150	0.150	0.150	0.150
17	0.149	0.149	0.149	0.149	0.150	0.150	0.150	0.150	0.150	0.150	0.150	0.150	0.150	0.150	0.150	0.150	0.150

cont'd

TABLE C-4

Time = 640 Days

WATER PRESSURE MATRIX (PSIA)

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	3087.	3090.	3092.	3072.	3018.	2948.	2886.	2842.	2823.	2829.	2855.	2906.	2981.	3024.	3034.	3029.	3025.
2	3084.	3090.	3103.	3096.	3024.	2943.	2876.	2830.	2810.	2819.	2845.	2901.	2986.	3043.	3041.	3028.	3023.
3	3069.	3085.	3160.	3361.	3069.	2922.	2842.	2791.	2770.	2781.	2815.	2877.	3035.	3239.	3080.	3023.	3010.
4	3026.	3061.	3381.	3574.	3377.	2879.	2776.	2722.	2699.	2712.	2758.	2849.	3259.	3387.	3241.	3000.	2975.
5	2955.	2962.	3024.	3396.	2897.	2764.	2683.	2619.	2594.	2618.	2676.	2746.	2864.	3262.	2971.	2919.	2914.
6	2878.	2876.	2864.	2854.	2750.	2662.	2565.	2478.	2438.	2485.	2570.	2656.	2729.	2820.	2828.	2844.	2848.
7	2814.	2808.	2786.	2738.	2665.	2563.	2432.	2286.	2192.	2301.	2449.	2567.	2653.	2714.	2756.	2779.	2787.
8	2770.	2762.	2736.	2686.	2607.	2486.	2303.	2031.	1731.	2055.	2326.	2496.	2599.	2666.	2710.	2735.	2743.
9	2745.	2737.	2710.	2661.	2583.	2453.	2232.	1785.	600.	1813.	2262.	2462.	2573.	2643.	2687.	2713.	2721.
10	2739.	2731.	2707.	2662.	2591.	2484.	2326.	2096.	1839.	2116.	2343.	2486.	2580.	2644.	2686.	2710.	2717.
11	2748.	2741.	2721.	2680.	2621.	2542.	2441.	2332.	2265.	2340.	2445.	2536.	2610.	2666.	2704.	2723.	2729.
12	2769.	2765.	2752.	2740.	2661.	2600.	2534.	2476.	2449.	2476.	2531.	2593.	2650.	2717.	2737.	2748.	2751.
13	2796.	2798.	2830.	2970.	2743.	2649.	2601.	2564.	2549.	2563.	2598.	2643.	2720.	2929.	2802.	2764.	2781.
14	2819.	2834.	2962.	3040.	2973.	2721.	2645.	2617.	2607.	2617.	2645.	2705.	2934.	3021.	2924.	2816.	2806.
15	2827.	2832.	2863.	2968.	2817.	2721.	2678.	2651.	2642.	2651.	2677.	2717.	2792.	2929.	2841.	2821.	2816.
16	2827.	2827.	2827.	2826.	2777.	2732.	2696.	2673.	2664.	2672.	2694.	2726.	2768.	2809.	2819.	2818.	2817.
17	2826.	2825.	2822.	2806.	2775.	2736.	2702.	2680.	2672.	2679.	2699.	2729.	2765.	2796.	2812.	2816.	2817.

OIL PRESSURE MATRIX (PSIA)

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	3150.	3153.	3155.	3134.	3078.	3005.	2940.	2895.	2875.	2882.	2908.	2962.	3039.	3085.	3094.	3089.	3066.
2	3147.	3154.	3167.	3151.	3085.	3000.	2930.	2882.	2862.	2871.	2898.	2956.	3045.	3098.	3103.	3089.	3083.
3	3131.	3149.	3209.	3365.	3107.	2978.	2895.	2842.	2820.	2832.	2867.	2931.	3064.	3243.	3129.	3083.	3070.
4	3086.	3105.	3385.	3576.	3380.	2910.	2826.	2771.	2747.	2761.	2808.	2873.	3263.	3389.	3245.	3048.	3033.
5	3012.	3020.	3045.	3399.	2912.	2814.	2730.	2665.	2639.	2663.	2723.	2795.	2876.	3265.	2991.	2975.	2970.
6	2932.	2930.	2918.	2872.	2799.	2709.	2609.	2519.	2479.	2527.	2615.	2703.	2777.	2836.	2881.	2897.	2901.
7	2866.	2860.	2837.	2788.	2712.	2608.	2473.	2323.	2227.	2339.	2490.	2611.	2700.	2762.	2806.	2830.	2838.
8	2820.	2812.	2785.	2733.	2652.	2528.	2341.	2064.	1759.	2088.	2367.	2538.	2644.	2713.	2759.	2784.	2792.
9	2795.	2786.	2758.	2708.	2628.	2494.	2269.	1815.	619.	1842.	2299.	2504.	2617.	2689.	2735.	2761.	2770.
10	2788.	2780.	2755.	2709.	2636.	2526.	2364.	2130.	1869.	2151.	2382.	2528.	2624.	2690.	2734.	2758.	2766.
11	2797.	2791.	2769.	2728.	2667.	2585.	2482.	2371.	2302.	2378.	2486.	2579.	2655.	2712.	2752.	2772.	2778.
12	2819.	2815.	2801.	2762.	2708.	2645.	2578.	2518.	2490.	2518.	2575.	2638.	2697.	2750.	2786.	2798.	2801.
13	2847.	2849.	2856.	2973.	2754.	2696.	2646.	2608.	2593.	2607.	2643.	2690.	2742.	2933.	2840.	2834.	2832.
14	2871.	2878.	2966.	3042.	2976.	2736.	2692.	2662.	2652.	2663.	2691.	2732.	2937.	3023.	2928.	2864.	2857.
15	2879.	2884.	2902.	2971.	2836.	2769.	2725.	2698.	2689.	2698.	2724.	2765.	2825.	2933.	2869.	2873.	2868.
16	2879.	2880.	2879.	2866.	2828.	2782.	2744.	2720.	2711.	2719.	2741.	2775.	2818.	2856.	2871.	2870.	2869.
17	2878.	2878.	2874.	2858.	2825.	2785.	2750.	2728.	2719.	2726.	2747.	2778.	2815.	2848.	2864.	2868.	2869.

MATRIX OF FRACTIONAL WATER SATURATION

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0.147	0.147	0.147	0.147	0.148	0.148	0.149	0.149	0.149	0.149	0.149	0.148	0.148	0.148	0.148	0.148	0.148
2	0.147	0.147	0.147	0.148	0.148	0.148	0.149	0.149	0.149	0.149	0.149	0.148	0.148	0.148	0.148	0.148	0.148
3	0.147	0.147	0.150	0.512	0.152	0.148	0.149	0.149	0.149	0.149	0.149	0.149	0.156	0.493	0.149	0.148	0.148
4	0.148	0.150	0.576	0.893	0.627	0.155	0.149	0.150	0.150	0.150	0.149	0.158	0.651	0.891	0.548	0.150	0.148
5	0.148	0.148	0.161	0.696	0.169	0.149	0.150	0.150	0.150	0.150	0.150	0.149	0.176	0.706	0.162	0.148	0.148
6	0.149	0.149	0.149	0.165	0.149	0.150	0.151	0.151	0.151	0.151	0.151	0.150	0.150	0.166	0.149	0.149	0.149
7	0.149	0.149	0.149	0.149	0.150	0.151	0.151	0.152	0.153	0.152	0.151	0.151	0.150	0.150	0.149	0.149	0.149
8	0.149	0.149	0.149	0.150	0.150	0.151	0.152	0.154	0.156	0.154	0.152	0.151	0.150	0.150	0.150	0.149	0.149
9	0.149	0.149	0.150	0.150	0.150	0.151	0.153	0.156	0.164	0.155	0.153	0.151	0.151	0.150	0.150	0.150	0.150
10	0.149	0.150	0.150	0.150	0.150	0.151	0.152	0.154	0.155	0.153	0.152	0.151	0.150	0.150	0.150	0.150	0.150
11	0.149	0.149	0.150	0.150	0.150	0.151	0.151	0.152	0.152	0.152	0.151	0.151	0.150	0.150	0.150	0.150	0.150
12	0.149	0.149	0.149	0.161	0.150	0.150	0.151	0.151	0.151	0.151	0.151	0.150	0.150	0.154	0.149	0.149	0.149
13	0.149	0.149	0.157	0.665	0.180	0.150	0.150	0.151	0.151	0.151	0.150	0.150	0.160	0.587	0.152	0.149	0.149
14	0.149	0.151	0.531	0.892	0.704	0.170	0.150	0.150	0.150	0.150	0.150	0.156	0.616	0.687	0.445	0.150	0.149
15	0.149	0.149	0.152	0.558	0.164	0.150	0.150	0.150	0.150	0.150	0.150	0.150	0.154	0.476	0.150	0.149	0.149
16	0.149	0.149	0.149	0.152	0.149	0.149	0.150	0.150	0.150	0.150	0.150	0.150	0.149	0.150	0.149	0.149	0.149
17	0.149	0.149	0.149	0.149	0.149	0.149	0.150	0.150	0.150	0.150	0.150	0.150	0.149	0.149	0.149	0.149	0.149

cont'd

TABLE C-4

Time = 1280 Days

WATER PRESSURE MATRIX (PSIA)

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	3202.	3203.	3202.	3181.	3134.	3068.	3006.	2964.	2946.	2953.	2980.	3029.	3096.	3132.	3143.	3142.	3140.
2	3201.	3205.	3212.	3255.	3142.	3065.	2997.	2951.	2933.	2943.	2971.	3027.	3105.	3197.	3150.	3142.	3138.
3	3191.	3205.	3321.	3480.	3263.	3052.	2963.	2912.	2892.	2905.	2943.	3018.	3242.	3360.	3236.	3140.	3129.
4	3156.	3239.	3465.	3548.	3429.	3075.	2893.	2842.	2821.	2835.	2882.	3065.	3317.	3404.	3347.	3172.	3102.
5	3099.	3119.	3334.	3408.	3347.	2910.	2800.	2740.	2716.	2739.	2794.	2898.	3260.	3301.	3235.	3068.	3054.
6	3027.	3028.	3045.	3189.	2907.	2781.	2686.	2601.	2562.	2608.	2691.	2773.	2888.	3144.	3005.	2993.	2993.
7	2960.	2954.	2932.	2886.	2794.	2690.	2558.	2413.	2319.	2427.	2574.	2692.	2779.	2863.	2900.	2923.	2930.
8	2913.	2905.	2877.	2822.	2739.	2616.	2433.	2162.	1864.	2186.	2457.	2624.	2728.	2799.	2848.	2874.	2882.
9	2887.	2879.	2849.	2798.	2716.	2585.	2364.	1920.	743.	1947.	2393.	2592.	2704.	2775.	2822.	2849.	2856.
10	2880.	2872.	2846.	2797.	2723.	2615.	2457.	2229.	1974.	2249.	2474.	2616.	2710.	2776.	2820.	2844.	2852.
11	2888.	2882.	2862.	2817.	2750.	2670.	2571.	2464.	2397.	2471.	2575.	2665.	2738.	2796.	2836.	2856.	2862.
12	2908.	2906.	2901.	2935.	2822.	2724.	2663.	2607.	2581.	2608.	2661.	2720.	2797.	2901.	2872.	2881.	2884.
13	2931.	2935.	3012.	3064.	3034.	2811.	2730.	2696.	2683.	2696.	2729.	2792.	3016.	3063.	2980.	2913.	2911.
14	2949.	3002.	3087.	3113.	3059.	2966.	2795.	2752.	2742.	2753.	2782.	2914.	3057.	3112.	3084.	2983.	2933.
15	2957.	2961.	3029.	3085.	3023.	2886.	2821.	2790.	2780.	2789.	2815.	2862.	2989.	3083.	3012.	2947.	2942.
16	2958.	2959.	2960.	2998.	2924.	2879.	2840.	2814.	2803.	2810.	2832.	2866.	2905.	2980.	2946.	2945.	2944.
17	2958.	2958.	2954.	2941.	2916.	2880.	2846.	2822.	2811.	2817.	2837.	2868.	2901.	2927.	2940.	2943.	2943.

OIL PRESSURE MATRIX (PSIA)

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	3271.	3273.	3271.	3249.	3199.	3130.	3066.	3021.	3003.	3011.	3038.	3090.	3159.	3198.	3209.	3208.	3206.
2	3270.	3274.	3282.	3264.	3208.	3127.	3056.	3009.	2990.	3000.	3029.	3087.	3167.	3208.	3216.	3209.	3204.
3	3260.	3275.	3327.	3483.	3267.	3113.	3020.	2968.	2947.	2961.	3000.	3075.	3246.	3363.	3244.	3206.	3195.
4	3224.	3246.	3467.	3550.	3432.	3080.	2947.	2895.	2873.	2887.	2935.	3069.	3320.	3406.	3349.	3180.	3166.
5	3163.	3175.	3338.	3411.	3350.	2932.	2851.	2789.	2764.	2788.	2845.	2914.	3263.	3304.	3239.	3125.	3116.
6	3087.	3089.	3090.	3193.	2927.	2832.	2734.	2646.	2606.	2653.	2739.	2824.	2901.	3148.	3048.	3052.	3052.
7	3018.	3011.	2988.	2929.	2845.	2737.	2602.	2453.	2357.	2468.	2618.	2740.	2830.	2900.	2955.	2979.	2986.
8	2968.	2960.	2931.	2875.	2789.	2662.	2474.	2197.	1894.	2221.	2498.	2670.	2777.	2850.	2901.	2928.	2936.
9	2941.	2932.	2902.	2849.	2765.	2629.	2403.	1951.	763.	1978.	2433.	2637.	2752.	2826.	2874.	2902.	2911.
10	2934.	2926.	2899.	2848.	2772.	2660.	2499.	2265.	2006.	2286.	2516.	2661.	2758.	2826.	2872.	2897.	2905.
11	2943.	2937.	2915.	2867.	2800.	2717.	2615.	2505.	2437.	2513.	2619.	2712.	2787.	2847.	2889.	2909.	2916.
12	2963.	2962.	2955.	2939.	2836.	2773.	2710.	2653.	2626.	2653.	2708.	2768.	2826.	2906.	2926.	2936.	2936.
13	2987.	2990.	3017.	3067.	3037.	2823.	2779.	2744.	2730.	2744.	2778.	2821.	3020.	3066.	2986.	2969.	2967.
14	3006.	3011.	3090.	3115.	3061.	2970.	2831.	2802.	2792.	2802.	2830.	2918.	3060.	3114.	3087.	2995.	2989.
15	3015.	3019.	3036.	3088.	3027.	2929.	2873.	2841.	2831.	2840.	2867.	2914.	2993.	3087.	3021.	3004.	2999.
16	3016.	3017.	3017.	3006.	2976.	2934.	2893.	2866.	2855.	2862.	2885.	2920.	2960.	2991.	3002.	3002.	3000.
17	3015.	3015.	3012.	2998.	2972.	2935.	2899.	2874.	2863.	2869.	2890.	2922.	2956.	2983.	2997.	3000.	3000.

MATRIX OF FRACTIONAL WATER SATURATION

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0.147	0.147	0.147	0.147	0.147	0.147	0.148	0.148	0.148	0.148	0.148	0.148	0.147	0.147	0.147	0.147	0.147
2	0.147	0.147	0.146	0.195	0.147	0.147	0.148	0.148	0.148	0.148	0.148	0.148	0.147	0.182	0.147	0.147	0.147
3	0.147	0.147	0.248	0.772	0.323	0.148	0.148	0.148	0.148	0.148	0.148	0.148	0.409	0.742	0.223	0.147	0.147
4	0.147	0.238	0.802	0.893	0.823	0.341	0.149	0.149	0.149	0.149	0.149	0.393	0.818	0.892	0.784	0.213	0.147
5	0.147	0.148	0.510	0.842	0.651	0.160	0.149	0.149	0.150	0.149	0.149	0.169	0.696	0.841	0.480	0.148	0.147
6	0.148	0.148	0.150	0.541	0.162	0.149	0.150	0.150	0.151	0.150	0.150	0.149	0.175	0.579	0.151	0.148	0.148
7	0.148	0.148	0.148	0.151	0.149	0.150	0.151	0.152	0.152	0.151	0.151	0.150	0.149	0.153	0.148	0.148	0.148
8	0.148	0.148	0.149	0.149	0.149	0.150	0.151	0.153	0.155	0.153	0.151	0.150	0.150	0.149	0.149	0.149	0.149
9	0.149	0.149	0.149	0.149	0.150	0.150	0.152	0.155	0.163	0.155	0.152	0.150	0.150	0.149	0.149	0.149	0.149
10	0.149	0.149	0.149	0.149	0.150	0.150	0.151	0.153	0.154	0.153	0.151	0.150	0.150	0.149	0.149	0.149	0.149
11	0.149	0.149	0.149	0.149	0.149	0.150	0.151	0.151	0.152	0.151	0.151	0.150	0.149	0.149	0.149	0.149	0.149
12	0.148	0.148	0.149	0.399	0.173	0.150	0.150	0.150	0.150	0.150	0.150	0.150	0.155	0.334	0.149	0.149	0.149
13	0.148	0.148	0.360	0.829	0.704	0.179	0.150	0.150	0.150	0.150	0.150	0.156	0.601	0.821	0.279	0.148	0.148
14	0.148	0.203	0.775	0.892	0.835	0.562	0.153	0.149	0.149	0.149	0.150	0.419	0.831	0.888	0.730	0.179	0.148
15	0.148	0.148	0.243	0.782	0.449	0.151	0.149	0.149	0.149	0.149	0.149	0.149	0.348	0.753	0.203	0.148	0.148
16	0.148	0.148	0.148	0.217	0.149	0.149	0.149	0.149	0.149	0.149	0.149	0.149	0.148	0.188	0.148	0.148	0.148
17	0.148	0.148	0.148	0.148	0.148	0.149	0.149	0.149	0.149	0.149	0.149	0.149	0.148	0.148	0.148	0.148	0.148

cont'd

TABLE C-4

Time = 1920 Days

WATER PRESSURE MATRIX (PSIA)

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	3339.	3340.	3337.	3314.	3265.	3201.	3139.	3094.	3074.	3079.	3103.	3151.	3215.	3256.	3270.	3270.	3269.
2	3338.	3342.	3351.	3404.	3297.	3199.	3130.	3082.	3061.	3069.	3094.	3147.	3267.	3334.	3279.	3271.	3267.
3	3328.	3350.	3482.	3552.	3475.	3234.	3097.	3042.	3020.	3031.	3066.	3194.	3408.	3450.	3387.	3274.	3259.
4	3294.	3402.	3531.	3613.	3495.	3380.	3060.	2969.	2948.	2960.	3052.	3325.	3405.	3489.	3434.	3319.	3230.
5	3218.	3283.	3472.	3477.	3429.	3146.	2929.	2868.	2844.	2867.	2929.	3193.	3351.	3388.	3386.	3235.	3175.
6	3147.	3149.	3231.	3374.	3278.	2921.	2814.	2729.	2690.	2735.	2817.	2928.	3265.	3304.	3168.	3117.	3117.
7	3081.	3076.	3058.	3067.	2929.	2816.	2686.	2541.	2448.	2555.	2699.	2814.	2930.	3048.	3031.	3050.	3056.
8	3034.	3026.	2998.	2943.	2863.	2741.	2560.	2291.	1995.	2314.	2582.	2747.	2850.	2922.	2974.	3000.	3008.
9	3008.	2999.	2971.	2920.	2839.	2708.	2490.	2049.	880.	2075.	2518.	2715.	2826.	2897.	2947.	2975.	2984.
10	3001.	2993.	2968.	2919.	2846.	2737.	2580.	2353.	2100.	2374.	2597.	2738.	2831.	2897.	2944.	2970.	2978.
11	3010.	3004.	2984.	2982.	2883.	2788.	2690.	2584.	2519.	2593.	2696.	2766.	2859.	2953.	2962.	2983.	2989.
12	3029.	3027.	3066.	3125.	3059.	2871.	2778.	2725.	2699.	2726.	2780.	2841.	2982.	3105.	3038.	3009.	3011.
13	3052.	3085.	3166.	3170.	3140.	3091.	2873.	2812.	2798.	2812.	2847.	3012.	3126.	3154.	3139.	3050.	3036.
14	3073.	3137.	3194.	3217.	3164.	3117.	2956.	2868.	2857.	2866.	2939.	3103.	3150.	3202.	3180.	3116.	3054.
15	3080.	3088.	3167.	3191.	3164.	3048.	2941.	2908.	2896.	2904.	2931.	3027.	3145.	3178.	3143.	3068.	3063.
16	3081.	3082.	3087.	3131.	3079.	2995.	2958.	2931.	2920.	2927.	2949.	2984.	3051.	3108.	3065.	3064.	3063.
17	3081.	3080.	3076.	3063.	3032.	2996.	2963.	2939.	2928.	2934.	2955.	2985.	3017.	3044.	3059.	3062.	3063.

OIL PRESSURE MATRIX (PSIA)

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	3416.	3418.	3414.	3390.	3338.	3271.	3205.	3158.	3136.	3142.	3167.	3217.	3285.	3328.	3343.	3344.	3342.
2	3415.	3420.	3425.	3409.	3344.	3268.	3195.	3145.	3123.	3131.	3158.	3214.	3289.	3340.	3351.	3345.	3341.
3	3405.	3423.	3487.	3555.	3478.	3259.	3160.	3103.	3080.	3091.	3129.	3205.	3411.	3453.	3391.	3345.	3332.
4	3363.	3406.	3534.	3614.	3497.	3383.	3080.	3027.	3005.	3017.	3065.	3328.	3407.	3491.	3437.	3324.	3299.
5	3288.	3294.	3475.	3479.	3432.	3150.	2984.	2922.	2897.	2920.	2980.	3197.	3354.	3391.	3389.	3248.	3242.
6	3213.	3215.	3237.	3377.	3282.	2966.	2866.	2778.	2738.	2784.	2869.	2953.	3268.	3307.	3194.	3182.	3181.
7	3144.	3139.	3120.	3073.	2974.	2868.	2733.	2585.	2489.	2599.	2747.	2866.	2954.	3053.	3091.	3111.	3117.
8	3095.	3086.	3058.	3000.	2916.	2790.	2604.	2329.	2027.	2352.	2627.	2797.	2903.	2978.	3032.	3059.	3068.
9	3068.	3059.	3029.	2976.	2892.	2757.	2532.	2082.	901.	2109.	2561.	2764.	2878.	2952.	3004.	3033.	3042.
10	3060.	3052.	3026.	2975.	2899.	2786.	2624.	2392.	2134.	2413.	2642.	2787.	2884.	2952.	3001.	3028.	3036.
11	3069.	3063.	3043.	2995.	2928.	2839.	2738.	2629.	2561.	2638.	2744.	2836.	2912.	2971.	3020.	3041.	3048.
12	3090.	3088.	3081.	3128.	3063.	2891.	2828.	2773.	2747.	2775.	2831.	2891.	2986.	3108.	3064.	3069.	3070.
13	3114.	3116.	3169.	3172.	3143.	3095.	2896.	2863.	2850.	2864.	2898.	3016.	3128.	3156.	3143.	3099.	3096.
14	3134.	3142.	3196.	3219.	3166.	3119.	2962.	2922.	2911.	2919.	2948.	3106.	3153.	3204.	3183.	3122.	3116.
15	3143.	3148.	3171.	3193.	3168.	3055.	2998.	2963.	2951.	2959.	2987.	3037.	3148.	3180.	3149.	3129.	3125.
16	3144.	3145.	3146.	3136.	3095.	3054.	3015.	2988.	2976.	2983.	3006.	3042.	3080.	3114.	3126.	3126.	3126.
17	3144.	3143.	3139.	3123.	3092.	3055.	3021.	2995.	2984.	2991.	3012.	3043.	3077.	3105.	3120.	3124.	3125.

MATRIX OF FRACTIONAL WATER SATURATION

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0.146	0.146	0.146	0.146	0.146	0.147	0.147	0.147	0.147	0.147	0.147	0.147	0.146	0.146	0.146	0.146	0.146
2	0.146	0.146	0.146	0.283	0.150	0.147	0.147	0.147	0.147	0.147	0.147	0.147	0.160	0.253	0.146	0.146	0.146
3	0.146	0.146	0.407	0.812	0.580	0.158	0.147	0.148	0.148	0.148	0.147	0.187	0.677	0.793	0.368	0.146	0.146
4	0.147	0.402	0.827	0.893	0.830	0.662	0.162	0.148	0.148	0.148	0.175	0.704	0.821	0.892	0.818	0.336	0.147
5	0.146	0.187	0.748	0.842	0.805	0.372	0.148	0.149	0.149	0.149	0.149	0.474	0.810	0.841	0.728	0.177	0.147
6	0.147	0.147	0.288	0.807	0.529	0.150	0.149	0.150	0.150	0.149	0.149	0.158	0.636	0.813	0.283	0.147	0.147
7	0.147	0.147	0.147	0.278	0.150	0.149	0.150	0.151	0.151	0.151	0.150	0.149	0.158	0.320	0.148	0.147	0.147
8	0.148	0.148	0.148	0.148	0.149	0.149	0.151	0.152	0.154	0.152	0.150	0.149	0.149	0.148	0.148	0.148	0.148
9	0.148	0.148	0.148	0.148	0.149	0.150	0.151	0.154	0.162	0.154	0.151	0.150	0.149	0.148	0.148	0.148	0.148
10	0.148	0.148	0.148	0.148	0.149	0.149	0.150	0.152	0.154	0.152	0.150	0.149	0.149	0.148	0.148	0.148	0.148
11	0.148	0.148	0.148	0.176	0.150	0.149	0.150	0.150	0.151	0.150	0.150	0.149	0.149	0.165	0.148	0.148	0.148
12	0.148	0.148	0.169	0.698	0.512	0.162	0.149	0.150	0.150	0.150	0.149	0.149	0.353	0.670	0.157	0.148	0.148
13	0.147	0.155	0.609	0.833	0.812	0.644	0.160	0.149	0.149	0.149	0.149	0.429	0.803	0.831	0.508	0.149	0.148
14	0.147	0.296	0.812	0.892	0.836	0.800	0.293	0.149	0.149	0.149	0.210	0.760	0.834	0.888	0.790	0.246	0.147
15	0.147	0.148	0.394	0.813	0.692	0.240	0.148	0.148	0.148	0.148	0.148	0.185	0.618	0.800	0.319	0.147	0.147
16	0.147	0.147	0.148	0.333	0.168	0.148	0.148	0.148	0.148	0.148	0.148	0.148	0.155	0.266	0.148	0.147	0.147
17	0.147	0.147	0.147	0.148	0.148	0.148	0.148	0.148	0.148	0.148	0.148	0.148	0.148	0.148	0.147	0.147	0.147

cont'd

TABLE C-4

Time = 2880 Days

WATER PRESSURE MATRIX (PSIA)

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	3471.	3472.	3468.	3463.	3391.	3334.	3276.	3234.	3217.	3228.	3255.	3301.	3357.	3403.	3408.	3410.	3408.
2	3471.	3475.	3522.	3564.	3464.	3332.	3268.	3221.	3204.	3218.	3250.	3309.	3430.	3492.	3446.	3411.	3407.
3	3463.	3531.	3616.	3643.	3593.	3414.	3242.	3180.	3160.	3179.	3242.	3421.	3516.	3554.	3531.	3450.	3400.
4	3490.	3578.	3625.	3704.	3586.	3498.	3251.	3098.	3079.	3096.	3269.	3441.	3508.	3593.	3539.	3493.	3410.
5	3370.	3459.	3562.	3570.	3519.	3458.	3100.	2983.	2963.	2983.	3112.	3407.	3455.	3495.	3491.	3395.	3319.
6	3300.	3354.	3491.	3480.	3447.	3343.	2937.	2840.	2803.	2842.	2954.	3347.	3397.	3422.	3429.	3311.	3264.
7	3241.	3237.	3288.	3395.	3286.	2955.	2799.	2656.	2563.	2664.	2799.	2964.	3322.	3355.	3252.	3202.	3206.
8	3195.	3188.	3160.	3150.	3014.	2869.	2682.	2413.	2117.	2432.	2697.	2864.	3020.	3122.	3127.	3152.	3159.
9	3170.	3161.	3131.	3072.	2982.	2839.	2615.	2176.	1014.	2200.	2640.	2839.	2959.	3040.	3095.	3125.	3134.
10	3164.	3157.	3131.	3078.	2989.	2864.	2702.	2479.	2229.	2499.	2719.	2861.	2963.	3042.	3093.	3119.	3128.
11	3173.	3169.	3158.	3190.	3095.	2954.	2803.	2709.	2648.	2719.	2815.	2921.	3045.	3142.	3117.	3133.	3138.
12	3190.	3190.	3254.	3274.	3251.	3218.	2947.	2854.	2835.	2859.	2921.	3166.	3228.	3250.	3214.	3156.	3158.
13	3210.	3271.	3318.	3313.	3282.	3248.	3199.	2996.	2946.	2961.	3085.	3222.	3259.	3289.	3293.	3235.	3181.
14	3253.	3311.	3338.	3361.	3308.	3265.	3224.	3078.	3015.	3060.	3191.	3241.	3285.	3337.	3316.	3277.	3209.
15	3239.	3280.	3329.	3335.	3310.	3264.	3169.	3078.	3062.	3073.	3137.	3229.	3289.	3313.	3304.	3237.	3212.
16	3240.	3241.	3282.	3311.	3258.	3185.	3131.	3102.	3089.	3097.	3122.	3155.	3236.	3279.	3237.	3215.	3214.
17	3240.	3240.	3236.	3258.	3195.	3165.	3134.	3109.	3098.	3104.	3125.	3151.	3176.	3211.	3211.	3214.	3214.

OIL PRESSURE MATRIX (PSIA)

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	3559.	3560.	3556.	3530.	3473.	3411.	3350.	3305.	3288.	3298.	3328.	3376.	3436.	3478.	3491.	3493.	3491.
2	3559.	3563.	3567.	3568.	3474.	3410.	3341.	3292.	3274.	3288.	3322.	3376.	3438.	3496.	3498.	3494.	3490.
3	3550.	3566.	362C.	3645.	3596.	3419.	3313.	3248.	3227.	3247.	3303.	3425.	3519.	3557.	3534.	3495.	3483.
4	3513.	3582.	3627.	3705.	3589.	3501.	3256.	3162.	3142.	3158.	3273.	3444.	3511.	3594.	3542.	3497.	3452.
5	3448.	3464.	3565.	3572.	3521.	3461.	3107.	3041.	3020.	3042.	3118.	3410.	3457.	3497.	3493.	3401.	3395.
6	3375.	3375.	3495.	3483.	3449.	3346.	2974.	2893.	2855.	2895.	2969.	3351.	3400.	3424.	3432.	3338.	3337.
7	3312.	3308.	3298.	3398.	3290.	2987.	2850.	2703.	2607.	2711.	2850.	2974.	3325.	3359.	3261.	3272.	3276.
8	3264.	3256.	3227.	3161.	3064.	2922.	2729.	2453.	2151.	2472.	2745.	2917.	3039.	3130.	3192.	3218.	3226.
9	3237.	3228.	3196.	3135.	3040.	2892.	2661.	2211.	1036.	2236.	2686.	2892.	3016.	3101.	3159.	3190.	3199.
10	3231.	3223.	3197.	3139.	3047.	2917.	2750.	2521.	2265.	2542.	2768.	2915.	3021.	3102.	3157.	3184.	3193.
11	3241.	3237.	3223.	3195.	3100.	2964.	2854.	2757.	2694.	2768.	2867.	2956.	3051.	3147.	3182.	3199.	3204.
12	3259.	3258.	3259.	3277.	3254.	3221.	2955.	2907.	2888.	2912.	2954.	3169.	3231.	3252.	3221.	3223.	3225.
13	3280.	3280.	3321.	3315.	3285.	3251.	3202.	3016.	3003.	3017.	3090.	3225.	3262.	3291.	3296.	3250.	3249.
14	3301.	3315.	3341.	3363.	3310.	3267.	3227.	3088.	3075.	3084.	3194.	3244.	3287.	3339.	3318.	3282.	3271.
15	3310.	3315.	3333.	3338.	3313.	3268.	3186.	3140.	3125.	3136.	3176.	3233.	3292.	3316.	3307.	3286.	3282.
16	3312.	3313.	3314.	3314.	3265.	3232.	3196.	3166.	3153.	3161.	3187.	3217.	3245.	3283.	3287.	3285.	3284.
17	3312.	3311.	3308.	3293.	3264.	3232.	3200.	3174.	3162.	3168.	3190.	3217.	3244.	3269.	3281.	3284.	3284.

MATRIX OF FRACTIONAL WATER SATURATION

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	C.145	0.145	0.145	C.147	0.145	0.146	0.146	0.146	0.146	0.146	0.146	0.146	0.146	0.146	0.145	0.145	0.145
2	C.145	0.145	0.150	C.455	0.189	0.146	0.146	0.146	0.147	0.146	0.146	0.147	0.223	0.379	0.149	0.145	0.145
3	0.145	0.153	C.561	C.823	C.750	0.300	0.146	0.147	0.147	0.147	0.148	0.487	0.780	0.810	0.525	0.150	0.145
4	C.16C	0.609	0.830	C.893	0.831	0.798	0.330	0.147	0.147	0.148	0.393	0.795	0.822	0.892	0.825	0.528	0.151
5	C.146	0.347	0.799	C.842	0.808	0.766	0.242	0.148	0.148	0.148	0.292	0.778	0.811	0.842	0.788	0.281	0.146
6	C.146	0.162	0.686	C.823	0.808	0.580	0.152	0.149	0.149	0.149	0.171	0.703	0.812	0.824	0.655	0.157	0.146
7	C.146	0.146	0.190	C.735	0.512	0.154	0.149	0.150	0.151	0.150	0.149	0.192	0.673	0.755	0.192	0.147	0.147
8	C.147	0.147	0.147	0.189	0.149	0.149	0.150	0.152	0.153	0.151	0.150	0.149	0.163	0.215	0.147	0.147	0.147
9	C.147	0.147	0.147	C.147	0.148	0.149	0.150	0.153	0.161	0.153	0.150	0.149	0.148	0.148	0.147	0.147	0.147
10	C.147	C.147	C.147	0.148	0.148	0.149	0.150	0.151	0.153	0.151	0.150	0.149	0.148	0.148	0.147	0.147	0.147
11	C.147	C.147	0.147	C.372	0.349	0.196	0.149	0.150	0.150	0.150	0.149	0.153	0.269	0.353	0.147	0.147	0.147
12	C.147	0.147	0.314	C.798	0.788	0.753	0.224	0.149	0.149	0.149	0.154	0.583	0.777	0.797	0.254	0.147	0.147
13	C.146	0.199	0.753	0.833	0.812	0.812	0.668	0.162	0.148	0.148	0.397	0.801	0.808	0.831	0.702	0.171	0.147
14	C.15C	0.46C	0.822	0.892	0.836	0.819	0.711	0.189	0.148	0.159	0.622	0.815	0.834	0.888	0.809	0.380	0.147
15	C.146	0.153	C.543	C.82C	0.769	0.541	0.166	0.147	0.147	0.147	0.152	0.416	0.749	0.816	0.482	0.149	0.146
16	C.146	0.146	0.154	0.516	0.241	0.150	0.147	0.147	0.147	0.147	0.147	0.147	0.202	0.412	0.149	0.146	0.146
17	0.146	0.146	0.146	C.153	0.147	0.147	0.147	0.147	0.147	0.147	0.147	0.147	0.147	0.148	0.146	0.148	0.146

cont'd

TABLE C-4

Time = 3840 Days

WATER PRESSURE MATRIX (PSIA)

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	3569.	357C.	3567.	3614.	3502.	3451.	3399.	3358.	3340.	3348.	3373.	3413.	3469.	3545.	3516.	3518.	3517.
2	3569.	3572.	3652.	3688.	3588.	3470.	3394.	3346.	3327.	3338.	3368.	3472.	3551.	3616.	3586.	3519.	3516.
3	3562.	3654.	37C9.	3726.	3675.	3586.	3414.	3307.	3282.	3300.	3420.	3557.	3611.	3651.	3637.	3590.	3512.
4	3619.	3677.	37C9.	3786.	3667.	3583.	3492.	3258.	3197.	3260.	3475.	3542.	3604.	3689.	3638.	3614.	3564.
5	3544.	3626.	3644.	3651.	3596.	3533.	3433.	3134.	3076.	3144.	3429.	3498.	3547.	3590.	3587.	3554.	3473.
6	3429.	3504.	3581.	3562.	3522.	3467.	3235.	2947.	2907.	2975.	3342.	3438.	3484.	3517.	3535.	3478.	3407.
7	3379.	3385.	3483.	3490.	3459.	3401.	2902.	2739.	2654.	2751.	2972.	3379.	3427.	3456.	3448.	3365.	3363.
8	3338.	3333.	3331.	3375.	3212.	2996.	2756.	2497.	2207.	2519.	2776.	3038.	3325.	3381.	3324.	3321.	3326.
9	3312.	3304.	3275.	3223.	3107.	2942.	2704.	2265.	1113.	2297.	2743.	2968.	3135.	3231.	3268.	3295.	3303.
10	3303.	3296.	3271.	3254.	3172.	3024.	2795.	2564.	2322.	2594.	2820.	3019.	3166.	3247.	3264.	3289.	3296.
11	3309.	3306.	3329.	3356.	3324.	3278.	3123.	2789.	2744.	2808.	2944.	3266.	3329.	3358.	3317.	3301.	3305.
12	3323.	3340.	34C2.	3394.	3365.	3324.	3278.	3037.	2953.	3011.	3296.	3338.	3373.	3399.	3396.	3324.	3320.
13	3340.	3411.	3438.	3432.	3397.	3358.	3321.	3235.	3104.	3160.	3337.	3370.	3406.	3437.	3444.	3407.	3338.
14	3413.	3446.	3459.	3480.	3426.	3382.	3346.	3291.	3185.	3258.	3358.	3392.	3433.	3485.	3464.	3444.	3397.
15	3363.	3428.	3454.	3456.	3429.	3398.	3339.	3249.	3227.	3239.	3332.	3402.	3436.	3461.	3458.	3419.	3363.
16	3364.	3365.	3429.	3444.	3406.	3364.	3280.	3261.	3250.	3257.	3278.	3349.	3398.	3443.	3418.	3364.	3364.
17	3364.	3364.	3362.	3416.	3337.	3305.	3282.	3265.	3257.	3263.	3281.	3303.	3327.	3395.	3360.	3363.	3363.

CIL PRESSURE MATRIX (PSIA)

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	3666.	3667.	3664.	3642.	3592.	3537.	3482.	3437.	3418.	3426.	3453.	3496.	3554.	3594.	3608.	3609.	3609.
2	3665.	3669.	3673.	3692.	3595.	3537.	3475.	3425.	3404.	3416.	3448.	3493.	3556.	3620.	3614.	3611.	3608.
3	3658.	367C.	3712.	3729.	3678.	3590.	3457.	3382.	3356.	3375.	3435.	3560.	3613.	3654.	3641.	3512.	3603.
4	3628.	3680.	3712.	3787.	3670.	3586.	3495.	3291.	3266.	3282.	3478.	3545.	3606.	3691.	3641.	3617.	3580.
5	3578.	3629.	3647.	3653.	3599.	3536.	3436.	3166.	3139.	3164.	3433.	3501.	3549.	3592.	3590.	3558.	3540.
6	3513.	3511.	3583.	3564.	3525.	3470.	3239.	3000.	2962.	3005.	3345.	3441.	3486.	3519.	3538.	3487.	3490.
7	3460.	3458.	3487.	3493.	3461.	34C4.	2914.	2788.	2701.	2800.	2978.	3382.	3429.	3459.	3453.	3441.	3443.
8	3416.	3411.	3394.	3378.	3217.	3010.	2806.	2539.	2243.	2562.	2826.	3044.	3328.	3385.	3384.	3398.	3403.
9	3388.	3379.	3349.	3282.	3171.	2998.	2752.	2302.	1135.	2335.	2793.	3025.	3179.	3278.	3341.	3370.	3378.
10	3379.	3371.	3345.	3284.	3185.	3034.	2841.	2608.	2361.	2639.	2872.	3051.	3183.	3276.	3337.	3364.	3371.
11	3385.	3381.	3369.	3359.	3327.	3282.	3127.	2834.	2793.	2860.	2954.	3269.	3332.	3362.	3366.	3377.	3381.
12	3400.	3399.	3406.	3397.	3368.	3327.	3281.	3042.	3010.	3024.	3299.	3341.	3376.	3402.	3400.	3396.	3397.
13	3417.	3417.	3441.	3434.	3400.	3361.	3323.	3238.	3157.	3166.	3340.	3373.	3409.	3440.	3447.	3415.	3416.
14	3434.	3449.	3461.	3482.	3428.	3385.	3349.	3295.	3247.	3264.	3361.	3395.	3436.	3487.	3467.	3448.	3434.
15	3442.	3446.	3458.	3459.	3432.	3401.	3346.	3319.	3298.	3310.	3342.	3406.	3439.	3464.	3461.	3446.	3442.
16	3444.	3445.	3445.	3448.	3411.	3379.	3354.	3333.	3322.	3330.	3352.	3378.	3404.	3446.	3445.	3444.	3443.
17	3444.	3444.	3442.	3430.	3407.	3380.	3357.	3339.	3330.	3336.	3355.	3379.	3403.	3427.	3440.	3443.	3443.

MATRIX OF FRACTIONAL WATER SATURATION

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0.144	0.144	0.144	C.157	C.145	0.145	0.145	0.146	0.146	0.146	0.145	0.145	0.145	0.149	0.145	0.145	0.145
2	C.144	0.144	C.161	C.594	C.262	0.147	0.145	0.146	0.146	0.146	0.146	0.161	0.304	0.487	0.156	0.145	0.145
3	C.144	0.167	0.636	C.824	0.787	0.519	0.151	0.146	0.146	0.146	0.170	0.696	0.798	0.814	0.602	0.161	0.145
4	C.193	0.678	0.830	C.893	0.831	0.808	0.586	0.154	0.147	0.161	0.626	0.799	0.822	0.892	0.826	0.623	0.169
5	C.153	0.573	C.803	C.842	0.809	0.792	0.593	0.155	0.147	0.162	0.630	0.792	0.811	0.842	0.798	0.443	0.147
6	C.145	0.239	0.786	C.823	0.809	0.806	0.469	0.149	0.148	0.155	0.609	0.807	0.813	0.824	0.762	0.201	C.145
7	C.145	0.146	0.452	C.806	0.796	0.713	0.179	0.149	0.150	0.149	0.287	0.801	0.804	0.809	0.408	0.146	0.146
8	C.146	0.146	C.147	C.475	0.356	0.173	0.149	0.151	0.153	0.151	0.149	0.266	0.564	0.519	0.148	0.146	0.146
9	C.146	0.146	0.146	C.148	0.147	0.148	0.150	0.152	0.160	0.152	0.149	0.148	0.151	0.150	0.146	0.146	0.146
10	C.146	0.146	C.146	C.155	0.174	0.186	0.150	0.151	0.152	0.150	0.149	0.154	0.166	0.156	0.146	0.146	0.146
11	C.146	0.146	0.151	C.556	0.660	0.719	0.491	0.150	0.149	0.149	0.192	0.616	0.649	0.564	0.150	0.146	0.146
12	C.146	C.148	C.472	C.804	0.795	0.813	0.814	0.328	0.148	0.173	0.716	0.811	0.795	0.807	0.382	0.146	0.146
13	C.146	C.26C	0.783	C.833	0.813	0.812	0.795	0.492	0.149	0.268	0.767	0.804	0.808	0.832	0.754	0.206	0.146
14	C.162	0.576	C.823	C.892	0.836	0.819	0.796	0.402	0.147	0.289	0.769	0.816	0.834	0.888	0.812	C.501	0.152
15	C.146	0.165	C.610	0.821	0.785	0.699	0.254	0.147	0.146	0.146	0.193	0.620	0.775	0.818	0.566	0.157	0.146
16	C.146	0.146	C.168	C.621	C.361	0.170	0.146	0.146	0.146	0.146	0.146	0.156	0.268	0.530	0.157	0.146	0.146
17	0.146	0.146	C.146	C.172	0.146	0.146	0.146	0.146	0.146	0.146	0.146	0.146	0.146	0.154	0.146	0.146	0.146

cont'd

TABLE C-4

Time = 4800 Days

WATER PRESSURE MATRIX (PSIA)

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	3644.	3644.	3641.	3713.	3591.	3529.	3480.	3441.	3423.	3429.	3450.	3484.	3554.	3636.	3575.	3576.	3576.
2	3643.	3646.	3741.	3769.	3690.	3594.	3474.	3431.	3412.	3421.	3445.	3559.	3634.	3684.	3660.	3577.	3575.
3	3639.	3740.	3783.	3796.	3742.	3680.	3531.	3397.	3371.	3391.	3516.	3613.	3662.	3706.	3695.	3660.	3571.
4	3710.	3752.	3782.	3856.	3736.	3652.	3582.	3388.	3290.	3392.	3538.	3593.	3656.	3743.	3694.	3673.	3637.
5	3660.	3715.	3718.	3722.	3664.	3597.	3530.	3264.	3159.	3295.	3493.	3546.	3599.	3645.	3644.	3637.	3585.
6	3535.	3623.	3655.	3636.	3589.	3524.	3448.	3060.	2943.	3153.	3422.	3464.	3535.	3573.	3592.	3560.	3482.
7	3483.	3540.	3600.	3574.	3527.	3455.	3359.	2735.	2596.	2875.	3343.	3422.	3480.	3518.	3541.	3484.	3446.
8	3450.	3446.	3501.	3521.	3472.	3387.	2871.	2366.	2064.	2358.	3260.	3371.	3434.	3474.	3465.	3413.	3417.
9	3428.	3422.	3401.	3416.	3318.	3117.	2744.	2136.	950.	2145.	2745.	3119.	3326.	3403.	3377.	3395.	3401.
10	3420.	3415.	3397.	3424.	3369.	3274.	3184.	2844.	2147.	2409.	3010.	3258.	3361.	3408.	3375.	3391.	3396.
11	3423.	3420.	3472.	3466.	3428.	3373.	3300.	3225.	2810.	3138.	3285.	3354.	3409.	3446.	3444.	3398.	3401.
12	3433.	3482.	3514.	3497.	3463.	3418.	3368.	3317.	3171.	3301.	3355.	3400.	3444.	3476.	3490.	3437.	3411.
13	3453.	3527.	3541.	3533.	3497.	3456.	3418.	3377.	3318.	3371.	3408.	3441.	3480.	3514.	3522.	3500.	3424.
14	3532.	3553.	3561.	3582.	3528.	3484.	3449.	3420.	3366.	3417.	3439.	3470.	3510.	3563.	3543.	3531.	3503.
15	3463.	3540.	3558.	3558.	3531.	3502.	3461.	3395.	3354.	3373.	3450.	3487.	3515.	3540.	3539.	3514.	3442.
16	3463.	3464.	3540.	3549.	3524.	3483.	3395.	3377.	3370.	3374.	3384.	3466.	3497.	3529.	3514.	3444.	3443.
17	3463.	3463.	3461.	3529.	3476.	3412.	3394.	3381.	3374.	3377.	3387.	3400.	3423.	3500.	3441.	3443.	3443.

OIL PRESSURE MATRIX (PSIA)

1	3748.	3748.	3745.	3724.	3680.	3622.	3568.	3527.	3507.	3513.	3536.	3573.	3628.	3661.	3672.	3673.	3673.
2	3747.	3750.	3753.	3772.	3695.	3619.	3562.	3516.	3495.	3505.	3531.	3568.	3638.	3688.	3678.	3674.	3672.
3	3742.	3751.	3786.	3798.	3745.	3683.	3542.	3479.	3452.	3472.	3523.	3616.	3665.	3706.	3698.	3674.	3667.
4	3718.	3755.	3765.	3858.	3739.	3655.	3585.	3397.	3365.	3399.	3541.	3596.	3659.	3745.	3697.	3676.	3647.
5	3672.	3718.	3721.	3725.	3666.	3599.	3533.	3271.	3225.	3301.	3496.	3549.	3601.	3647.	3646.	3640.	3613.
6	3620.	3627.	3657.	3639.	3592.	3527.	3451.	3065.	2999.	3158.	3425.	3486.	3538.	3575.	3594.	3567.	3570.
7	3572.	3568.	3603.	3576.	3529.	3458.	3362.	2742.	2642.	2879.	3346.	3425.	3482.	3520.	3544.	3529.	3531.
8	3536.	3531.	3517.	3524.	3475.	3390.	2876.	2404.	2097.	2370.	3263.	3374.	3438.	3477.	3484.	3496.	3500.
9	3513.	3506.	3483.	3430.	3331.	3131.	2757.	2168.	971.	2180.	2762.	3125.	3333.	3415.	3458.	3477.	3483.
10	3504.	3498.	3479.	3438.	3376.	3278.	3188.	2847.	2180.	2433.	3014.	3262.	3367.	3420.	3456.	3473.	3478.
11	3507.	3504.	3493.	3470.	3431.	3376.	3303.	3227.	2816.	3142.	3288.	3357.	3412.	3449.	3471.	3480.	3483.
12	3517.	3516.	3518.	3500.	3466.	3421.	3371.	3320.	3177.	3305.	3357.	3403.	3446.	3478.	3494.	3492.	3494.
13	3531.	3532.	3544.	3536.	3500.	3459.	3420.	3381.	3329.	3374.	3411.	3444.	3483.	3517.	3525.	3507.	3507.
14	3544.	3556.	3564.	3584.	3530.	3487.	3451.	3424.	3396.	3421.	3442.	3473.	3513.	3565.	3546.	3535.	3521.
15	3550.	3552.	3561.	3561.	3534.	3505.	3466.	3447.	3433.	3444.	3457.	3491.	3518.	3543.	3542.	3531.	3528.
16	3550.	3551.	3551.	3552.	3528.	3493.	3474.	3458.	3450.	3454.	3465.	3481.	3501.	3533.	3530.	3529.	3528.
17	3550.	3550.	3548.	3539.	3520.	3496.	3476.	3462.	3455.	3458.	3468.	3482.	3500.	3517.	3526.	3528.	3528.

MATRIX OF FRACTIONAL WATER SATURATION

1	0.144	0.144	0.144	0.179	0.145	0.145	0.145	0.145	0.145	0.145	0.145	0.145	0.146	0.156	0.144	0.144	0.144
2	0.144	0.144	0.177	0.673	0.385	0.159	0.145	0.145	0.145	0.145	0.145	0.199	0.425	0.564	0.166	0.144	0.144
3	0.144	0.183	0.681	0.825	0.797	0.673	0.186	0.145	0.145	0.145	0.244	0.788	0.804	0.815	0.649	0.173	0.144
4	0.234	0.706	0.830	0.893	0.831	0.808	0.729	0.205	0.146	0.223	0.740	0.799	0.822	0.892	0.826	0.661	0.191
5	0.181	0.673	0.803	0.842	0.809	0.792	0.738	0.238	0.147	0.267	0.749	0.792	0.811	0.842	0.799	0.575	0.156
6	0.145	0.373	0.799	0.824	0.809	0.807	0.763	0.278	0.148	0.344	0.772	0.807	0.813	0.824	0.783	0.259	0.145
7	0.145	0.156	0.643	0.808	0.801	0.819	0.789	0.247	0.151	0.407	0.803	0.820	0.804	0.810	0.585	0.151	0.145
8	0.145	0.145	0.169	0.662	0.677	0.715	0.344	0.152	0.154	0.179	0.698	0.789	0.749	0.680	0.164	0.145	0.145
9	0.145	0.145	0.145	0.172	0.174	0.171	0.178	0.154	0.161	0.153	0.167	0.273	0.242	0.180	0.145	0.145	0.145
10	0.145	0.145	0.145	0.172	0.244	0.438	0.704	0.489	0.154	0.159	0.479	0.441	0.258	0.179	0.145	0.145	0.145
11	0.145	0.145	0.161	0.625	0.740	0.788	0.818	0.801	0.256	0.567	0.819	0.790	0.743	0.647	0.157	0.145	0.145
12	0.145	0.154	0.568	0.804	0.795	0.814	0.820	0.760	0.264	0.638	0.812	0.811	0.795	0.807	0.487	0.148	0.145
13	0.146	0.334	0.792	0.833	0.813	0.812	0.797	0.673	0.185	0.518	0.786	0.804	0.809	0.832	0.773	0.250	0.145
14	0.180	0.640	0.823	0.892	0.836	0.819	0.798	0.519	0.155	0.420	0.781	0.816	0.835	0.888	0.813	0.589	0.164
15	0.145	0.179	0.647	0.821	0.788	0.743	0.323	0.149	0.146	0.146	0.227	0.692	0.783	0.818	0.612	0.167	0.145
16	0.145	0.145	0.182	0.662	0.482	0.194	0.146	0.145	0.146	0.145	0.145	0.170	0.339	0.599	0.167	0.145	0.145
17	0.145	0.145	0.145	0.197	0.151	0.145	0.145	0.145	0.145	0.145	0.145	0.145	0.146	0.167	0.145	0.145	0.145

TABLE C-5

MATERIAL BALANCE

TWO-PHASE FLOW

Time Step Δt days	$\frac{\Delta t}{\Delta t_{max}}$	Time days	Percent Deviation Water	Percent Deviation Oil
10	86	320	0.30	-4.8
		960	0.30	-2.3
		1920	0.30	-1.6
20	172	320	1.5	-7.0
		960	1.1	-3.7

APPENDIX D

SOURCE LISTINGS OF COMPUTER PROGRAMS

Computer: IBM 7040

Language : Fortran IV

Annex 1. Three-Dimensional Diffusion Equation

a) Analytical Solution

b) Brian-Douglas-Rachford Method

c) Alternating Direction Explicit Procedure

Annex 2. Two-Dimensional Model of a Liquid Reservoir

a) Alternating Direction Implicit Procedure

b) Alternating Direction Explicit Procedure

Annex 3. Nonlinear Model of a Gas Reservoir:

Alternating Direction Explicit Procedure

Annex 4. Simulation of Two-Phase Flow in Oil Reservoirs:

Alternating Direction Explicit Procedure

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C                                                        APPENDIX D    ANNX 1A
C
C
C                                       *******
C      THREE-CIMENSIONAL DIFFUSICN EQUATICN IS ANALYTICALLY
C      SCLVED BY THE METHOD CF PRODUCTS OF SOLUTIONS.
C                                       *******
C
C
      CIMENSICN TS(1C),TAX(12),TA(12,12,12)
  124 FCRMAT(1X,I2,2X,F4.1)
  125 FORMAT(1H1,51HANALYTICAL SOLUTION UNIDIRECTIONAL HEAT
     1FLCW,TAX(I))
  126 FORMAT(1HC,66HANALYTICAL SCLUTICN THREE DIMENSIONAL HE
     1AT FLCW,TEMP
     SERATURES AFTER,1X,I4,6H HOURS)
  128 FCRMAT(1H ,E14.8)
  123 FCRMAT(1HJ,10F1C.7)
  157 FCRMAT(1HJ,62H IS JS KS LS    DT    DX     PK       RHO
     1CP        DF
     S      ALFA )
  14C FCRMAT(1X,4I3,1X,F5.1,1X,F4.1,1X,F6.3,1X,F6.2,1X,F5.3)
  141 FCRMAT(1HJ,1X,4I3,1X,F5.1,1X,F4.1,1X,F6.3,1X,F6.1,1X,F
     16.3,1X,E11.5
     S,1X,F6.3)
      READ(5,14C) IS,JS,KS,LS,DT,DX,PK,RHC,CP
      READ  (5      ,124    ) NT,V
      PI=3.14159265
      PB=PI/(2.C*V)
      CF=PK/(RHC*CP)
      ZETA=DF*PB*PB
      ALFA=DX*DX/(DF*DT)
      ISM=IS-1
      JSM=JS-1
      KSM=KS-1
      WRITE(6,157)
      WRITE(6,141)IS,JS,KS,LS,DT,DX,PK,RHC,CP,DF,ALFA
      CC 2C0 L=1,LS
      LT=3*L
      FLT=LT
      CLT=CT*FLT
      WRITE (6      ,125    )
      CC 155 I=2,IS
      TAX(I)=C.C
      FI=I
      CC 153 NS=1,NT
      N=NS-1
      NP=2*N+1
      FNP=NP
      FX=(FI-1.5)*DX
      W=PB*FNP*FX
      XP=FNP*FNP*ZETA*DLT
      CW=CCS (W)
      FEP=EXP (XP)
      TS(NS)=((-1.0)**N)*CW/(FNP*FEP)
```

```
  153 TAX(I)=TAX(I)+TS(NS)
      TAX(I)=4.C*TAX(I)/PI
      WRITE (6      ,128    ) TAX(I)
  155 CCNTINUE
      CC 158 K=2,KS
      CC 158 J=2,JS
      CC 158 I=2,IS
  158 TA(I,J,K)=TAX(I)*TAX(J)*TAX(K)
      KCLT=CLT
      WRITE (6      ,126    ) KCLT
      CC 87 K=2,KSM
   87 WRITE(6,123) ((TA(I,J,K),J=2,JSM),I=2,ISM)
  2CC CCNTINUE
      ENC
```

```
C                                                   APPENDIX D   ANNX 1B
C
C
C                              *******
C     THREE-CIMENSIONAL CIFFUSICN EQUATICN SOLVED BY BRIAN-
C     CCUGLAS-RACHFORC METHCD, 10.5*10.5 GRID
C                              *******
C
C
      CIMENSICN  A(12),B(12),C(12),D(12,12,12),TX(12,12,12),
     1TY(12,12,12)
      CIMENSICN T(12,12,12),E(12),F(12),G(12,12,12)
  1C1 FCRMAT(1H1,25HCCNTINUED  K=7 TC 11       )
  1C2 FORMAT(1H/,46HJ=2 TO 11 HORIZONTAL, I=2 TO 11 VERTICAL
     1, K=   ,I2)
  103 FCRMAT(1H1,43HTEMPS. AFTER CCMPLETICN OF CYCLE AFTER
     1    ,I4,2HHR)
  123 FCRMAT(1H ,10F11.7)
  14C FORMAT(1X,4I3,1X,F5.1,1X,F4.1,1X,F6.3,1X,F6.2,1X,F5.3,
     11X,I2)
  141 FCRMAT(1HJ,1X,4I3,1X,F5.1,1X,F4.1,1X,F6.3,1X,F6.1,1X,F
     16.3,1X,E11.5
     S,1X,F6.3)
  157 FCRMAT(1HJ,62H IS JS KS LS   DT   CX    PK     RHO
     1CP      CF
     S      ALFA )
  164 FCRMAT(1X,F4.2)
      READ (5,164) FZ
      READ(5,14C) IS,JS,KS,LS,DT,DX,PK,RHC,CP,LST
      PI=3.14159265
      ISM=IS-1
      JSM=JS-1
      KSM=KS-1
      ISC=IS-2
      JSC=JS-2
      KSC=KS-2
      CF=PK/(RHC*CP)
      ALFA=CX*CX/(FZ*CT*CF)
      RF=ALFA*(1.C-FZ)/FZ
      BETA=-2.0-ALFA
      GAMA=-1.C-ALFA
      BELTA=2.C-RF
      CELTA=2.0-ALFA
      ETA=4.C-ALFA
      A(2)=GAMA
      CC 1C N=3,ISM
      A(N)=BETA
   1C C(N)=1.C
      CC 12 N=2,ISD
   12 B(N)=1.C
      E(2)=A(2)
      CC 18 N=2,ISC
      F(N)=B(N)/E(N)
   18 E(N+1)=A(N+1)-C(N+1)*F(N)
C     INITIAL CCNDITICN
```

```
      DO 15 K=2,KSM
      DO 15 J=2,JSM
      DO 15 I=2,ISM
   15 T(I,J,K)=1.0
C     BOUNDARY CONDITIONS
      DO 20 K=2,KSM
      DO 20 J=2,JSM
      TX(12,J,K)=0.
      TY(12,J,K)=0.
   20 T(12,J,K)=0.
      DO 21 I=2,ISM
      DO 21 K=2,KSM
      TX(I,12,K)=0.
      TY(I,12,K)=0.
   21 T(I,12,K)=0.
      DO 22 I=2,ISM
      DO 22 J=2,JSM
      TX(I,J,12)=0.
      TY(I,J,12)=0.
   22 T(I,J,12)=0.
      FLS=LS
      WRITE(6,157)
      WRITE(6,141) IS,JS,KS,LS,DT,DX,PK,RHO,CP,DF,ALFA
      DO 201 LA=1,LST
      TQ=DT*FLS*FLOAT(LA)
      ITQ=TQ
      DO 200 L=1,LS
C     CONDITION OF SYMMETRY
      DO 30 K=2,KSM
      DO 30 J=2,JSM
   30 T(1,J,K)=T(2,J,K)
      DO 32 K=2,KSM
      DO 32 I=2,ISM
   32 T(I,1,K)=T(I,2,K)
      DO 33 I=2,ISM
      DO 33 J=2,JSM
   33 T(I,J,1)=T(I,J,2)
C     IMPLICIT IN X-DIRECTION
      DO 36 K=2,KSM
      DO 36 J=2,JSM
      DO 36 I=2,ISM
   36 D(I,J,K)=ETA*T(I,J,K)-T(I,J-1,K)-T(I,J+1,K)-T(I,J,K-1)
     1-T(I,J,K+1)
  508 DO 37 K=2,KSM
      DO 37 J=2,JSM
      G(2,J,K)=D(2,J,K)/E(2)
      DO 37 I=3,ISM
   37 G(I,J,K)=(D(I,J,K)-C(I)*G(I-1,J,K))/E(I)
      DO 40 K=2,KSM
      DO 40 J=2,JSM
      TX(ISM,J,K)=G(ISM,J,K)
      DO 40 I=2,ISD
      IP=IS-I
   40 TX(IP,J,K)=G(IP,J,K)-F(IP)*TX(IP+1,J,K)
C     CONDITION OF SYMM.REVIVED
```

```
      CC 53 K=2,KSM
      CC 53 J=2,JSM
   53 TX(1,J,K)=TX(2,J,K)
      CC 54 K=2,KSM
      CC 54 I=2,ISM
   54 TX(I,1,K)=TX(I,2,K)
      CC 55 J=2,JSM
      CC 55 I=2,ISM
   55 TX(I,J,1)=TX(I,J,2)
C     IMPLICIT IN Y-CIRECTICN
      CC 6C K=2,KSM
      CC 6C I=2,ISM
      CC 6C J=2,JSM
   6C C(I,J,K)=T(I,J-1,K)-2.0*T(I,J,K)+T(I,J+1,K)-ALFA*TX(I,
     1J,K)
  517 CC 63 K=2,KSM
      CC 63 I=2,ISM
      G(I,2,K)=C(I,2,K)/E(2)
      CC 63 J=3,JSM
   63 G(I,J,K)=(C(I,J,K)-C(J)*G(I,J-1,K))/E(J)
      CC 65 K=2,KSM
      CC 65 I=2,ISM
      TY(I,JSM,K)=G(I,JSM,K)
      CC 65 J=2,JSD
      JP=JS-J
   65 TY(I,JP,K)=G(I,JP,K)-F(JP)*TY(I,JP+1,K)
C     CCNCITICN CF SYMM.REVIVED
      CC 76 K=2,KSM
      CC 76 J=2,JSM
   76 TY(1,J,K)=TY(2,J,K)
      CC 77 K=2,KSM
      CC 77 I=2,ISM
   77 TY(I,1,K)=TY(I,2,K)
      CC 78 J=2,JSM
      CC 78 I=2,ISM
   78 TY(I,J,1)=TY(I,J,2)
C     IMPLICIT IN Z-CIRECTICN
      CC 8C I=2,ISM
      CC 8C J=2,JSM
      CC 8C K=2,KSM
      ZG=-ALFA*TY(I,J,K)/FZ
   8C C(I,J,K)=T(I,J,K-1)-BELTA*T(I,J,K)+T(I,J,K+1)+ZQ
  525 CC 82 I=2,ISM
      CC 82 J=2,JSM
      G(I,J,2)=C(I,J,2)/E(2)
      CC 82 K=3,KSM
   82 G(I,J,K)=(C(I,J,K)-C(K)*G(I,J,K-1))/E(K)
      CC 84 I=2,ISM
      CC 84 J=2,JSM
      T(I,J,KSM)=G(I,J,KSM)
      CC 84 K=2,KSD
      KP=KS-K
   84 T(I,J,KP)=G(I,J,KP)-F(KP)*T(I,J,KP+1)
  2CC CCNTINUE
      WRITE(6,1C3) ITC
```

```
      DO 87 K=2,6
      WRITE(6,102) K
   87 WRITE(6,123) ((T(I,J,K),J=2,JSM),I=2,ISM)
      WRITE(6,101)
      DO 88 K=7,KSM
      WRITE(6,102) K
   88 WRITE(6,123) ((T(I,J,K),J=2,JSM),I=2,ISM)
  201 CONTINUE
      WRITE(6,123) T(3,3,3),T(6,6,6),T(10,10,10)
      STOP
      END
```

```
C
C
C
C                        *******
C      THREE-DIMENSIONAL DIFFUSION EQUATION SOLVED BY ADEP
C                        *******
C
C
      DIMENSION TF(12,12,12),TR(12,12,12)
    5 FORMAT(1H/,12F10.6)
    6 FORMAT(1H1,34HFORWARD SWEEP, TEMPS. AFTER          ,I5,3H
     1HRS)
    7 FORMAT(1H1,34HREVERSE SWEEP, TEMPS. AFTER          ,I5,3H
     1HRS)
    8 FORMAT(1H1,46HK=7 TO KSM,J=1,JS(HORIZONT.),I=1,ISM(VER
     1TICAL))
    9 FORMAT(1H0,40HK=2 TO 6,J=1,JS(HORIZ.),I=2,ISM(VERTIC.)
     1)
  140 FORMAT(1X,4I3,1X,F5.1,1X,F4.1,1X,F6.3,1X,F6.2,1X,F5.3,
     11X,I2)
  141 FORMAT(1H0,1X,4I3,1X,F5.1,1X,F4.1,1X,F6.3,1X,F6.1,1X,F
     16.3,1X,E11.5
     S,1X,F6.3)
  157 FORMAT(1H0,62H IS JS KS LS    DT    DX     PK      RHO
     1CP      DF
     S     ALFA )
      READ(5,140) IS,JS,KS,LS,DT,DX,PK,RHO,CP,LST
      DF=PK/(RHO*CP)
      ALFA=DX*DX/(DF*DT)
      BETA=ALFA
      WRITE(6,157)
      WRITE(6,141)IS,JS,KS,LS,DT,DX,PK,RHO,CP,DF,ALFA
      BET3=3.-BETA
      BP3=3.+BETA
      KSM=KS-1
      JSM=JS-1
      ISM=IS-1
      MDT=DT
      DO 21 K=1,KSM
      DO 21 J=1,JSM
      DO 21 I=1,ISM
      TR(I,J,K)=1.0
   21 TF(I,J,K)=1.
      DO 22 K=2,KS
      DO 22 J=2,JS
      TF(IS,J,K)=0.
   22 TR(IS,J,K)=0.
      DO 23 K=2,KS
      DO 23 I=2,IS
      TF(I,JS,K)=0.
   23 TR(I,JS,K)=0.
      DO 24 I=2,IS
      DO 24 J=2,JS
      TF(I,J,KS)=0.
   24 TR(I,J,KS)=0.
```

```
      IDT=DT
      DO 101 LA=1,LST
      MT=LA*2*LS*IDT
      NT=MT-IDT
      DO 100 L=1,LS
C     FORWARD SWEEP
      TF(2,2,2)=(TR(3,2,2)+TR(2,3,2)+TR(2,2,3)-BET3*TR(2,2,2
     1))/BETA
      DO 30 I=3,ISM
   30 TF(I,2,2)=(TF(I-1,2,2)+TR(I+1,2,2)+TR(I,3,2)+TR(I,2,3)
     1-BET3*TR(I,2
     S,2))/(1.+BETA)
      K=2
      DO 33 J=3,JSM
      I=2
      TF(I,J,K)=(TF(I,J-1,K)+TR(I+1,J,K)+TR(I,J+1,K)+TR(I,J,
     1K+1)-BET3*TR
     S(I,J,K))/(1.+BETA)
      DO 33 I=3,ISM
   33 TF(I,J,K)=(TF(I-1,J,K)+TF(I,J-1,K)+TR(I+1,J,K)+TR(I,J+
     11,K)+TR(I,J,
     SK+1)-BET3*TR(I,J,K))/(2.+BETA)
      DO 40 K=3,KSM
      J=2
      I=2
      TF(I,J,K)=(TF(I,J,K-1)+TR(I+1,J,K)+TR(I,J+1,K)+TR(I,J,
     1K+1)-BET3*TR
     S(I,J,K))/(1.+BETA)
      DO 36 I=3,ISM
   36 TF(I,J,K)=(TF(I-1,J,K)+TF(I,J,K-1)+TR(I+1,J,K)+TR(I,J+
     11,K)+TR(I,J,
     SK+1)-BET3*TR(I,J,K))/(2.+BETA)
      DO 39 J=3,JSM
      I=2
      TF(I,J,K)=(TF(I,J-1,K)+TF(I,J,K-1)+TR(I+1,J,K)+TR(I,J+
     11,K)+TR(I,J,
     SK+1)-BET3*TR(I,J,K))/(2.+BETA)
      DO 39 I=3,ISM
   39 TF(I,J,K)=(TF(I-1,J,K)+TF(I,J-1,K)+TF(I,J,K-1)+TR(I+1,
     1J,K)+TR(I,J+
     S1,K)+TR(I,J,K+1)-BET3*TR(I,J,K))/(3.+BETA)
   40 CONTINUE
      DO 41 K=1,KS
      DO 41 J=1,JS
   41 TF(1,J,K)=TF(2,J,K)
      DO 42 K=1,KS
      DO 42 I=1,IS
   42 TF(I,1,K)=TF(I,2,K)
      DO 43 I=1,IS
      DO 43 J=1,JS
   43 TF(I,J,1)=TF(I,J,2)
C     REVERSE SWEEP
      DO 50 KZ=2,KSM
      K=KS-KZ+1
      DO 50 JZ=2,JSM
```

```
      J=JS-JZ+1
      DO 50 IZ=2,ISM
      I=IS-IZ+1
   50 TR(I,J,K)=(TR(I+1,J,K)+TR(I,J+1,K)+TR(I,J,K+1)+TF(I-1,
     1J,K)+TF(I,J-
     S1,K)+TF(I,J,K-1)-BET3*TF(I,J,K))/BP3
      DO 57 K=1,KS
      DO 57 J=1,JS
   57 TR(1,J,K)=TR(2,J,K)
      DO 58 K=1,KS
      DO 58 I=1,IS
   58 TR(I,1,K)=TR(I,2,K)
      DO 60 I=1,IS
      DO 60 J=1,JS
   60 TR(I,J,1)=TR(I,J,2)
  100 CONTINUE
      WRITE(6,7) MT
      WRITE(6,9)
      DO 55 K=2,6
   55 WRITE(6,5) ((TR(I,J,K),J=1,JS),I=2,ISM)
      WRITE(6,8)
      DO 56 K=7,KSM
   56 WRITE(6,5) ((TR(I,J,K),J=1,JS),I=2,ISM)
  101 CONTINUE
      WRITE(6,5) TR(3,3,3),TR(6,6,6),TR(10,10,10)
      STOP
      END
```

```
C                                                  APPENDIX D   ANNX 2A
C
C
C      TWC-CIMENSICNAL MCDEL CF AN CIL RESERVOIR
C      SCLVED BY ACIP
C
C
C      THIS PRCGRAM PREDICTS THE BEHAVIOR OF A SINGLE PHASE,
C      SLIGHTLY CCMPRESSIBLE, CIL RESERVOIR AS OIL IS WITH-
C      DRAWN AT A KNOWN RATE FRCM FOURTEEN ARBITRARILY
C      LCCATED WELLS. THE RESERVOIR CONSIDERED IS CF IRREG-
C      ULAR PCLYGCNAL SHAPE WITH ZERO MASS FLUX ACROSS THE
C      BOUNDARY.
C      THE RESERVCIR IS SIMULATED BY A TWC-CIMENSICNAL
C      MATHEMATICAL MCDEL AND THE RESULTING PARTIAL
C      DIFFERENTIAL ECUATICN IS SCLVED BY ACIP.
C      VISCCSITY AT P(N*DT) IS USED TO CALCULATE P((N+1)*DT)
C      THE PRCGRAM IS GENERAL AND IS DESIGNED TO COMPUTE
C      PRESSURE DISTRIBUTION IN THE RESERVCIR AND MATERIAL
C      BALANCE AT ANY REQUIRED TIME AND FCR ANY KNCWN
C      PRODUCTICN RATE.
C
C                           ************
C                           NCMENCLATURE
C                           ************
C
C      A           RATIC CF CONVERSION CONSTANTS
C      B           FORMATION VCLUME FACTOR, BS,BO AT PS,PO
C      B1,B2       CONSTANTS IN B VS P RELATION
C      C           COMPRESSIBILITY OF LIQUID (1/PSIA)
C      D,E         CONSTANTS IN VISCOSITY-PRESSURE RELATION
C      DT          TIME STEP (DAYS) PER SWEEP
C      DX          =DY=SPACE GRID SIZE (FT)
C      FACTCR      CHANGES PRCDUCTION RATES CF WELLS
C      FDAY        REAL TIME FOR WHICH RESERVOIR HISTORY IS
C                  DESIRED(DAYS)
C      MU          VISCCSITY (CP) , USED WITH PRE-SCRIPTS
C      NSET        NUMBER CF DATA SET
C      NT          NUMBER OF DATA SETS USED
C      P( , )      PRESSURE (PSIA)   , USED AS PS,PO
C      PERM        PERMEABILITY (DARCY)
C      PRINT       TIME INTERVAL BETWEEN CCNSECUTIVE PRINTOUTS
C      PX( , )     PRESSURE OBTAINED BY CHANGE IN X-DIRECTION
C      PY( , )     PRESSURE OBTAINED BY CHANGE IN Y-DIRECTION
C      Q( , )      PRODUCTION RATE OF A WELL (S.T.B./DAY)
C      TIME        REAL TIME FCR WHICH RESERVOIR HISTCRY HAS
C                  BEEN COMPUTED (DAYS)
C      XKH( , )    X-DIRECTIONAL AVERAGE OF K*H(DARCY-FT)
C      YKH( , )    Y-DIRECTIONAL AVERAGE OF K*H(DARCY-FT)
C      XMU( , )    X-DIRECTIONAL AVERAGE CF VISCOSITY (CP)
C      YMU( , )    Y-DIRECTIONAL AVERAGE OF VISCOSITY (CP)
C
C      SUBSCRIPTS AND/CR PRE-SCRIPTS
C      Z,C         ORIGINAL
C      S           SATURATION
```

```
C       X,Y            DIRECTIONS
C       AT( )          ELEMENTS OF TRIDIAGONAL MATRIX
C       BT( )          * OF THE SET OF SIMULTANEOUS EQUATIONS
C       CT( )          *
C       DR( )          RIGHT HAND SIDE OF EQUATIONS
C       GT( )          INTERMEDIATE VARIABLES USED IN ADIP
C       WT( )          INTERMEDIATE VARIABLES USED IN ADIP
C
C
      COMMON  I,PX(21,21),PY(21,21),XKH(21,21),YKH(21,21),XM
     1U(21,21),YMU
     S(21,21),FIH(21,21),Q(21,21),AT(21),BT(21),CT(21),DR(21
     1),WT(21),GT(
     S21),IR,IRM,IRP,IRD,IRDP,IQ,IQM,IQP,IQPP,IQM1,JR,JRM,JR
     1P,JRD,JQ,JQM
     S,JQP,JT,JTM,IT,ITM,ITMM,ITD,JB,JE,BETA,B1,B2,TOA
      DIMENSION TIM(5,99),ACT(5,99),CAL(5,99),RATI(5,99)
    1 FORMAT(1H1,2H  )
    2 FORMAT(1HL,10X,38H   IR   IQ   IT   JR   JQ   JT   DX(FT)
     1  )
    3 FORMAT(1HK,17X,46HA                 B1               B2
     1D          E)
    4 FORMAT(1HL,14X,58HSMU     ZMU      PS       PO        BS
     1  BO     COMP
     CRESS   PERM)
    5 FORMAT(1HJ,10X,2F7.3,2F8.1,2F8.4,E10.3,F6.3)
    6 FORMAT(1HJ,10X,5E12.4)
    7 FORMAT(1HK,18X,51HMATRIX OF PRESSURE IN OIL RESERVOIR,
     1(PSIA). BY A
     1DIP)
    8 FORMAT(1HL,2H  )
    9 FORMAT(1HK,35X,6HTIME =,F6.0,5H DAYS)
   10 FORMAT(1HK,32X,12H(TIME STEP =,F6.2,6H DAYS))
   11 FORMAT(1HL,9X,66H  J  1      2      3      4      5      6
     1     7      8
     C     9     10      11)
   12 FORMAT(1H ,9X,2H I)
   13 FORMAT(1HL,29X,33HPHI*H MATRIX (FT) - OIL RESERVOIR)
   14 FORMAT(1HL,27X,37HK*H MATRIX (DARCY-FT) - OIL RESERVOI
     1R)
   15 FORMAT(1HJ,10X,6I4,F8.0)
   16 FORMAT(1HL,12X,38HPRODUCTION RATES OF WELLS (S.T.B./DA
     1Y))
   17 FORMAT(1HK,12X,66HQ(4,4)  Q(7,4)  Q(10,4)  Q(13,4)  Q(
     14,7)  Q(7,7)
     C  Q(10,7)  Q(13,7))
   18 FORMAT(1HK,12X,56HQ(16,7)  Q(19,7)  Q(10,10)  Q(13,10)
     1  Q(16,10)
     CQ(19,10))
   19 FORMAT(1X,2F7.3,2F8.1,2F8.4,E10.3,F6.3)
   20 FORMAT(1H2,3H   )
   21 FORMAT(1X,6I4,F8.0)
   22 FORMAT(1HJ,12X,F5.0,F8.0,2F9.0,2F8.0,2F9.0)
   23 FORMAT(1HJ,12X,F6.0,F9.0,4F10.0)
   24 FORMAT(1HS,9X,2H I)
```

```
 25 FORMAT(1HK,2H  )
 26 FORMAT(1H ,9X,I2,8F6.1)
 27 FORMAT(1HJ,2H  )
 28 FORMAT(1H ,9X,I2,11F6.1)
 29 FORMAT(1H3,39X,8HTABLE A-,I2)
 30 FORMAT(1H ,9X,I2,18X,8F6.1)
 31 FORMAT(1X,12F6.3/5F6.3)
 33 FORMAT(1X,12F6.3/9F6.3)
 34 FORMAT(1H2,9X,6HCONTD.,22X,8HTABLE A-,I2)
 35 FORMAT(1X,12F6.3/3F6.3)
 36 FORMAT(1HL,15X,26HINITIAL RESERVOIR PRESSURE,9X,3H PO,
   1F9.2,5X,
  C 4HPSIA)
 37 FORMAT(1HJ,15X,26HSATURATION PRESSURE OF OIL,9X,3H PS,
   1F9.2,5X,
  C 4HPSIA)
 38 FORMAT(1HJ,15X,29HFORMATION VOLUME FACTOR AT PO,6X,3H
   1BO,F12.5)
 39 FORMAT(1HJ,15X,29HFORMATION VOLUME FACTOR AT PS,6X,3H
   1BS,F12.5)
 40 FORMAT(1H ,9X,I2,8F6.0)
 41 FORMAT(1H ,9X,I2,11F6.0)
 42 FORMAT(1H ,9X,I2,18X,8F6.0)
 43 FORMAT(1X,12F6.2/5F6.2)
 44 FORMAT(1X,12F6.2/9F6.2)
 45 FORMAT(1X,12F6.2/3F6.2)
 48 FORMAT(1HA,I2,1X,14F9.0)
 49 FORMAT(1H=,27X,36HPY(10,7) IS LESS THAN SATN. PRESSURE
   1)
 50 FORMAT(1X,F7.2,2F7.0,F5.2)
 51 FORMAT(1HL,12X,24HDT   PRINT  FDAY  FACTOR)
 52 FORMAT(1X,I3)
 54 FORMAT(1H3 ,24X,42HCUMULATIVE PRODUCTION (STOCK TANK B
   1ARRELS))
 55 FORMAT(1HK,24X,6HACTUAL, E12.4,4X,10HCALCULATED, E12.4
   1)
 56 FORMAT(1HJ, 8X,3F7.0,F6.2)
 57 FORMAT(1HK,12X,65HTWO DIMENSIONAL MODEL OF A SLIGHTLY
   1COMPRESSIBLE
  C LIQUID RESERVOIR)
 58 FORMAT(1HL,15X,38HDIGITAL SIMULATION OF AN OIL RESERVO
   1IR)
 59 FORMAT(1H9,3H   )
340 FORMAT(1HJ,15X,15HVISCOSITY AT PO,20X,3HZMU,F12.5,2X,2
   1HCP)
341 FORMAT(1HJ,15X,15HVISCOSITY AT PS,20X,3HSMU,F12.5,2X,2
   1HCP)
342 FORMAT(1HJ,15X,15HCOMPRESSIBILITY,20X,3H  C,E16.5,2X,6
   1H1/PSIA)
343 FORMAT(1HK,40X,9HTABLE A-3)
344 FORMAT(1H2,37X,8HTABLE A-,I2)
345 FORMAT(1HL,28X,35HPRODUCTION PATTERN OF OIL RESERVOIR)
346 FORMAT(1HL,27X,16HLOCATION OF WELL,5X,10HPRODUCTION)
347 FORMAT(1HS,31X,7H I , J ,10X,17HQ(I,J) S.T.B./DAY)
348 FORMAT(1HJ,31X,I2,I4,13X,F6.0)
```

```
      IBB1=1
      IBB2=2
      IBC=5
C     PARAMETERS OF RESERVOIR GEOMETRY, DX=DY
      READ (5,21) IR,IQ,IT,JR,JQ,JT,DX
      READ(5,19) SMU,ZMU,PS,PO,BS,BO,C,PERM
      C=(BS-BO)*2.0/((BS+BO)*(PO-PS))
C     1 BBL= 5.6146 C.FT., 1 DARCY= 6.327 SQ.FT.*CP/( DAY*PS
     1I )
      A=5.6146/6.327
      TOA=2.0*A
      B1=(BS*PO-BO*PS)/(PO-PS)
      B2=(BO-BS)/(PO-PS)
      POPS=(PO+PS)/2.0
      BR=B1+B2*POPS
      D=(SMU*PO-ZMU*PS)/(PO-PS)
      E=(ZMU-SMU)/(2.0*(PO-PS))
      IRM=IR-1
      IRP=IR+1
      IRD=IR+2
      IQP=IQ+1
      IQM=IQ-1
      ITM=IT-1
      JRP=JR+1
      JRM=JR-1
      JRD=JR+2
      JQP=JQ+1
      JTM=JT-1
      JQM=JQ-1
      IQM1=IQM-1
      IQPP=IQP+1
      ITMM=ITM-1
      ITD=IT-2
      IRDP=IRD+1
      JTD=JT-2
      JTMM=JT-2
      JQD=JQ+2
C     PHI*H (FT) DATA
      READ(5,43) ((FIH(I,J),I=1,IQ),J=1,JRM)
      READ(5,44) ((FIH(I,J),I=1,IT),J=JR,JQ)
      READ(5,45) ((FIH(I,J),I=IR,IT),J=JQP,JT)
      WRITE(4,1)
      WRITE(4,29) IBB1
      WRITE(4,13)
      WRITE(4,11)
      WRITE(4,12)
      DO 60 I=2,IR
      IM=I-1
   60 WRITE(4,26) IM,(FIH(I,J),J=2,JQM)
      DO 61 I=IRP,IQM
      IM=I-1
   61 WRITE(4,28) IM,(FIH(I,J),J=2,JTM)
      DO 62 I=IQ,ITM
      IM=I-1
   62 WRITE(4,30) IM,(FIH(I,J),J=JRP,JTM)
```

```
      VI=0.0
      DO 801 J=2,JR
      DO 801 I=2,IQM
  801 VI=VI+FIH(I,J)
      DO 803 J=JRP,JQM
      DO 803 I=2,ITM
  803 VI=VI+FIH(I,J)
      DO 805 J=JQ,JTM
      DO 805 I=IRP,ITM
  805 VI=VI+FIH(I,J)
      VI=VI*DX*DX/(B1+B2*PO)
C     READ IN FI (=PHI) VALUES USING PY MEMORY
      READ(5,31) ((PY(I,J),I=1,IQ),J=1,JRM)
      READ(5,33) ((PY(I,J),I=1,IT),J=JR,JQ)
      READ(5,35) ((PY(I,J),I=IR,IT),J=JQP,JT)
C     CALCULATE H, K*H USING Q,PX MEMORIES RESPY., K=CONSTAN
     1T
      DO 70 J=1,JRM
      DO 70 I=1,IQ
      Q(I,J)=FIH(I,J)/PY(I,J)
   70 PX(I,J)=Q(I,J)*PERM
      DO 71 J=JR,JQ
      DO 71 I=1,IT
      Q(I,J)=FIH(I,J)/PY(I,J)
   71 PX(I,J)=Q(I,J)*PERM
      DO 72 J=JQP,JT
      DO 72 I=IR,IT
      Q(I,J)=FIH(I,J)/PY(I,J)
   72 PX(I,J)=Q(I,J)*PERM
      WRITE(4,29) IBB2
      WRITE(4,14)
      WRITE(4,11)
      WRITE(4,12)
      DO 80 I= 2,IR
      IM=I-1
   80 WRITE(4,26) IM,(PX(I,J),J=2,JQM)
      DO 82 I=IRP,IQM
      IM=I-1
   82 WRITE(4,28) IM,(PX(I,J),J=2,JTM)
      DO 84 I=IQ,ITM
      IM=I-1
   84 WRITE(4,30) IM,(PX(I,J),J=JRP,JTM)
C     CALCULATE 2*ARITH.AV. OF K*H IN X- AND Y-DIRECTIONS
      DO 101 J=1,JRM
      DO 101 I=2,IQ
  101 XKH(I,J)=PX(I,J)+PX(I-1,J)
      DO 103 J=JR,JQ
      DO 103 I=2,IT
  103 XKH(I,J)=PX(I,J)+PX(I-1,J)
      DO 105 J=JQP,JT
      DO 105 I=IRP,IT
  105 XKH(I,J)=PX(I,J)+PX(I-1,J)
      DO 109 I=1,IRM
      DO 109 J=2,JQ
  109 YKH(I,J)=PX(I,J)+PX(I,J-1)
```

```
      CC 110 I=IR,IQ
      CC 110 J=2,JT
  11C YKF(I,J)=FX(I,J)+PX(I,J-1)
      CC 112 I=IGP,IT
      CC 112 J=JRP,JT
  112 YKF(I,J)=PX(I,J)+PX(I,J-1)
      REAC(5,52) NT
      CC 888 NSET=1,NT
      REAC(5,50) CT,PRINT,FCAY,FACTOR
      WRITE(6,1)
      WRITE(6,2C)
      WRITE(6,57)
      WRITE(6,2)
      WRITE(6,15) IR,IQ,IT,JR,JG,JT,DX
      WRITE(6,4)
      WRITE(6,5) SMU,ZMU,PS,PC,BS,BO,C,PERM
      WRITE(6,3)
      WRITE(6, 6) A,B1,B2,D,E
      BETA=(C*DX*CX/CT)*(   2.0/6.327)
      WRITE(6,1)
      WRITE(6,2C)
      WRITE(6,58)
      WRITE(6,8)
      WRITE(6,8)
      WRITE(6,36) PC
      WRITE(6,37) PS
      WRITE(6,38) BO
      WRITE(6,39) BS
      WRITE(6,34C) ZMU
      WRITE(6,341) SMU
      WRITE(6,342) C
      CT2=CT*2.C
C     INITIALIZE PRESSURES, CLEAR Q MEMORY
      CO 120 J=1,JRM
      CC 120 I=1,IQ
      G(I,J)=0.C
  12C PY(I,J)=PC
      CO 123 J=JR,JQ
      CC 123 I=1,IT
      G(I,J)=0.C
  123 PY(I,J)=PC
      CO 125 J=JGP,JT
      CC 125 I=IR,IT
      G(I,J)=0.0
  125 PY(I,J)=PC
C     SPECIFY PRCDUCTION RATES CF CIFFERENT WELLS
      G(1C,1C)=75.0*FACTOR
      G(13,1C)=75.0*FACTOR
      G(16,10)=75.0*FACTOR
      G(19,1C)=75.0*FACTOR
      G(4,7)=125.C*FACTCR
      G(7,7)=125.C*FACTCR
      G(1C,7)=125.0*FACTOR
      G(13,7)=125.0*FACTOR
      G(16,7)=125.0*FACTCR
```

```
      G(19,7)=125.0*FACTOR
      G(4,4)=75.C*FACTOR
      G(7,4)=75.C*FACTOR
      G(1C,4)=75.C*FACTCR
      G(13,4)=75.0*FACTCR
      WRITE(4,2C)
      WRITE(4,25)
      WRITE(4,343)
      WRITE(4,345)
      WRITE(4,346)
      WRITE(4,347)
      CC 465 J=2,JTM
      CC 465 I=2,ITM
      IF(G(I,J) .GT. C.C) GC TO 463
      GC TC 465
  463 IM=I-1
      JM=J-1
      WRITE(4,348) IM,JM,G(I,J)
  465 CCNTINUE
      WRITE(6,2C)
      WRITE(6,16)
      WRITE(6,17)
      WRITE(6,22) Q(4,4),Q(7,4),G(10,4),G(13,4),Q(4,7),Q(7,7
     1),
     C G(1C,7),G(13,7)
      WRITE(6,18)
      WRITE(6,23) Q(16,7),Q(19,7),G(1C,1C),Q(13,10),Q(16,10)
     1,G(19,10)
      WRITE(6,51)
      WRITE(6,56)CT,PRINT,FCAY,FACTOR
      WRITE(4,344) IBC
      PROC=(Q(1C,10)+G(13,1C)+Q(16,10)+ G(19,10)+Q(4,7)+Q(7,
     17)+
     C G(1C,7)+G(13,7)+G(16,7)+G(19,7)+G(4,4)+Q(7,4)+G(10,4)
     1+
     C G(13,4))
      NML=C
      TIME=C.C
 3CCC CCNTINUE
      CUTPLT=C.C
C     PRINTCUT CYCLE
  5CO CUTPLT=CUTPUT+CT2
      TIME=TIME+CT2
C     IMPLICIT IN X-CIRECTICN
C     CALCULATE MU(I,J) IN THE X- AND Y-CIRECTIONS
      CC 150 J=1,JRM
      CC 15C I=2,IQ
  15C XMU(I,J)=C+E*(PY(I,J)+PY(I-1,J))
      CC 151 J=JR,JQ
      CC 151 I=2,IT
  151 XMU(I,J)=C+E*(PY(I,J)+PY(I-1,J))
      CC 152 J=JGP,JT
      CC 152 I=IRP,IT
  152 XMU(I,J)=C+E*(PY(I,J)+PY(I-1,J))
      CC 154 I=1,IRM
```

```
      CC 154 J=2,JQ
  154 YMU(I,J)=C+E*(PY(I,J)+PY(I,J-1))
      CC 155 I=IR,IQ
      CC 155 J=2,JT
  155 YMU(I,J)=C+E*(PY(I,J)+PY(I,J-1))
      CC 157 I=IQP,IT
      CC 157 J=JRP,JT
  157 YMU(I,J)=C+E*(PY(I,J)+PY(I,J-1))
      J=2
      I=2
      R=BETA*FIH(I,J)
      T=YKH(I,J+1)/YMU(I,J+1)
      U=B1+B2*PY(I,J)
      BT(I)=-XKH(I+1,J)/XMU(I+1,J)
      AT(I)=R-BT(I)
      CR(I)=(R-T)*PY(I,J)+T*PY(I,J+1)-TCA*U*Q(I,J)
      CC 2CC I=3,IQM1
      R=BETA*FIH(I,J)
      S=YKH(I,J)/YMU(I,J)
      T=YKH(I,J+1)/YMU(I,J+1)
      U=B1+B2*PY(I,J)
      CT(I)=-XKH(I,J)/XMU(I,J)
      BT(I)=-XKH(I+1,J)/XMU(I+1,J)
      AT(I)=R-CT(I)-BT(I)
  2CC CR(I)=(R-T)*PY(I,J)+T*PY(I,J+1)-TCA*U*Q(I,J)
      I=IQM
      R=BETA*FIH(I,J)
      CT(I)=-XKH(I,J)/XMU(I,J)
      AT(I)=R-CT(I)
      S=YKH(I,J)/YMU(I,J)
      T=YKH(I,J+1)/YMU(I,J+1)
      U=B1+B2*PY(I,J)
      CR(I)=(R-T)*PY(I,J)+T*PY(I,J+1)-TCA*U*Q(I,J)
      WT(2)=AT(2)
      GT(2)=CR(2)/WT(2)
      CC 2C2 I=3,IQM
      WT(I)=AT(I)-CT(I)*BT(I-1)/WT(I-1)
  2C2 GT(I)=(CR(I)-CT(I)*GT(I-1))/WT(I)
      PX(IQM,2)=GT(IQM)
      PX(IQM,1)=PX(IQM,2)
      CC 2C4 IZ=3,IQM
      I=IQM+2-IZ
      PX(I,J)=GT(I)-PX(I+1,J)*BT(I)/WT(I)
  2C4 PX(I,1)=PX(I,2)
      CC 212 J=3,JR
      I=2
      R=BETA*FIH(I,J)
      BT(I)=-XKH(I+1,J)/XMU(I+1,J)
      AT(I)=R-BT(I)
      S=YKH(I,J)/YMU(I,J)
      T=YKH(I,J+1)/YMU(I,J+1)
      U=B1+B2*PY(I,J)
      CR(I)=S*PY(I,J-1)+(R-S-T)*PY(I,J)+T*PY(I,J+1)-TCA*U*Q(
     1I,J)
      CC 2C6 I=3,IQM1
```

```
      R=BETA*FIH(I,J)
      CT(I)=-XKH(I,J)/XMU(I,J)
      BT(I)=-XKH(I+1,J)/XMU(I+1,J)
      AT(I)=R-CT(I)-BT(I)
      S=YKH(I,J)/YMU(I,J)
      T=YKH(I,J+1)/YMU(I,J+1)
      U=B1+B2*PY(I,J)
  206 CR(I)=S*PY(I,J-1)+(R-S-T)*PY(I,J)+T*PY(I,J+1)-TOA*U*Q(
     1I,J)
      I=IQM
      R=BETA*FIH(I,J)
      CT(I)=-XKH(I,J)/XMU(I,J)
      AT(I)=R-CT(I)
      S=YKH(I,J)/YMU(I,J)
      T=YKH(I,J+1)/YMU(I,J+1)
      U=B1+B2*PY(I,J)
      CR(I)=S*PY(I,J-1)+(R-S-T)*PY(I,J)+T*PY(I,J+1)-TOA*U*Q(
     1I,J)
      WT(2)=AT(2)
      GT(2)=CR(2)/WT(2)
      DO 208 I=3,IQM
      WT(I)=AT(I)-CT(I)*BT(I-1)/WT(I-1)
  208 GT(I)=(CR(I)-CT(I)*GT(I-1))/WT(I)
      PX(IQM,J)=GT(IQM)
      DO 210 IZ=3,IQM
      I=IQ+1-IZ
  210 PX(I,J)=GT(I)-PX(I+1,J)*BT(I)/WT(I)
  212 CONTINUE
      DO 213 J=1,JR
  213 PX(IQ,J)=PX(IQM,J)
      PX(IQ,JRP)=PX(IQ,JR)
      J=JRP
      I=2
      R=BETA*FIH(I,J)
      BT(I)=-XKH(I+1,J)/XMU(I+1,J)
      AT(I)=R-BT(I)
      S=YKH(I,J)/YMU(I,J)
      T=YKH(I,J+1)/YMU(I,J+1)
      U=B1+B2*PY(I,J)
      CR(I)=S*PY(I,J-1)+(R-S-T)*PY(I,J)+T*PY(I,J+1)-TOA*U*Q(
     1I,J)
      DO 215 I=3,IQM1
      R=BETA*FIH(I,J)
      CT(I)=-XKH(I,J)/XMU(I,J)
      BT(I)=-XKH(I+1,J)/XMU(I+1,J)
      AT(I)=R-CT(I)-BT(I)
      S=YKH(I,J)/YMU(I,J)
      T=YKH(I,J+1)/YMU(I,J+1)
      U=B1+B2*PY(I,J)
  215 CR(I)=S*PY(I,J-1)+(R-S-T)*PY(I,J)+T*PY(I,J+1)-TOA*U*Q(
     1I,J)
      I=IQM
      R=BETA*FIH(I,J)
      CT(I)=-XKH(I,J)/XMU(I,J)
      BT(I)=-XKH(I+1,J)/XMU(I+1,J)
```

```
      AT(I)=R-CT(I)-BT(I)
      DR(I)=S*PY(I,J-1)+(R-S-T)*PY(I,J)+T*PY(I,J+1)-TOA*U*Q(
     1I,J)-BT(I)*P
     SX(I+1,J)
      I=2
      WT(2)=AT(2)
      GT(2)=DR(2)/WT(2)
      DO 216 I=3,IQM
      WT(I)=AT(I)-CT(I)*BT(I-1)/WT(I-1)
  216 GT(I)=(DR(I)-CT(I)*GT(I-1))/WT(I)
      PX(IQM,J)=GT(IQM)
      DO 217 IZ=3,IQM
      I=IQP-IZ
  217 PX(I,J)=GT(I)-PX(I+1,J)*BT(I)/WT(I)
      I=IQP
      R=BETA*FIH(I,J)
      CT(I)=-XKH(I,J)/XMU(I,J)
      BT(I)=-XKH(I+1,J)/XMU(I+1,J)
      AT(I)=R-CT(I)-BT(I)
      T=YKH(I,J+1)/YMU(I,J+1)
      U=B1+B2*PY(I,J)
      DR(I)=(R-T)*PY(I,J)+T*PY(I,J+1)-TOA*U*Q(I,J)-CT(I)*PX(
     1I-1,J)
      DO 218 I=IQPP,ITMM
      R=BETA*FIH(I,J)
      CT(I)=-XKH(I,J)/XMU(I,J)
      BT(I)=-XKH(I+1,J)/XMU(I+1,J)
      AT(I)=R-CT(I)-BT(I)
      S=YKH(I,J)/YMU(I,J)
      T=YKH(I,J+1)/YMU(I,J+1)
      U=B1+B2*PY(I,J)
      DR(I)=S*PY(I,J-1)+(R-S-T)*PY(I,J)+T*PY(I,J+1)-TOA*U*Q(
     1I,J)
  218 CONTINUE
      I=ITM
      R=BETA*FIH(I,J)
      CT(I)=-XKH(I,J)/XMU(I,J)
      AT(I)=R-CT(I)
      S=YKH(I,J)/YMU(I,J)
      T=YKH(I,J+1)/YMU(I,J+1)
      U=B1+B2*PY(I,J)
      DR(I)=S*PY(I,J-1)+(R-S-T)*PY(I,J)+T*PY(I,J+1)-TOA*U*Q(
     1I,J)
      WT(IQP)=AT(IQP)
      GT(IQP)=DR(IQP)/WT(IQP)
      DO 220 I=IQPP,ITM
      WT(I)=AT(I)-CT(I)*BT(I-1)/WT(I-1)
  220 GT(I)=(DR(I)-CT(I)*GT(I-1))/WT(I)
      PX(ITM,J)=GT(ITM)
      PX(ITM,JR)=PX(ITM,JRP)
      ITQ2=IT-IQ-2
      DO 225 IZ=1,ITQ2
      I=ITM-IZ
      PX(I,J)=GT(I)-PX(I+1,J)*BT(I)/WT(I)
  225 PX(I,JR)=PX(I,JRP)
```

```
      DO 230 J=JRD,JQM
      I=2
      R=BETA*FIH(I,J)
      BT(I)=-XKH(I+1,J)/XMU(I+1,J)
      AT(I)=R-BT(I)
      S=YKH(I,J)/YMU(I,J)
      T=YKH(I,J+1)/YMU(I,J+1)
      U=B1+B2*PY(I,J)
      DR(I)=S*PY(I,J-1)+(R-S-T)*PY(I,J)+T*PY(I,J+1)-TOA*U*Q(
     1I,J)
      DO 227 I=3,ITD
      R=BETA*FIH(I,J)
      CT(I)=-XKH(I,J)/XMU(I,J)
      BT(I)=-XKH(I+1,J)/XMU(I+1,J)
      AT(I)=R-CT(I)-BT(I)
      S=YKH(I,J)/YMU(I,J)
      T=YKH(I,J+1)/YMU(I,J+1)
      U=B1+B2*PY(I,J)
  227 DR(I)=S*PY(I,J-1)+(R-S-T)*PY(I,J)+T*PY(I,J+1)-TOA*U*Q(
     1I,J)
      I=ITM
      R=BETA*FIH(I,J)
      CT(I)=-XKH(I,J)/XMU(I,J)
      AT(I)=R-CT(I)
      S=YKH(I,J)/YMU(I,J)
      T=YKH(I,J+1)/YMU(I,J+1)
      U=B1+B2*PY(I,J)
      DR(I)=S*PY(I,J-1)+(R-S-T)*PY(I,J)+T*PY(I,J+1)-TOA*U*Q(
     1I,J)
      I=2
      WT(I)=AT(I)
      GT(I)=DR(I)/WT(I)
      DO 228 I=3,ITM
      WT(I)=AT(I)-CT(I)*BT(I-1)/WT(I-1)
  228 GT(I)=(DR(I)-CT(I)*GT(I-1))/WT(I)
      PX(ITM,J)=GT(ITM)
      DO 229 IZ=3,ITM
      I=ITM+2-IZ
  229 PX(I,J)=GT(I)-PX(I+1,J)*BT(I)/WT(I)
  230 CONTINUE
      DO 232 J=1,JQM
  232 PX(1,J)=PX(2,J)
      DO 233 I=1,IR
  233 PX(I,JQ)=PX(I,JQM)
      PX(IRP,JQ)=PX(IR,JQ)
      J=JQ
      I=IRD
      R=BETA*FIH(I,J)
      BT(I)=-XKH(I+1,J)/XMU(I+1,J)
      CT(I)=-XKH(I,J)/XMU(I,J)
      AT(I)=R-CT(I)-BT(I)
      S=YKH(I,J)/YMU(I,J)
      T=YKH(I,J+1)/YMU(I,J+1)
      U=B1+B2*PY(I,J)
      DR(I)=S*PY(I,J-1)+(R-S-T)*PY(I,J)+T*PY(I,J+1)-TOA*U*Q(
     1I,J)-CT(I)*P
```

```
     SX(I-1,J)
      DO 235 I=IRDP,ITD
      R=BETA*FIH(I,J)
      CT(I)=-XKH(I,J)/XMU(I,J)
      BT(I)=-XKH(I+1,J)/XMU(I+1,J)
      AT(I)=R-CT(I)-BT(I)
      S=YKH(I,J)/YMU(I,J)
      T=YKH(I,J+1)/YMU(I,J+1)
      U=B1+B2*PY(I,J)
  235 DR(I)=S*PY(I,J-1)+(R-S-T)*PY(I,J)+T*PY(I,J+1)-TCA*U*Q(
     1I,J)
      I=ITM
      R=BETA*FIH(I,J)
      CT(I)=-XKH(I,J)/XMU(I,J)
      AT(I)=R-CT(I)
      S=YKH(I,J)/YMU(I,J)
      T=YKH(I,J+1)/YMU(I,J+1)
      U=B1+B2*PY(I,J)
      DR(I)=S*PY(I,J-1)+(R-S-T)*PY(I,J)+T*PY(I,J+1)-TCA*U*Q(
     1I,J)
      WT(IRD)=AT(IRD)
      GT(IRD)=DR(IRD)/WT(IRD)
      IR3=IR+3
      DO 237 I=IR3,ITM
      WT(I)=AT(I)-CT(I)*BT(I-1)/WT(I-1)
  237 GT(I)=(DR(I)-CT(I)*GT(I-1))/WT(I)
      PX(ITM,J)=GT(ITM)
      ITR=ITM-IRD
      DO 238 IZ=1,ITR
      I=ITM-IZ
  238 PX(I,J)=GT(I)-PX(I+1,J)*BT(I)/WT(I)
      DO 244 J=JQP,JTM
      I=IRP
      R=BETA*FIH(I,J)
      BT(I)=-XKH(I+1,J)/XMU(I+1,J)
      AT(I)=R-BT(I)
      S=YKH(I,J)/YMU(I,J)
      T=YKH(I,J+1)/YMU(I,J+1)
      U=B1+B2*PY(I,J)
      DR(I)=S*PY(I,J-1)+(R-S-T)*PY(I,J)+T*PY(I,J+1)-TCA*U*Q(
     1I,J)
      DO 240 I=IRD,ITD
      R=BETA*FIH(I,J)
      CT(I)=-XKH(I,J)/XMU(I,J)
      BT(I)=-XKH(I+1,J)/XMU(I+1,J)
      AT(I)=R-BT(I)-CT(I)
      S=YKH(I,J)/YMU(I,J)
      T=YKH(I,J+1)/YMU(I,J+1)
      U=B1+B2*PY(I,J)
  240 DR(I)=S*PY(I,J-1)+(R-S-T)*PY(I,J)+T*PY(I,J+1)-TCA*U*Q(
     1I,J)
      I=ITM
      R=BETA*FIH(I,J)
      CT(I)=-XKH(I,J)/XMU(I,J)
      AT(I)=R-CT(I)
```

```
      S=YKF(I,J)/YMU(I,J)
      T=YKF(I,J+1)/YMU(I,J+1)
      U=B1+B2*PY(I,J)
      CR(I)=S*PY(I,J-1)+(R-S-T)*PY(I,J)+T*PY(I,J+1)-TCA*U*Q(
     1I,J)
      I=IRP
      WT(I)=AT(I)
      GT(I)=CR(I)/WT(I)
      DO 242 I=IRD,ITM
      WT(I)=AT(I)-CT(I)*BT(I-1)/WT(I-1)
  242 GT(I)=(CR(I)-CT(I)*GT(I-1))/WT(I)
      PX(ITM,J)=GT(ITM)
      ITRM=ITM-IRM
      DO 244 IZ=3,ITRM
      I=ITM+2-IZ
  244 PX(I,J)=GT(I)-PX(I+1,J)*BT(I)/WT(I)
C     BOUNDARY CONDITION OF 0.0 FLUX, USE REFLECTION
      DO 246 J=JQP,JTM
  246 PX(IR,J)=PX(IRP,J)
      DO 247 J=JR,JTM
  247 PX(IT,J)=PX(ITM,J)
      DO 248 I=IR,IT
  248 PX(I,JT)=PX(I,JTM)
C     IMPLICIT IN Y-DIRECTION
C     CALCULATE MU(I,J) IN X- AND Y-DIRECTIONS
      DO 251 J=1,JRM
      DO 251 I=2,IQ
  251 XMU(I,J)=D+E*(PX(I,J)+PX(I-1,J))
      DO 252 J=JR,JQ
      DO 252 I=2,IT
  252 XMU(I,J)=D+E*(PX(I,J)+PX(I-1,J))
      DO 253 J=JQP,JT
      DO 253 I=IRP,IT
  253 XMU(I,J)=D+E*(PX(I,J)+PX(I-1,J))
      DO 256 I=1,IRM
      DO 256 J=2,JQ
  256 YMU(I,J)=D+E*(PX(I,J)+PX(I,J-1))
      DO 257 I=IR,IQ
      DO 257 J=2,JT
  257 YMU(I,J)=D+E*(PX(I,J)+PX(I,J-1))
      DO 258 I=IQP,IT
      DO 258 J=JRP,JT
  258 YMU(I,J)=D+E*(PX(I,J)+PX(I,J-1))
      I=2
      J=2
      R=BETA*FIF(I,J)
      BT(J)=-YKF(I,J+1)/YMU(I,J+1)
      AT(J)=R-BT(J)
      T=XKF(I+1,J)/XMU(I+1,J)
      U=B1+B2*PX(I,J)
      CR(J)=(R-T)*PX(I,J)+T*PX(I+1,J)-TCA*U*Q(I,J)
      WT(J)=AT(J)
      GT(J)=CR(J)/WT(J)
      JE=JQ-2
      DO 350 J=3,JE
```

```
      R=BETA*FIH(I,J)
      BT(J)=-YKH(I,J+1)/YMU(I,J+1)
      CT(J)=-YKH(I,J)/YMU(I,J)
      AT(J)=R-BT(J)-CT(J)
      T=XKH(I+1,J)/XMU(I+1,J)
      U=B1+B2*PX(I,J)
      DR(J)=(R-T)*PX(I,J)+T*PX(I+1,J)-TOA*U*Q(I,J)
      WT(J)=AT(J)-CT(J)*BT(J-1)/WT(J-1)
  350 GT(J)=(DR(J)-CT(J)*GT(J-1))/WT(J)
      J=JQM
      R=BETA*FIH(I,J)
      CT(J)=-YKH(I,J)/YMU(I,J)
      AT(J)=R-CT(J)
      DR(J)=(R-T)*PX(I,J)+T*PX(I+1,J)-TOA*U*Q(I,J)
      WT(J)=AT(J)-CT(J)*BT(J-1)/WT(J-1)
      GT(J)=(DR(J)-CT(J)*GT(J-1))/WT(J)
      JB=3
      CALL TOMASY
      DO 353 J=2,JQM
  353 PY(1,J)=PY(2,J)
      DO 355 I=3,IR
      CALL TRIDIY
      CALL TOMASY
  355 CONTINUE
      DO 357 I=1,IR
  357 PY(I,JQ)=PY(I,JQM)
      PY(IRP,JQ)=PY(IR,JQ)
      I=IRP
      J=2
      R=BETA*FIH(I,J)
      BT(J)=-YKH(I,J+1)/YMU(I,J+1)
      AT(J)=R-BT(J)
      S=XKH(I,J)/XMU(I,J)
      T=XKH(I+1,J)/XMU(I+1,J)
      U=B1+B2*PX(I,J)
      DR(J)=S*PX(I-1,J)+(R-S-T)*PX(I,J)+T*PX(I+1,J)-TOA*U*Q(
     1I,J)
      WT(J)=AT(J)
      GT(J)=DR(J)/WT(J)
      DO 359 J=3,JE
      R=BETA*FIH(I,J)
      CT(J)=-YKH(I,J)/YMU(I,J)
      BT(J)=-YKH(I,J+1)/YMU(I,J+1)
      AT(J)=R-BT(J)-CT(J)
      S=XKH(I,J)/XMU(I,J)
      T=XKH(I+1,J)/XMU(I+1,J)
      U=B1+B2*PX(I,J)
      DR(J)=S*PX(I-1,J)+(R-S-T)*PX(I,J)+T*PX(I+1,J)-TOA*U*Q(
     1I,J)
      WT(J)=AT(J)-CT(J)*BT(J-1)/WT(J-1)
      GT(J)=(DR(J)-CT(J)*GT(J-1))/WT(J)
  359 CONTINUE
      J=JE+1
      R=BETA*FIH(I,J)
      BT(J)=-YKH(I,J+1)/YMU(I,J+1)
```

```
      CT(J)=-YKH(I,J)/YMU(I,J)
      AT(J)=R-BT(J)-CT(J)
      S=XKH(I,J)/XMU(I,J)
      T=XKH(I+1,J)/XMU(I+1,J)
      DR(J)=S*PX(I-1,J)+(R-S-T)*PX(I,J)+T*PX(I+1,J)-TOA*U*Q(
     1I,J)-BT(J)*P
     SY(I,J+1)
      WT(J)=AT(J)-CT(J)*BT(J-1)/WT(J-1)
      GT(J)=(DR(J)-CT(J)*GT(J-1))/WT(J)
      CALL TOMASY
      J=JQP
      R=BETA*FIH(I,J)
      BT(J)=-YKH(I,J+1)/YMU(I,J+1)
      CT(J)=-YKH(I,J)/YMU(I,J)
      AT(J)=R-BT(J)-CT(J)
      T=XKH(I+1,J)/XMU(I+1,J)
      U=B1+B2*PX(I,J)
      DR(J)=(R-T)*PX(I,J)+T*PX(I+1,J)-TOA*U*Q(I,J)-CT(J)*PY(
     1I,J-1)
      WT(J)=AT(J)
      GT(J)=DR(J)/WT(J)
      IF(JTMM.LT.JQD) GO TO 361
      DO 360 J=JQD,JTMM
      R=BETA*FIH(I,J)
      BT(J)=-YKH(I,J+1)/YMU(I,J+1)
      CT(J)=-YKH(I,J)/YMU(I,J)
      AT(J)=R-BT(J)-CT(J)
      T=XKH(I+1,J)/XMU(I+1,J)
      U=B1+B2*PX(I,J)
      DR(J)=(R-T)*PX(I,J)+T*PX(I+1,J)-TOA*U*Q(I,J)
      WT(J)=AT(J)-CT(J)*BT(J-1)/WT(J-1)
  360 GT(J)=(DR(J)-CT(J)*GT(J-1))/WT(J)
  361 J=JTM
      R=BETA*FIH(I,J)
      CT(J)=-YKH(I,J)/YMU(I,J)
      AT(J)=R-CT(J)
      T=XKH(I+1,J)/XMU(I+1,J)
      U=B1+B2*PX(I,J)
      DR(J)=(R-T)*PX(I,J)+T*PX(I+1,J)-TOA*U*Q(I,J)
      WT(J)=AT(J)-CT(J)*BT(J-1)/WT(J-1)
      GT(J)=(DR(J)-CT(J)*GT(J-1))/WT(J)
      JB=JG+2
      JE=JT-2
      CALL TOMASY
      DO 363 J=JQP,JTM
  363 PY(IR,J)=PY(IRP,J)
      JB=3
      JE=JT-2
      DO 365 I=IRD,IGM
      CALL TRIDIY
      CALL TOMASY
  365 CONTINUE
      DO 368 I=1,IQM
  368 PY(I,1)=PY(I,2)
      DO 370 J=1,JR
```

```
 37C PY(IG,J)=PY(IGM,J)
     PY(IG,JRP)=PY(IG,JR)
     I=IG
     J=JRC
     R=BETA*FIF(I,J)
     BT(J)=-YKF(I,J+1)/YMU(I,J+1)
     CT(J)=-YKF(I,J)/YMU(I,J)
     AT(J)=R-BT(J)-CT(J)
     S=XKF(I,J)/XMU(I,J)
     T=XKF(I+1,J)/XMU(I+1,J)
     U=B1+B2*PX(I,J)
     DR(J)=S*PX(I-1,J)+(R-S-T)*PX(I,J)+T*PX(I+1,J)-TCA*U*Q(
    1I,J)-CT(J)*P
    SY(I,J-1)
     WT(J)=AT(J)
     GT(J)=DR(J)/WT(J)
     JB=JR+3
     JE=JT-2
     DO 373 J=JB,JE
     R=BETA*FIF(I,J)
     BT(J)=-YKF(I,J+1)/YMU(I,J+1)
     CT(J)=-YKF(I,J)/YMU(I,J)
     AT(J)=R-BT(J)-CT(J)
     S=XKF(I,J)/XMU(I,J)
     T=XKF(I+1,J)/XMU(I+1,J)
     U=B1+B2*PX(I,J)
     DR(J)=S*PX(I-1,J)+(R-S-T)*PX(I,J)+T*PX(I+1,J)-TCA*U*Q(
    1I,J)
     WT(J)=AT(J)-CT(J)*BT(J-1)/WT(J-1)
 373 GT(J)=(DR(J)-CT(J)*GT(J-1))/WT(J)
     J=JE+1
     R=BETA*FIF(I,J)
     CT(J)=-YKF(I,J)/YMU(I,J)
     AT(J)=R-CT(J)
     S=XKF(I,J)/XMU(I,J)
     T=XKF(I+1,J)/XMU(I+1,J)
     U=B1+B2*PX(I,J)
     DR(J)=S*PX(I-1,J)+(R-S-T)*PX(I,J)+T*PX(I+1,J)-TCA*U*Q(
    1I,J)
     WT(J)=AT(J)-CT(J)*BT(J-1)/WT(J-1)
     GT(J)=(DR(J)-CT(J)*GT(J-1))/WT(J)
     CALL TOMASY
     JB=JR+2
     JE=JTD
     DO 375 I=IGP,ITD
     CALL TRIDIY
     CALL TOMASY
 375 CONTINUE
     I=ITM
     J=JRP
     R=BETA*FIF(I,J)
     BT(J)=-YKF(I,J+1)/YMU(I,J+1)
     AT(J)=R-BT(J)
     S=XKF(I,J)/XMU(I,J)
     U=B1+B2*PX(I,J)
```

```
      CR(J)=S*PX(I-1,J)+(R-S)*PX(I,J)-TCA*U*Q(I,J)
      WT(J)=AT(J)
      GT(J)=CR(J)/WT(J)
      CO 378 J=JRC,JTC
      R=BETA*FIH(I,J)
      BT(J)=-YKH(I,J+1)/YMU(I,J+1)
      CT(J)=-YKH(I,J)/YMU(I,J)
      AT(J)=R-BT(J)-CT(J)
      S=XKH(I,J)/XMU(I,J)
      U=B1+B2*PX(I,J)
      CR(J)=S*PX(I-1,J)+(R-S)*PX(I,J)-TCA*U*Q(I,J)
      WT(J)=AT(J)-CT(J)*BT(J-1)/WT(J-1)
  378 GT(J)=(CR(J)-CT(J)*GT(J-1))/WT(J)
      J=JTM
      R=BETA*FIH(I,J)
      CT(J)=-YKH(I,J)/YMU(I,J)
      AT(J)=R-CT(J)
      S=XKH(I,J)/XMU(I,J)
      U=B1+B2*PX(I,J)
      CR(J)=S*PX(I-1,J)+(R-S)*PX(I,J)-TCA*U*Q(I,J)
      WT(J)=AT(J)-CT(J)*BT(J-1)/WT(J-1)
      GT(J)=(CR(J)-CT(J)*GT(J-1))/WT(J)
      JB=JRC
      JE=JTC
      CALL TOMASY
C     BOUNCARY CCNDITICN CF 0.0 FLUX, USE REFLECTION
      CO 380 I=IGP,ITM
  38C PY(I,JR)=PY(I,JRP)
      CC 383 J=JR,JTM
  383 PY(IT,J)=PY(ITM,J)
      CC 385 I=IR,IT
  385 PY(I,JT)=PY(I,JTM)
      IF(CLTPUT .LT. PRINT) GO TC 500
      WRITE(4,7)
      WRITE(4,1C) DT2
      WRITE(4,27)
      WRITE(4,9) TIME
      WRITE(4,11)
      WRITE(4,24)
      CO 39C I=2,IR
      IM=I-1
  390 WRITE(4,4C) IM,(PY(I,J),J=2,JQM)
      CC 291 I=IRP,IGM
      IM=I-1
  291 WRITE(4,41) IM,(PY(I,J),J=2,JTM)
      CC 292 I=IG,ITM
      IM=I-1
  292 WRITE(4,42) IM,(PY(I,J),J=JRP,JTM)
      WRITE(4,34) IBC
      VF=C.C
      CC 9C1 J=2,JR
      CC 9C1 I=2,IQM
  9C1 VF=VF+FIH(I,J)/(B1+B2*PY(I,J))
      CC 9C3 J=JRP,JGM
      CC 9C3 I=2,ITM
```

```
903 VF=VF+FIH(I,J)/(B1+B2*PY(I,J))
    DO 905 J=JG,JTM
    DO 905 I=IRP,ITM
905 VF=VF+FIH(I,J)/(B1+B2*PY(I,J))
    VF=VF*DX*DX
    DV=(VI-VF)/5.6146
    PRC =PROD*TIME
    NML=NML+1
    TIM(NSET,NML)=TIME
    ACT(NSET,NML)=PRC
    CAL(NSET,NML)=DV
    RATI(NSET,NML)=PRC/DV
    IF( PY(10,7) .LT. PS) GO TO 555
    IF(TIME .LT. FDAY) GO TO 3000
    GO TO 887
555 WRITE(6,49)
    GO TO 888
887 NMT=NML
    WRITE(6,54)
    WRITE(6,349) (TIM(NSET,NML),ACT(NSET,NML),CAL(NSET,NML
   1),
   C RATI(NSET,NML),NML=1,NMT)
349 FORMAT(1H ,18X,F6.0,2E14.4,F8.4)
    IBC=IBC+2
888 CONTINUE
    STOP
    END
```

```
      SUBRCUTINE TRICIY
C
C
C      TRICIY CALCULATES THE TRIDIAGONAL ELEMENTS OF THE COEF
     1FICIENT
C      MATRIX ANC THE RIGHT HAND SIDE CF ECLATIONS, TRANSFORM
     1S THEM
C      INTC INTERMEDIATE VARIABLES
C
      COMMCN  I,PX(21,21),PY(21,21),XKH(21,21),YKH(21,21),XM
     1U(21,21),YMU
     S(21,21),FIH(21,21),Q(21,21),AT(21),BT(21),CT(21),DR(21
     1),WT(21),GT(
     S21),IR,IRM,IRP,IRD,IRDP,IQ,IQM,IQP,IQPP,IQM1,JR,JRM,JR
     1P,JRD,JQ,JQM
     S,JQP,JT,JTM,IT,ITM,ITMM,ITD,JB,JE,BETA,B1,B2,TOA
      J=JB-1
      R=BETA*FIH(I,J)
      BT(J)=-YKH(I,J+1)/YMU(I,J+1)
      AT(J)=R-BT(J)
      S=XKH(I,J)/XMU(I,J)
      T=XKH(I+1,J)/XMU(I+1,J)
      U=B1+B2*PX(I,J)
      DR(J)=S*PX(I-1,J)+(R-S-T)*PX(I,J)+T*PX(I+1,J)-TOA*U*Q(
     1I,J)
      WT(J)=AT(J)
      GT(J)=DR(J)/WT(J)
      DO 300 J=JB,JE
      R=BETA*FIH(I,J)
      BT(J)=-YKH(I,J+1)/YMU(I,J+1)
      CT(J)=-YKH(I,J)/YMU(I,J)
      AT(J)=R-BT(J)-CT(J)
      S=XKH(I,J)/XMU(I,J)
      T=XKH(I+1,J)/XMU(I+1,J)
      U=B1+B2*PX(I,J)
      DR(J)=S*PX(I-1,J)+(R-S-T)*PX(I,J)+T*PX(I+1,J)-TOA*U*Q(
     1I,J)
      WT(J)=AT(J)-CT(J)*BT(J-1)/WT(J-1)
      GT(J)=(DR(J)-CT(J)*GT(J-1))/WT(J)
  300 CONTINUE
      J=JE+1
      R=BETA*FIH(I,J)
      CT(J)=-YKH(I,J)/YMU(I,J)
      AT(J)=R-CT(J)
      DR(J)=S*PX(I-1,J)+(R-S-T)*PX(I,J)+T*PX(I+1,J)-TOA*U*Q(
     1I,J)
      WT(J)=AT(J)-CT(J)*BT(J-1)/WT(J-1)
      GT(J)=(DR(J)-CT(J)*GT(J-1))/WT(J)
      RETURN
      END
```

```
      SUBRCUTINE TCMASY
C
C
C      THIS SUBRCUTINE SOLVES A SET OF SIMULTANEOUS EQUATIONS
     1 INVCLVING
C      TRICIAGONAL COEFFICIENT MATRIX BY THCMAS METHOD
C
      CCMMCN  I,PX(21,21),PY(21,21),XKH(21,21),YKH(21,21),XM
     1U(21,21),YMU
     S(21,21),FIH(21,21),Q(21,21),AT(21),BT(21),CT(21),DR(21
     1),WT(21),GT(
     S21),IR,IRM,IRP,IRC,IRCP,IQ,IQM,IQP,IQPP,IQM1,JR,JRM,JR
     1P,JRC,JQ,JQM
     S,JQP,JT,JTM,IT,ITM,ITMM,ITD,JB,JE,BETA,B1,B2,TOA
      JEB2=JE-JB+2
      PY(I,JE+1)=GT(JE+1)
      DC 3C3 JZ=1,JEB2
      J=JE+1-JZ
  303 PY(I,J)=GT(J)-PY(I,J+1)*BT(J)/WT(J)
      RETURN
      END
```

```
C                                               APPENDIX D   ANNX 2B
C
C
C       TWC-CIMENSICNAL MCDEL OF AN CIL RESERVOIR
C       SCLVEC BY ACEP
C
C
C       THIS PRCGRAM PREDICTS THE BEHAVIOR CF A SINGLE PHASE,
C       SLIGHTLY CCMPRESSIBLE, OIL RESERVOIR AS OIL IS WITH-
C       CRAWN AT A KNOWN RATE FROM FOURTEEN ARBITRARILY
C       LCCATEC WELLS. THE RESERVOIR CONSIDERED IS CF IRREG-
C       LLAR PCLYGCNAL SHAPE WITH ZERO MASS FLUX ACROSS THE
C       BCUNCARY.
C       THE RESERVCIR IS SIMULATED BY A TWO-CIMENSICNAL
C       MATHEMATICAL MCCEL AND THE RESULTING PARTIAL
C       CIFFERENTIAL ECUATION IS SCLVED BY ACEP.
C       VISCCSITY AT P(N*CT) IS USED TO CALCULATE P((N+1)*DT)
C       THE PRCGRAM IS CENERAL AND IS DESIGNED TO COMPUTE
C       PRESSURE CISTRIBUTICN IN THE RESERVOIR AND MATERIAL
C       BALANCE AT ANY REQUIRED TIME AND FCR ANY KNCWN
C       PRCCUCTICN RATE.
C
C                           ************
C                           NCMENCLATURE
C                           ************
C
C       A           RATIC OF CCNVERSION CONSTANTS
C       B           FORMATION VOLUME FACTCR, BS,BO AT PS,PO
C       B1,B2       CONSTANTS IN B VS P RELATION
C       C           COMPRESSIBILITY OF LIQUID (1/PSIA)
C       C,E         CONSTANTS IN VISCOSITY-PRESSURE RELATICN
C       CT          TIME STEP (DAYS) PER SWEEP
C       CX          =DY=SPACE GRID SIZE (FT)
C       FACTCR      CHANCES PRCDUCTION RATES CF WELLS
C       FCAY        REAL TIME FOR WHICH RESERVOIR HISTORY IS
C                   CESIREC(CAYS)
C       MU          VISCCSITY (CP) , USEC WITH PRE-SCRIPTS
C       NSET        NUMBER OF CATA SET
C       NT          NUMBER OF CATA SETS USED
C       P( , )      PRESSURE (PSIA)   , USEC AS PS,PO
C       PERM        PERMEABILITY (DARCY)
C       PRINT       TIME INTERVAL BETWEEN CONSECUTIVE PRINTOUTS
C       PX( , )     PRESSURE AFTER FORWARD SWEEP (PSIA)
C       PY( , )     PRESSURE AFTER REVERSE SWEEP (PSIA)
C       Q( , )      PRODLCTION RATE OF A WELL (S.T.B./DAY)
C       TIME        REAL TIME FOR WHICH RESERVOIR HISTORY HAS
C                   BEEN COMPUTED (DAYS)
C       XKH( , )    X-DIRECTIONAL AVERAGE OF K*H(DARCY-FT)
C       YKH( , )    Y-DIRECTIONAL AVERAGE OF K*H(DARCY-FT)
C       XMU( , )    X-DIRECTIONAL AVERAGE OF VISCOSITY (CP)
C       YMU( , )    Y-DIRECTIONAL AVERAGE OF VISCOSITY (CP)
C
C       SUBSCRIPTS AND/CR PRE-SCRIPTS
C       Z,C         CRIGINAL
C       S           SATURATION
```

```
C       X,Y          DIRECTIONS
C
      DIMENSION PX(21,21),PY(21,21),XKH(21,21),YKH(21,21),XM
     1U(21,21)
      DIMENSION YMU(21,21),FIH(21,21),Q(21,21)
      DIMENSION TIM(5,99),ACT(5,99),CAL(5,99) ,RATI(5,99)
    1 FORMAT(1H1,2H  )
    2 FORMAT(1HL,10X,38H   IR   IQ   IT   JR   JQ   JT   DX(FT)
     1   )
    3 FORMAT(1HK,17X,46HA                B1            B2
     1C          E)
    4 FORMAT(1HL,14X,58HSMU     ZMU      PS      PO       BS
     1  BO     COMP
     CRESS  PERM)
    5 FORMAT(1HJ,10X,2F7.3,2F8.1,2F8.4,E10.3,F6.3)
    6 FORMAT(1HJ,10X,5E12.4)
    7 FORMAT(1HL,2H  )
    8 FORMAT(1HK,18X,51HMATRIX OF PRESSURE IN OIL RESERVOIR,
     1(PSIA). BY A
     1DEP)
    9 FORMAT(1HK,35X,6HTIME =,F6.0,5H DAYS)
   10 FORMAT(1HK,32X,12H(TIME STEP =,F6.2,6H DAYS))
   11 FORMAT(1HL,9X,66H  J  1      2      3      4      5      6
     1     7      8
     C     9     10     11)
   12 FORMAT(1H ,9X,2H I)
   13 FORMAT(1HL,37X,17HPHI*H MATRIX (FT))
   14 FORMAT(1HL,35X,21HK*H MATRIX (DARCY-FT))
   15 FORMAT(1HJ,10X,6I4,F8.0)
   16 FORMAT(1HL,12X,38HPRODUCTION RATES OF WELLS (S.T.B./DA
     1Y))
   17 FORMAT(1HK,12X,66HQ(4,4)  Q(7,4)  Q(10,4)  Q(13,4)  Q(
     14,7)  Q(7,7)
     C  Q(10,7)  Q(13,7))
   18 FORMAT(1HK,12X,56HQ(16,7)  Q(19,7)  Q(10,10)  Q(13,10)
     1  Q(16,10)
     CQ(19,10))
   19 FORMAT(1X,2F7.3,2F8.1,2F8.4,E10.3,F6.3)
   20 FORMAT(1H2,3H   )
   21 FORMAT(1X,6I4,F8.0)
   22 FORMAT(1HJ,12X,F5.0,F8.0,2F9.0,2F8.0,2F9.0)
   23 FORMAT(1HJ,12X,F6.0,F9.0,4F10.0)
   24 FORMAT(1HS,9X,2H I)
   25 FORMAT(1HK,2H  )
   26 FORMAT(1H ,9X,I2,8F6.1)
   27 FORMAT(1HJ,2H  )
   28 FORMAT(1H ,9X,I2,11F6.1)
   29 FORMAT(1H3,39X,8HTABLE A-,I2)
   30 FORMAT(1H ,9X,I2,18X,8F6.1)
   31 FORMAT(1X,12F6.3/5F6.3)
   33 FORMAT(1X,12F6.3/9F6.3)
   34 FORMAT(1H2,9X,6HCONTD.,22X,8HTABLE A-,I2)
   35 FORMAT(1X,12F6.3/3F6.3)
   36 FORMAT(1HL,15X,26HINITIAL RESERVOIR PRESSURE,9X,3H PO,
     1F9.2,5X,
```

```
  C 4HPSIA)
 37 FORMAT(1HJ,15X,26HSATURATION PRESSURE OF OIL,9X,3H PS,
   1F9.2,5X,
  C 4HPSIA)
 38 FORMAT(1HJ,15X,29HFORMATION VOLUME FACTOR AT PO,6X,3H
   1BO,F12.5)
 39 FORMAT(1HJ,15X,29HFORMATION VOLUME FACTOR AT PS,6X,3H
   1BS,F12.5)
 40 FORMAT(1H ,9X,I2,8F6.0)
 41 FORMAT(1H ,9X,I2,11F6.0)
 42 FORMAT(1H ,9X,I2,18X,8F6.0)
 43 FORMAT(1X,12F6.2/5F6.2)
 44 FORMAT(1X,12F6.2/9F6.2)
 45 FORMAT(1X,12F6.2/3F6.2)
 47 FORMAT(1HK,129HLT    Q(4,4)   Q(7,4)  Q(10,4)  Q(13,4)
   1   Q(4,7)
   SQ(7,7)  Q(10,7)  Q(13,7)  Q(16,7)  Q(19,7) Q(10,10) Q(
   113,10) Q(16,
   S10) Q(19,10))
 48 FORMAT(1HA,I2,1X,14F9.0)
 49 FORMAT(1H=,27X,36HPY(10,7) IS LESS THAN SATN. PRESSURE
   1)
 50 FORMAT(1X,F7.2,2F7.0,F5.2)
 51 FORMAT(1HL,12X,24HDT   PRINT  FDAY  FACTOR)
 52 FORMAT(1X,I3)
 54 FORMAT(1H3 ,24X,42HCUMULATIVE PRODUCTION (STOCK TANK B
   1ARRELS))
 55 FORMAT(1HK,24X,6HACTUAL, E12.4,4X,10HCALCULATED, E12.4
   1)
 56 FORMAT(1HJ, 8X,3F7.0,F6.2)
 57 FORMAT(1HK,12X,65HTWO DIMENSIONAL MODEL OF A SLIGHTLY
   1COMPRESSIBLE
   C LIQUID RESERVOIR)
 58 FORMAT(1HL,15X,38HDIGITAL SIMULATION OF AN OIL RESERVO
   1IR)
 59 FORMAT(1H9,3H   )
340 FORMAT(1HJ,15X,15HVISCOSITY AT PO,20X,3HZMU,F12.5,2X,2
   1HCP)
341 FORMAT(1HJ,15X,15HVISCOSITY AT PS,20X,3HSMU,F12.5,2X,2
   1HCP)
342 FORMAT(1HJ,15X,15HCOMPRESSIBILITY,20X,3H  C,E16.5,2X,6
   1H1/PSIA)
343 FORMAT(1HK,40X,9HTABLE A-3)
344 FORMAT(1H2,37X,8HTABLE A-,I2)
345 FORMAT(1HL,28X,35HPRODUCTION PATTERN OF OIL RESERVOIR)
346 FORMAT(1HL,27X,16HLOCATION OF WELL,5X,10HPRODUCTION)
347 FORMAT(1HS,31X,7H I , J ,10X,17HQ(I,J) S.T.B./DAY)
348 FORMAT(1HJ,31X,I2,I4,13X,F6.0)
    IBB1=1
    IBB2=2
    IBC=4
C   PARAMETERS OF RESERVOIR GEOMETRY, DX=DY
    READ (5,21) IR,IQ,IT,JR,JQ,JT,DX
    READ(5,19) SMU,ZMU,PS,PO,BS,BO,C,PERM
    C=(BS-BO)*2.0/((BS+BO)*(PO-PS))
```

```
C       1 BBL= 5.6146 C.FT., 1 DARCY= 6.327 SQ.FT.*CP/( DAY*PS
       1I )
        A=5.6146/6.327
        TOA=2.0*A
        B1=(BS*PO-BO*PS)/(PO-PS)
        B2=(BO-BS)/(PO-PS)
        POPS=(PO+PS)/2.0
        BR=B1+B2*POPS
        D=(SMU*PO-ZMU*PS)/(PO-PS)
        E=(ZMU-SMU)/(2.0*(PO-PS))
        IRM=IR-1
        IRP=IR+1
        IRD=IR+2
        IQP=IQ+1
        IQM=IQ-1
        ITM=IT-1
        JRP=JR+1
        JRM=JR-1
        JRD=JR+2
        JQP=JQ+1
        JTM=JT-1
        JQM=JQ-1
C       PHI*H (FT) DATA
        READ(5,43) ((FIH(I,J),I=1,IQ),J=1,JRM)
        READ(5,44) ((FIH(I,J),I=1,IT),J=JR,JQ)
        READ(5,45) ((FIH(I,J),I=IR,IT),J=JQP,JT)
        WRITE(6,1)
        WRITE(6,29) IBB1
        WRITE(6,13)
        WRITE(6,11)
        WRITE(6,12)
        DO 60 I=2,IR
        IM=I-1
   60   WRITE(6,26) IM,(FIH(I,J),J=2,JQM)
        DO 61 I=IRP,IQM
        IM=I-1
   61   WRITE(6,28) IM,(FIH(I,J),J=2,JTM)
        DO 62 I=IQ,ITM
        IM=I-1
   62   WRITE(6,30) IM,(FIH(I,J),J=JRP,JTM)
        VI=0.0
        DO 801 J=2,JR
        DO 801 I=2,IQM
  801   VI=VI+FIH(I,J)
        DO 803 J=JRP,JQM
        DO 803 I=2,ITM
  803   VI=VI+FIH(I,J)
        DO 805 J=JQ,JTM
        DO 805 I=IRP,ITM
  805   VI=VI+FIH(I,J)
        VI=VI*DX*DX/(B1+B2*PO)
C       READ IN FI (=PHI) VALUES USING PY MEMORY
        READ(5,31) ((PY(I,J),I=1,IQ),J=1,JRM)
        READ(5,33) ((PY(I,J),I=1,IT),J=JR,JQ)
        READ(5,35) ((PY(I,J),I=IR,IT),J=JQP,JT)
```

```
C       CALCULATE H, K*H USING Q,PX MEMORIES RESPY., K=CONSTAN
       1T
        DO 70 J=1,JRM
        DO 70 I=1,IQ
        Q(I,J)=FIH(I,J)/PY(I,J)
     70 PX(I,J)=Q(I,J)*PERM
        DO 71 J=JR,JQ
        DO 71 I=1,IT
        Q(I,J)=FIH(I,J)/PY(I,J)
     71 PX(I,J)=Q(I,J)*PERM
        DO 72 J=JQP,JT
        DO 72 I=IR,IT
        Q(I,J)=FIH(I,J)/PY(I,J)
     72 PX(I,J)=Q(I,J)*PERM
C       WRITE  KH MATRIX
        WRITE(6,29) IBB2
        WRITE(6,14)
        WRITE(6,11)
        WRITE(6,12)
        DO 80 I= 2,IR
        IM=I-1
     80 WRITE(6,26) IM,(PX(I,J),J=2,JQM)
        DO 82 I=IRP,IQM
        IM=I-1
     82 WRITE(6,28) IM,(PX(I,J),J=2,JTM)
        DO 84 I=IQ,ITM
        IM=I-1
     84 WRITE(6,30) IM,(PX(I,J),J=JRP,JTM)
C       CALCULATE 2*ARITH.AV. OF K*H IN X- AND Y-DIRECTIONS
        DO 101 J=1,JRM
        DO 101 I=2,IQ
    101 XKH(I,J)=PX(I,J)+PX(I-1,J)
        DO 103 J=JR,JQ
        DO 103 I=2,IT
    103 XKH(I,J)=PX(I,J)+PX(I-1,J)
        DO 105 J=JQP,JT
        DO 105 I=IRP,IT
    105 XKH(I,J)=PX(I,J)+PX(I-1,J)
        DO 109 I=1,IRM
        DO 109 J=2,JQ
    109 YKH(I,J)=PX(I,J)+PX(I,J-1)
        DO 110 I=IR,IQ
        DO 110 J=2,JT
    110 YKH(I,J)=PX(I,J)+PX(I,J-1)
        DO 112 I=IQP,IT
        DO 112 J=JRP,JT
    112 YKH(I,J)=PX(I,J)+PX(I,J-1)
        READ(5,52) NT
        DO 888 NSET=1,NT
        READ(5,50) DT,PRINT,FDAY,FACTOR
        WRITE(6,1)
        WRITE(6,20)
        WRITE(6,57)
        WRITE(6,2)
        WRITE(6,15) IR,IQ,IT,JR,JQ,JT,DX
```

```
      WRITE(6,4)
      WRITE(6,5) SMU,ZMU,PS,PO,BS,BO,C,PERM
      WRITE(6,3)
      WRITE(6, 6) A,B1,B2,D,E
      DT2=DT*2.0
      BETA=(C*DX*DX/DT)*(  2.0/6.327)
      WRITE(6,1)
      WRITE(6,20)
      WRITE(6,58)
      WRITE(6,7)
      WRITE(6,7)
      WRITE(6,36) PO
      WRITE(6,37) PS
      WRITE(6,38) BO
      WRITE(6,39) BS
      WRITE(6,340) ZMU
      WRITE(6,341) SMU
      WRITE(6,342) C
C     INITIALIZE PRESSURES, CLEAR Q MEMORY
      DO 120 J=1,JRM
      DO 120 I=1,IQ
      Q(I,J)=0.0
  120 PY(I,J)=PO
      DO 123 J=JR,JQ
      DO 123 I=1,IT
      Q(I,J)=0.0
  123 PY(I,J)=PO
      DO 125 J=JQP,JT
      DO 125 I=IR,IT
      Q(I,J)=0.0
  125 PY(I,J)=PO
C     SPECIFY PRODUCTION RATES OF DIFFERENT WELLS
      Q(10,10)=75.0*FACTOR
      Q(13,10)=75.0*FACTOR
      Q(16,10)=75.0*FACTOR
      Q(19,10)=75.0*FACTOR
      Q(4,7)=125.0*FACTOR
      Q(7,7)=125.0*FACTOR
      Q(10,7)=125.0*FACTOR
      Q(13,7)=125.0*FACTOR
      Q(16,7)=125.0*FACTOR
      Q(19,7)=125.0*FACTOR
      Q(4,4)=75.0*FACTOR
      Q(7,4)=75.0*FACTOR
      Q(10,4)=75.0*FACTOR
      Q(13,4)=75.0*FACTOR
      WRITE(6,20)
      WRITE(6,25)
      WRITE(6,343)
      WRITE(6,345)
      WRITE(6,346)
      WRITE(6,347)
      DO 465 J=2,JTM
      DO 465 I=2,ITM
      IF(Q(I,J) .GT. 0.0) GO TO 463
```

```
      GC TC 465
  463 IM=I-1
      JM=J-1
      WRITE(6,348) IM,JM,Q(I,J)
  465 CONTINUE
      WRITE(6,20)
      WRITE(6,16)
      WRITE(6,17)
      WRITE(6,22) Q(4,4),Q(7,4),Q(10,4),Q(13,4),Q(4,7),Q(7,7
     1),
     C Q(10,7),Q(13,7)
      WRITE(6,18)
      WRITE(6,23) Q(16,7),Q(19,7),Q(10,10),Q(13,10),Q(16,10)
     1,Q(19,10)
      WRITE(6,51)
      WRITE(6,56)DT,PRINT,FDAY,FACTOR
      WRITE(4,344) IBC
      PROD=(Q(10,10)+Q(13,10)+Q(16,10)+ Q(19,10)+Q(4,7)+Q(7,
     17)+
     C Q(10,7)+Q(13,7)+Q(16,7)+Q(19,7)+Q(4,4)+Q(7,4)+Q(10,4)
     1+
     C Q(13,4))
      NML=0
      TIME=0.0
 3000 CONTINUE
      OUTPUT=0.0
C     PRINTOUT CYCLE
  500 OUTPUT=OUTPUT+DT2
      TIME=TIME+DT2
C     CALCULATE MU(I,J) IN THE X- AND Y-DIRECTIONS
      DO 150 J=1,JRM
      DO 150 I=2,IQ
  150 XMU(I,J)=D+E*(PY(I,J)+PY(I-1,J))
      DO 151 J=JR,JQ
      DO 151 I=2,IT
  151 XMU(I,J)=D+E*(PY(I,J)+PY(I-1,J))
      DO 152 J=JQP,JT
      DO 152 I=IRP,IT
  152 XMU(I,J)=D+E*(PY(I,J)+PY(I-1,J))
      DO 154 I=1,IRM
      DO 154 J=2,JQ
  154 YMU(I,J)=D+E*(PY(I,J)+PY(I,J-1))
      DO 155 I=IR,IQ
      DO 155 J=2,JT
  155 YMU(I,J)=D+E*(PY(I,J)+PY(I,J-1))
      DO 157 I=IQP,IT
      DO 157 J=JRP,JT
  157 YMU(I,J)=D+E*(PY(I,J)+PY(I,J-1))
C     FORWARD SWEEP IN SOUTH-EAST DIRECTION TO FIND PX(I,J)
      U=BETA*FIH(2,2)
      V=XKH(3,2)/XMU(3,2)
      W=YKH(2,3)/YMU(2,3)
      R=B1+B2*PY(2,2)
      PX(2,2)=((U-V-W)*PY(2,2)+V*PY(3,2)+W*PY(2,3)-TOA*Q(2,2
     1)*R)/U
```

```
      PX(1,2)=PX(2,2)
      PX(2,1)=PX(2,2)
      DO 160 J=3,JQ
      R=BETA*FIH(2,J)
      S=YKH(2,J)/YMU(2,J)
      U=XKH(3,J)/XMU(3,J)
      V=YKH(2,J+1)/YMU(2,J+1)
      T=B1+B2*PY(2,J)
  160 PX(2,J)=(S*PX(2,J-1)+U*PY(3,J)+V*PY(2,J+1)+(R-U-V)*PY(
     12,J)-TOA*Q(2
     S,J)*T)/(R+S)
      DO 161 J=1,JQM
  161 PX(1,J)=PX(2,J)
      DO 162 I=3,IQM
      R=BETA*FIH(I,2)
      S=XKH(I,2)/XMU(I,2)
      T=B1+B2*PY(I,2)
      U=XKH(I+1,2)/XMU(I+1,2)
      V=YKH(I,3)/YMU(I,3)
  162 PX(I,2)=(S*PX(I-1,2)+(R-U-V)*PY(I,2)+U*PY(I+1,2)+V*PY(
     1I,3)-TOA*Q(I
     S,2)*T)/(R+S)
      PX(IQ,2)=PX(IQM,2)
      DO 163 I=3,IQ
  163 PX(I,1)=PX(I,2)
      DO 164 J=3,JRP
      DO 164 I=3,IQM
      R=BETA*FIH(I,J)
      S=XKH(I,J)/XMU(I,J)
      T=B1+B2*PY(I,J)
      U=YKH(I,J)/YMU(I,J)
      V=XKH(I+1,J)/XMU(I+1,J)
      W=YKH(I,J+1)/YMU(I,J+1)
  164 PX(I,J)=(S*PX(I-1,J)+U*PX(I,J-1)+(R-V-W)*PY(I,J)+V*PY(
     1I+1,J)+W*PY(
     SI,J+1)-TOA*Q(I,J)*T)/(R+U+S)
      DO 166 J=3,JR
  166 PX(IQ,J)=PX(IQM,J)
      PX(IQ,JR+1)=PX(IQ,JR)
      DO 168 I=IQP,ITM
      R=BETA*FIH(I,JRP)
      S=XKH(I,JRP)/XMU(I,JRP)
      T=B1+B2*PY(I,JRP)
      U=XKH(I+1,JRP)/XMU(I+1,JRP)
      V=YKH(I,JRP+1)/YMU(I,JRP+1)
  168 PX(I,JRP)=(S*PX(I-1,JRP)+(R-U-V)*PY(I,JRP)+U*PY(I+1,JR
     1P)+V*PY(I,JR
     SP+1)-TOA*Q(I,JRP)*T)/(R+S)
      DO 169 I=IQP,ITM
  169 PX(I,JR)=PX(I,JRP)
      DO 170 J=JRD,JQM
      DO 170 I=3,ITM
      R=BETA*FIH(I,J)
      S=XKH(I,J)/XMU(I,J)
      T=B1+B2*PY(I,J)
```

```
      U=YKF(I,J)/YMU(I,J)
      V=XKF(I+1,J)/XMU(I+1,J)
      W=YKF(I,J+1)/YMU(I,J+1)
  170 PX(I,J)=(S*PX(I-1,J)+U*PX(I,J-1)+(R-V-W)*PY(I,J)+V*PY(
     1I+1,J)+W*PY(
     SI,J+1)-TOA*Q(I,J)*T)/(R+U+S)
      DO 172 I=1,IR
  172 PX(I,JQ)=PX(I,JQM)
      PX(IRP,JQ)=PX(IR,JQ)
      DO 175 J=JQP,JTM
      R=BETA*FIF(IRP,J)
      S=YKF(IRP,J)/YMU(IRP,J)
      T=B1+B2*PY(IRP,J)
      U=XKF(IRD,J)/XMU(IRD,J)
      V=YKF(IRP,J+1)/YMU(IRP,J+1)
  175 PX(IRP,J)=(S*PX(IRP,J-1)+U*PY(IRD,J)+V*PY(IRP,J+1)+(R-
     1U-V)*PY(IRP,
     SJ)-TOA*Q(IRP,J)*T)/(R+S)
      DO 177 J=JQP,JTM
  177 PX(IR,J)=PX(IRP,J)
      DO 179 J=JQ,JTM
      DO 179 I=IRD,ITM
      R=BETA*FIF(I,J)
      S=XKF(I,J)/XMU(I,J)
      T=B1+B2*PY(I,J)
      U=YKF(I,J)/YMU(I,J)
      V=XKF(I+1,J)/XMU(I+1,J)
      W=YKF(I,J+1)/YMU(I,J+1)
  179 PX(I,J)=(S*PX(I-1,J)+U*PX(I,J-1)+(R-V-W)*PY(I,J)+V*PY(
     1I+1,J)+W*PY(
     SI,J+1)-TOA*Q(I,J)*T)/(R+U+S)
C     BOUNDARY CONDITION OF 0.0 FLUX, USE REFLECTION
      DO 181 I=IR,ITM
  181 PX(I,JT)=PX(I,JTM)
      DO 183 J=JR,JT
  183 PX(IT,J)=PX(ITM,J)
C     REVERSE SWEEP IN NORTH-WEST DIRECTION TO FIND PY(I,J)
C     CALCULATE MU(I,J) IN X- AND Y-DIRECTIONS
      DO 201 J=1,JRM
      DO 201 I=2,IQ
  201 XMU(I,J)=D+E*(PX(I,J)+PX(I-1,J))
      DO 202 J=JR,JQ
      DO 202 I=2,IT
  202 XMU(I,J)=D+E*(PX(I,J)+PX(I-1,J))
      DO 203 J=JQP,JT
      DO 203 I=IRP,IT
  203 XMU(I,J)=D+E*(PX(I,J)+PX(I-1,J))
      DO 206 I=1,IRM
      DO 206 J=2,JQ
  206 YMU(I,J)=D+E*(PX(I,J)+PX(I,J-1))
      DO 207 I=IR,IQ
      DO 207 J=2,JT
  207 YMU(I,J)=D+E*(PX(I,J)+PX(I,J-1))
      DO 208 I=IQP,IT
      DO 208 J=JRP,JT
```

```
208 YMU(I,J)=C+E*(PX(I,J)+PX(I,J-1))
    R=BETA*FIH(ITM,JTM)
    S=XKH(ITM,JTM)/XMU(ITM,JTM)
    U=YKH(ITM,JTM)/YMU(ITM,JTM)
    V=B1+B2*PX(ITM,JTM)
    PY(ITM,JTM)=((R-S-U)*PX(ITM,JTM)+S*PX(ITM-1,JTM)+U*PX(
   1ITM,JTM-1)-T
   SCA*Q(ITM,JTM)*V)/R
    PY(IT,JTM)=PY(ITM,JTM)
    PY(ITM,JT)=PY(ITM,JTM)
    PY(IT,JT)=PY(ITM,JTM)
    JTJR=JT-JR
    DO 215 JB=2,JTJR
    J=JT+1-JB
    R=BETA*FIH(ITM,J)
    S=YKH(ITM,J+1)/YMU(ITM,J+1)
    U=XKH(ITM,J)/XMU(ITM,J)
    V=YKH(ITM,J)/YMU(ITM,J)
    T=B1+B2*PX(ITM,J)
215 PY(ITM,J)=(S*PY(ITM,J+1)+(R-U-V)*PX(ITM,J)+U*PX(ITM-1,
   1J)+V*PX(ITM,
   SJ-1)-TCA*Q(ITM,J)*T)/(R+S)
    DO 216 J=JRP,JTM
216 PY(IT,J)=PY(ITM,J)
    PY(IT,JR)=PY(ITM,JRP)
    PY(ITM,JR)=PY(ITM,JRP)
    ITIR=IT-IR
    DO 218 IB=3,ITIR
    I=IT+1-IB
    R=BETA*FIH(I,JTM)
    S=XKH(I+1,JTM)/XMU(I+1,JTM)
    U=XKH(I,JTM)/XMU(I,JTM)
    V=YKH(I,JTM)/YMU(I,JTM)
    T=B1+B2*PX(I,JTM)
    PY(I,JTM)=(S*PY(I+1,JTM)+(R-U-V)*PX(I,JTM)+U*PX(I-1,JT
   1M)+V*PX(I,JT
   SM-1)-TCA*Q(I,JTM)*T)/(R+S)
218 PY(I,JT)=PY(I,JTM)
    PY(IR,JT)=PY(IRP,JT)
    PY(IR,JTM)=PY(IRP,JTM)
    JTJQT=JT-JQ+2
    DO 220 JB=3,JTJQT
    J=JT+1-JB
    DO 220 IB=3,ITIR
    I=IT+1-IB
    R=BETA*FIH(I,J)
    S=XKH(I+1,J)/XMU(I+1,J)
    T=B1+B2*PX(I,J)
    U=YKH(I,J+1)/YMU(I,J+1)
    V=XKH(I,J)/XMU(I,J)
    W=YKH(I,J)/YMU(I,J)
220 PY(I,J)=(S*PY(I+1,J)+U*PY(I,J+1)+(R-V-W)*PX(I,J)+V*PX(
   1I-1,J)+W*PX(
   SI,J-1)-TCA*Q(I,J)*T)/(R+S+U)
    JTJQ1=JT-JQ+1
```

```
      DO 222 JB=3,JTJQ1
      J=JT+1-JB
  222 PY(IR,J)=PY(IRP,J)
      PY(IR,JQM)=PY(IR,JQ)
      ITIRT=IT-IR+2
      DO 223 IB=ITIRT,ITM
      I=IT+1-IB
      R=BETA*FIH(I,JQM)
      S=XKH(I+1,JQM)/XMU(I+1,JQM)
      U=XKH(I,JQM)/XMU(I,JQM)
      V=YKH(I,JQM)/YMU(I,JQM)
      T=B1+B2*PX(I,JQM)
      PY(I,JQM)=(S*PY(I+1,JQM)+(R-U-V)*PX(I,JQM)+U*PX(I-1,JQ
     1M)+V*PX(I,JQ
     SM-1)-TOA*Q(I,JQM)*T)/(R+S)
  223 PY(I,JQ)=PY(I,JQM)
      JTJQ=JT-JQ+3
      JTJR=JT-JR
      DO 225 JB=JTJQ,JTJR
      J=JT+1-JB
      DO 225 IB=3,ITM
      I=IT+1-IB
      R=BETA*FIH(I,J)
      S=XKH(I+1,J)/XMU(I+1,J)
      T=B1+B2*PX(I,J)
      U=YKH(I,J+1)/YMU(I,J+1)
      V=XKH(I,J)/XMU(I,J)
      W=YKH(I,J)/YMU(I,J)
  225 PY(I,J)=(S*PY(I+1,J)+U*PY(I,J+1)+(R-V-W)*PX(I,J)+V*PX(
     1I-1,J)+W*PX(
     SI,J-1)-TOA*Q(I,J)*T)/(R+S+U)
      DO 226 I=IQ,ITM
  226 PY(I,JR)=PY(I,JRP)
      PY(IQM,JR)=PY(IQ,JR)
      JTJRP=JT-JR+2
      DO 228 JB=JTJRP,JTM
      J=JT+1-JB
      R=BETA*FIH(IQM,J)
      S=YKH(IQM,J+1)/YMU(IQM,J+1)
      T=B1+B2*PX(IQM,J)
      U=XKH(IQM,J)/XMU(IQM,J)
      V=YKH(IQM,J)/YMU(IQM,J)
      PY(IQM,J)=(S*PY(IQM,J+1)+(R-U-V)*PX(IQM,J)+U*PX(IQM-1,
     1J)+V*PX(IQM,
     SJ-1)-TOA*Q(IQM,J)*T)/(R+S)
  228 PY(IQ,J)=PY(IQM,J)
      PY(IQ,1)=PY(IQ,2)
      JTJR1=JT-JR+1
      ITIQ3=IT-IQ+3
      DO 230 JB=JTJR1,JTM
      J=JT+1-JB
      DO 230 IB=ITIQ3,ITM
      I=IT-IB+1
      R=BETA*FIH(I,J)
      S=XKH(I+1,J)/XMU(I+1,J)
```

```
      T=B1+B2*PX(I,J)
      U=YKF(I,J+1)/YMU(I,J+1)
      V=XKF(I,J)/XMU(I,J)
      W=YKF(I,J)/YMU(I,J)
  230 PY(I,J)=(S*PY(I+1,J)+U*PY(I,J+1)+(R-V-W)*PX(I,J)+V*PX(
     1I-1,J)+W*PX(
     SI,J-1)-TCA*Q(I,J)*T)/(R+S+U)
      ITIQ2=IT-IQ+2
C     BOUNDARY CONDITION OF 0.0 FLUX, USE REFLECTION
      DO 231 IB=ITIQ2,ITM
      I=IT+1-IB
  231 PY(I,1)=PY(I,2)
      JTJQ1=JT-JQ+1
      DO 232 JB=JTJQ1,JT
      J=JT+1-JB
  232 PY(1,J)=PY(2,J)
      IF(OUTPUT .LT. PRINT) GO TO 500
      WRITE(4,8)
      WRITE(4,10) DT2
      WRITE(4,27)
      WRITE(4,9) TIME
      WRITE(4,11)
      WRITE(4,24)
      DO 390 I=2,IR
      IM=I-1
  390 WRITE(4,40) IM,(PY(I,J),J=2,JQM)
      DO 291 I=IRP,IQM
      IM=I-1
  291 WRITE(4,41) IM,(PY(I,J),J=2,JTM)
      DO 292 I=IQ,ITM
      IM=I-1
  292 WRITE(4,42) IM,(PY(I,J),J=JRP,JTM)
      WRITE(4,34) IBC
      VF=0.0
      DO 901 J=2,JR
      DO 901 I=2,IQM
  901 VF=VF+FIH(I,J)/(B1+B2*PY(I,J))
      DO 903 J=JRP,JQM
      DO 903 I=2,ITM
  903 VF=VF+FIH(I,J)/(B1+B2*PY(I,J))
      DO 905 J=JQ,JTM
      DO 905 I=IRP,ITM
  905 VF=VF+FIH(I,J)/(B1+B2*PY(I,J))
      VF=VF*DX*DX
      DV=(VI-VF)/5.6146
      PRO =PROD*TIME
      NML=NML+1
      TIM(NSET,NML)=TIME
      ACT(NSET,NML)=PRO
      CAL(NSET,NML)=DV
      RATI(NSET,NML)=PRO/DV
      IF( PY(10,7) .LT. PS) GO TO 555
      IF(TIME .LT. FDAY) GO TO 3000
      GO TO 887
  555 WRITE(6,49)
```

```
      GO TO 888
  887 NMT=NML
      WRITE(6,54)
      WRITE(6,349) (TIM(NSET,NML),ACT(NSET,NML),CAL(NSET,NML
     1),
     C RATI(NSET,NML),NML=1,NMT)
  349 FORMAT(1H ,18X,F6.0,2E14.4,F8.4)
      IBC=IBC+2
  888 CONTINUE
      STOP
      END
```

```
C                                                 APPENDIX D   ANNX 3
C
C
C     TWO-DIMENSIONAL NONLINEAR MODEL OF A GAS RESERVOIR
C     SOLVED BY ADEP
C
C
C     THIS PROGRAM PREDICTS THE BEHAVIOR OF A GAS RESERVOIR
C     AS GAS IS PRODUCED FROM THIRTY SEVEN ARBITRARILY
C     LOCATED WELLS. THE RESERVOIR IS OF IRREGULAR POLYGONAL
C     SHAPE WITH ZERO MASSFLUX ACROSS THE BOUNDARY.
C     THE RESERVOIR IS SIMULATED BY A TWO-DIMENSIONAL
C     NONLINEAR MODEL. THE MODEL IS SOLVED BY ADEP.
C     IN THE PROBLEM UNDER CONSIDERATION, P/MU*Z AND P
C     COULD BE EXPRESSED SATISFACTORILY AS LINEAR FUNCTIONS
C     OF P/Z.
C
C
C                         ************
C                         NOMENCLATURE
C                         ************
C
C     A( , )      X-DIRECTIONAL AVERAGE OF H( , ) (DARCY-FT)
C     B( , )      Y-DIRECTIONAL AVERAGE OF H( , ) (DARCY-FT)
C     C1,C2       CONSTANTS IN  P/ MU*Z)   C1   C2* P/Z)
C     DT          TIME STEP (DAYS) PER SWEEP
C     DX          SIZE OF SPACE GRID (FT)
C     E( , )      CONSTANT*U( , )
C     F( , )      PRESSURE (PSIA)
C     FACTOR      FACTOR TO CHANGE PRODUCTION RATE
C     FDAY        REAL TIME DURING WHICH RESERVOIR HISTORY
C                 IS DESIRED
C     H( , )      PERMEABILITY*FORMATION HEIGHT (DARCY-FT)
C     NSET        NUMBER OF DATA SET
C     P           PRESSURE (PSIA)
C     P1,P2       CONSTANTS OF THE RELATION P=P1+P2*(P/Z)
C     PI          INITIAL PRESSURE (PSIA)
C     PRINT       COMPUTED RESULTS PRINTED AT INTERVALS OF
C                 THESE MANY DAYS OF RESERVOIR HISTORY
C     PZ( , )     PRESSURE/COMPRESSIBILITY FACTOR (PSIA)
C     R           GAS CONSTANT
C     T           TEMPERATURE (DEGREES RANKINE)
C     TIME        TIME FOR WHICH RESERVOIR HISTORY HAS BEEN
C                 COMPUTED
C     U( , )      POROSITY*FORMATION HEIGHT (FT)
C     W( , )      PRODUCTION RATE OF A WELL (MMSCF/DAY)
C     Z           COMPRESSIBILITY FACTOR
C     ZI          INITIAL Z
C
      DIMENSION A(21,13), B(21,13), E(21,13), F(21,13), H(21
     1,13),
     C PZ(21,13),W(21,13),U(21,13)
      DIMENSION GB(99,5)
    1 FORMAT (2X, 6I4,F6.0,F7.4,F5.0,F6.2,F5.2)
    2 FORMAT(1X,3F7.0)
```

```
  3 FORMAT (2X, 14F5.1)
  4 FORMAT(1H2,3H    )
  5 FORMAT (2X, 9F6.2)
  6 FORMAT(1H2,9X,6HCONTD.,22X,8HTABLE B-,I2)
  7 FORMAT(2X,2E16.8,2F7.3)
  8 FORMAT(1H2,37X,8HTABLE B-,I2)
381 FORMAT(1HL,15X,16HINITIAL PRESSURE,16X,2HPI,F9.2,6X,4H
   1PSIA)
382 FORMAT(1HJ,15X,34HINITIAL COMPRESSIBILITY FACTOR  ZI,F
   111.4)
383 FORMAT(1HJ,15X,21HRESERVOIR TEMPERATURE,11X,2HTF,F9.2,
   16X,9HDEGREES
    C F)
384 FORMAT(1HJ,15X,15HSPACE GRID SIZE,17X,2HDX,F9.2,6X,4HF
   1EET)
385 FORMAT(1HL,15X,40HPRESSURE,COMPRESSIBILITY FACTOR RELA
    1TION)
386 FORMAT(1HJ,31X,22HP = 80.00 + 0.80*(P/Z))
387 FORMAT(1HJ,15X,50HPRESSURE,VISCOSITY,COMPRESSIBILITY F
    1ACTOR RELATI
    CON)
388 FORMAT(1HJ,31X,32HP/(MU*Z) = 6815.0 + 3737.0*(P/Z))
407 FORMAT(1HS,9X,2H I)
408 FORMAT(1HK,18X,51HMATRIX OF PRESSURE IN GAS RESERVOIR,
    1(PSIA). BY A
    1DEP)
409 FORMAT(1HK,35X,6HTIME =,F6.0,5H DAYS)
410 FORMAT(1HK,32X,12H(TIME STEP =,F5.0,6H DAYS))
411 FORMAT(1HL,9X,66H  J  1     2     3     4     5     6
    1     7     8
    C     9     10     11)
412 FORMAT(1H ,9X,2H I)
413 FORMAT(1HL,37X,17HPHI*H MATRIX (FT))
414 FORMAT(1HL,35X,21HK*H MATRIX (DARCY-FT))
415 FORMAT(1H1,2H  )
416 FORMAT(1HK,19X,52H NA1  NB  NN MA1  MB  MM   P     Z
    1   T    R
    C  DX)
417 FORMAT(1HJ,19X,6I4,F6.0,F7.4,F5.0,F6.2,F5.2)
418 FORMAT(1HL,24X,30HD1          D2          P1       P2)
419 FORMAT(1HJ,19X,2E12.4,2F7.3)
420 FORMAT(1H3,20X,47HPRODUCTION RATES OF WELLS (MILLION S
    1.C.F./DAY) )
421 FORMAT(1HL,5H     )
422 FORMAT(1HL,25X,40HTWO DIMENSIONAL MODEL OF A GAS RESER
    1VOIR)
423 FORMAT(1HK,2H  )
426 FORMAT(1H ,9X,I2,8F6.2)
427 FORMAT(1HJ,2H  )
428 FORMAT(1H ,9X,I2,11F6.2)
429 FORMAT(1H3,39X, 8HTABLE B-,I2)
430 FORMAT(1H ,9X,I2,18X,8F6.2)
440 FORMAT(1H ,9X,I2,8F6.0)
441 FORMAT(1H ,9X,I2,11F6.0)
442 FORMAT(1H ,9X,I2,18X,8F6.0)
```

```
  449 FORMAT(1X,3F7.0,F6.2)
  450 FORMAT(1HJ,15X,3F7.0,F6.2)
  451 FORMAT(1H3,19X,24HDT    PRINT   FDAY   FACTOR)
  452 FORMAT(1X,I3)
  453 FORMAT(1HL,22X,38HCUMULATIVE PRODUCTION (MILLION S.C.F
     1.))
  454 FORMAT(1HK,22X,6HACTUAL, E12.4,4X,10HCALCULATED, E12.4
     1)
  455 FORMAT(1H ,9X,I2,8F6.1)
  456 FORMAT(1H ,9X,I2,11F6.1)
  457 FORMAT(1H ,9X,I2,18X,8F6.1)
  459 FORMAT(1H9,3H   )
C
      IEB1=1
      IEB2=2
      IBC=4
C     PARAMETERS OF RESERVOIR GEOMETRY AND INITIAL CONDITION
      READ(5,1)     NA,NB,NN,MA,MB,MM,PI,ZI,T,R,DX
      TF=T-460.0
      NA1=NA-1
      NB1=NB+1
      NN1=NN+1
      NN2=NN-1
      MA1=MA-1
      MB1=MB+1
      MM1=MM+1
      MM2=MM-1
      MB2=MB-1
      NB2=NB-1
      KN1=NN-NA1
      KM1=MM-MB
      KN2=NN-1
      KM2=MB-MA1
      KN3=NB-1
      KM3=MA1-1
      WRITE(6,4)
      WRITE(6,422)
      WRITE(6,421)
      WRITE(6,416)
      WRITE(6,417) NA1,NB,NN,MA1,MB,MM,PI,ZI,T,R,DX
      READ(5,7) D1,D2,P1,P2
      D2=D2/2.0
      WRITE(6,418)
      WRITE(6,419) D1,D2,P1,P2
C     CONVERSION FACTORS
C     1 DARCY=6.327 SQ.FT.*CP/(DAY*PSI)
      DX=DX*5280.0
      CU=1000000.0*492.0/(359.0*520.0)
      CW=CU*R*T/(6.327*P2)
      WRITE(6,415)
      WRITE(6,4)
      WRITE(6,422)
      WRITE(6,421)
      WRITE(6,381) PI
      WRITE(6,382) ZI
```

```
      WRITE(6,383) TF
      WRITE(6,384) DX
      WRITE(6,385)
      WRITE(6,386)
      WRITE(6,387)
      WRITE(6,388)
C     K*H (DARCY-FT) DATA
      READ (5,3) ((H(I,J), I=2,NN), J=2,MM)
C     PHI*H (FT) DATA
      READ(5,3) ((U(I,J),I=2,NN),J=2,MM)
      DO 300 I=1,NN1
      H(I,1)=0.00
      H(I,MM1)=0.00
      E(I,1)=0.00
  300 E(I,MM1)=0.00
      DO 301 J=2,MM1
      H(1,J)=0.00
      H(NN1,J)=0.00
      E(1,J)=0.00
  301 E(NN1,J)=0.00
C     ACTUAL DATA OF K*H AND PHI*H USED
      DO 111 I=1,NN1
      DO 111 J=1,MM1
      H(I,J)=H(I,J)/400.0
  111 U(I,J)=U(I,J)/10.0
      WRITE(4,415)
      WRITE(4,429) IEB1
      WRITE(4,413)
      WRITE(4,411)
      WRITE(4,412)
      DO 215 I=2,NA1
      IM=I-1
  215 WRITE(4,426) IM,(U(I,J),J=2,MB)
      DO 217 I=NA,NB
      IM=I-1
  217 WRITE(4,428) IM,(U(I,J),J= 2,MM)
      DO 219 I=NB1,NN
      IM=I-1
  219 WRITE(4,430) IM,(U(I,J),J=MA,MM)
      WRITE(4,415)
      WRITE(4,429) IEB2
      WRITE(4,414)
      WRITE(4,411)
      WRITE(4,412)
      DO 235 I=2,NA1
      IM=I-1
  235 WRITE(4,426) IM,(H(I,J),J=2,MB)
      DO 237 I=NA,NB
      IM=I-1
  237 WRITE(4,428) IM,(H(I,J),J=2,MM)
      DO 239 I=NB1,NN
      IM=I-1
  239 WRITE(4,430) IM,(H(I,J),J=MA,MM)
C
C     SETTING THE BOUNDARY CONDITIONS OF THE RESERVOIR.
```

```
      CC 11 I=2,NB
11    B(I,1)=0.CC
      CC 12 J=2,MA1
12    A(NB,J)=C.CC
      CC 13 I=NB1,NN
13    B(I,MA1)=C.CO
      CC 14 J=MA,MM
14    A(NN,J)=0.CC
      CC 15 J=2,MB
15    A(1,J)=C.CC
      CC 16 I=2,NA1
16    B(I,MB)=0.CC
      CC 17 J=MB1,MM
17    A(NA1,J)=C.CO
      CC 18 I=NA,NN
   18 B(I,MM)=C.CC
C     X-ANC Y-CIRECTICNAL AVERAGES OF K*H VALUES
      CC 23 J=2,MA1
      CC 24 I=2,NB2
      A(I,J)=(H(I,J)+H(I+1,J))*0.5
24    B(I,J)=(H(I,J)+H(I,J+1))*C.5
23    B(NB,J)=(H(NB,J)+H(NB,J+1))*0.5
      CC 25 J=MA,MB2
      CC 26 I=2,NN2
      A(I,J)=(H(I,J)+H(I+1,J))*0.5
26    B(I,J)=(H(I,J)+H(I,J+1))*0.5
25    B(NN,J)=(H(NN,J)+H(NN,J+1))*0.5
      CC 33C I=2,NA1
  33C A(I,MB)=(H(I,MB)+H(I+1,MB))*C.5
      CC 27 J=MB,MM2
      CC 28 I=NA,NN2
      A(I,J)=(H(I,J)+H(I+1,J))*0.5
28    B(I,J)=(H(I,J)+H(I,J+1))*0.5
27    B(NN,J)=(H(NN,J)+H(NN,J+1))*0.5
      CC 29 I=NA,NN2
29    A(I,MM)=(H(I,MM)+H(I+1,MM))*C.5
C     INITIAL MCLES IN THE RESERVOIR
      SUM=C.0
      CC 34C I=2,NN
      CC 34C J=2,MM
  34C SUM=SUM+U(I,J)
      BMCLS=SUM*(CX*CX/(R*T))*PI/ZI
C
      READ(5,452) NT
      CC 2CCC NSET=1,NT
      READ(5,449) DT,PRINT,FDAY,FACTOR
      WRITE(6,451)
      WRITE(6,45C) DT,PRINT,FDAY,FACTCR
      CT2=CT*2.C
      CE=CX*CX/(P2*6.327*CT)
      CC 342 I=1,NN1
      CC 342 J=1,MM1
      E(I,J)=U(I,J)*CE
  342 PZ(I,J)=PI/ZI
C     PRODUCTION RATES OF WELLS (MILLICN STD CU FT/DAY) I.E.
     1 AT
```

```
C       14.7 PSIA AND 520.0 DEGREES RANKINE
        DO 120 I=1,NN1
        DO 120 J=1,MM1
  120   W(I,J)=0.0
        READ (5,5) ((W(I,J), I=3,NN,2), J=3,MM,2)
        DO 125 I=3,NN,2
        DO 125 J=3,MM,2
  125   W(I,J)=W(I,J)*FACTOR
        WRITE(6,420)
        WRITE(6,411)
        WRITE(6,407)
        DO 460 I= 2,NA1
        IM=I-1
  460   WRITE(6,455) IM,(W(I,J),J=2,MB)
        DO 462 I=NA,NB
        IM=I-1
  462   WRITE(6,456) IM,(W(I,J),J=2,MM)
        DO 464 I=NB1,NN
        IM=I-1
  464   WRITE(6,457) IM,(W(I,J),J=MA,MM)
        RATE=0.0
        DO 118 I=3,NN,2
        DO 118 J=3,MM,2
        RATE=RATE+W(I,J)
  118   W(I,J)=W(I,J)*CW
C
        WRITE(4,8)IBC
        NKK=0
        TIME=0.00
  350   CONTINUE
        NKK=NKK+1
        OUTPUT=0.0
  352   OUTPUT=OUTPUT+DT2
        TIME=TIME+DT2
C       FORWARD SWEEP
C       C1*X*X + C2*X + C3 =0.0, X=P/Z =PZ(I,J)
        DO 31 J=2,MA1
        DO 31 I=2,NB
        PPZ=PZ(I,J)*PZ(I,J)
        AB=A(I-1,J)+B(I,J-1)
        C1=D2*AB
        C2=E(I,J)+D1*AB
        C3=A(I-1,J)*(D2*PZ(I-1,J)*PZ(I-1,J)+D1*PZ(I-1,J))+B(I,
       1J-1)*(D2*PZ(
       1I,J-1)*PZ(I,J-1)+D1*PZ(I,J-1))+A(I,J)*(D2*(PZ(I+1,J)*P
       1Z(I+1,J)-PPZ
       2)+D1*(PZ(I+1,J)-PZ(I,J)))+B(I,J)*(D2*(PZ(I,J+1)*PZ(I,J
       1+1)-PPZ)+D1*
       3(PZ(I,J+1)-PZ(I,J)))+E(I,J)*PZ(I,J)-W(I,J)
        C3=-C3
        IF(C1.GT.0.0) GO TO 32
        PZ(I,J)=-C3/C2
        GO TO 31
   32   ROOT=SQRT(C2*C2-4.0*C1*C3)
        C12=2.0*C1
```

```
      PZ(I,J)=(RCCT-C2)/C12
   31 CCNTINUE
      CC 33 J=MA,MB
      CO 33 I=2,NN
      PPZ=PZ(I,J)*PZ(I,J)
      AB=A(I-1,J)+B(I,J-1)
      C1=D2*AB
      C2=E(I,J)+D1*AB
      C3=A(I-1,J)*(D2*PZ(I-1,J)*PZ(I-1,J)+D1*PZ(I-1,J))+B(I,
     1J-1)*(D2*PZ(
     1I,J-1)*PZ(I,J-1)+D1*PZ(I,J-1))+A(I,J)*(D2*(PZ(I+1,J)*P
     1Z(I+1,J)-PPZ
     2)+D1*(PZ(I+1,J)-PZ(I,J)))+B(I,J)*(D2*(PZ(I,J+1)*PZ(I,J
     1+1)-PPZ)+D1*
     3(PZ(I,J+1)-PZ(I,J)))+E(I,J)*PZ(I,J)-W(I,J)
      C3=-C3
      RCCT=SQRT(C2*C2-4.0*C1*C3)
      C12=2.C*C1
      PZ(I,J)=(RCCT-C2)/C12
   33 CCNTINUE
      CC 34 J=MB1,MM
      CC 34 I=NA,NN
      PPZ=PZ(I,J)*PZ(I,J)
      AB=A(I-1,J)+B(I,J-1)
      C1=D2*AB
      C2=E(I,J)+D1*AB
      C3=A(I-1,J)*(D2*PZ(I-1,J)*PZ(I-1,J)+D1*PZ(I-1,J))+B(I,
     1J-1)*(D2*PZ(
     1I,J-1)*PZ(I,J-1)+D1*PZ(I,J-1))+A(I,J)*(D2*(PZ(I+1,J)*P
     1Z(I+1,J)-PPZ
     2)+D1*(PZ(I+1,J)-PZ(I,J)))+B(I,J)*(D2*(PZ(I,J+1)*PZ(I,J
     1+1)-PPZ)+D1*
     3(PZ(I,J+1)-PZ(I,J)))+E(I,J)*PZ(I,J)-W(I,J)
      C3=-C3
      RCCT=SQRT(C2*C2-4.0*C1*C3)
      C12=2.C*C1
      PZ(I,J)=(RCCT-C2)/C12
   34 CCNTINUE
C     REVERSE SWEEP
      NSW=NSW+1
      CC 4C JJ=1,KM1
      J=MM1-JJ
      CC 4C II=1,KN1
      I=NN1-II
      PPZ=PZ(I,J)*PZ(I,J)
      AB=A(I,J)+B(I,J)
      C1=D2*AB
      C2=E(I,J)+D1*AB
      C3=A(I,J)*(D2*PZ(I+1,J)*PZ(I+1,J)+D1*PZ(I+1,J))+B(I,J)
     1*(D2*PZ(I,J+
     11)*PZ(I,J+1)+D1*PZ(I,J+1))+A(I-1,J)*(D2*(PZ(I-1,J)*PZ(
     1I-1,J)-PPZ)+
     2D1*(PZ(I-1,J)-PZ(I,J)))+B(I,J-1)*(D2*(PZ(I,J-1)*PZ(I,J
     1-1)-PPZ)+D1*
     3(PZ(I,J-1)-PZ(I,J)))+E(I,J)*PZ(I,J)-W(I,J)
```

```
      C3=-C3
      IF(C1.GT.C.C) GC TC 39
      PZ(I,J)=-C3/C2
      GC TC 40
   39 RCCT=SQRT(C2*C2-4.0*C1*C3)
      C12=2.C*C1
      PZ(I,J)=(RCCT-C2)/C12
 4C   CCNTINUE
      CC 42 JJ=1,KM2
      J=MB1-JJ
      CC 42 II=1,KN2
      I=NN1-II
      PPZ=PZ(I,J)*PZ(I,J)
      AB=A(I,J)+B(I,J)
      C1=D2*AB
      C2=E(I,J)+C1*AB
      C3=A(I,J)*(D2*PZ(I+1,J)*PZ(I+1,J)+D1*PZ(I+1,J))+B(I,J)
     I*(D2*PZ(I,J+
     11)*PZ(I,J+1)+D1*PZ(I,J+1))+A(I-1,J)*(D2*(PZ(I-1,J)*PZ(
     1I-1,J)-PPZ)+
     2D1*(PZ(I-1,J)-PZ(I,J)))+B(I,J-1)*(D2*(PZ(I,J-1)*PZ(I,J
     1-1)-PPZ)+D1*
     3(PZ(I,J-1)-PZ(I,J)))+E(I,J)*PZ(I,J)-W(I,J)
      C3=-C3
      RCCT=SQRT(C2*C2-4.0*C1*C3)
      C12=2.C*C1
      PZ(I,J)=(RCCT-C2)/C12
   42 CCNTINUE
      CO 43 JJ=1,KM3
      J=MA-JJ
      CO 43 II=1,KN3
      I=NB1-II
      PPZ=PZ(I,J)*PZ(I,J)
      AB=A(I,J)+B(I,J)
      C1=D2*AB
      C2=E(I,J)+C1*AB
      C3=A(I,J)*(D2*PZ(I+1,J)*PZ(I+1,J)+D1*PZ(I+1,J))+B(I,J)
     1*(D2*PZ(I,J+
     11)*PZ(I,J+1)+D1*PZ(I,J+1))+A(I-1,J)*(D2*(PZ(I-1,J)*PZ(
     1I-1,J)-PPZ)+
     2D1*(PZ(I-1,J)-PZ(I,J)))+B(I,J-1)*(D2*(PZ(I,J-1)*PZ(I,J
     1-1)-PPZ)+D1*
     3(PZ(I,J-1)-PZ(I,J)))+E(I,J)*PZ(I,J)-W(I,J)
      C3=-C3
      RCOT=SQRT(C2*C2-4.0*C1*C3)
      C12=2.C*C1
      PZ(I,J)=(RCCT-C2)/C12
   43 CCNTINUE
      IF(CLTPUT .LT. PRINT) GO TO 352
C
C     CCNVERT P/Z TO P I.E.  PZ(I,J) TO F(I,J)
      CC 36C J=2,MM
      CC 360 I=2,NN
  36C F(I,J)=P2*PZ(I,J)+PI
      WRITE(4,408)
```

```
      WRITE(4,410) DT2
      WRITE(4,427)
      WRITE(4,409) TIME
      WRITE(4,411)
      WRITE(4,407)
      DO 362 I=2,NA1
      IM=I-1
  362 WRITE(4,440) IM,(F(I,J),J=2,MB)
      DO 363 I=NA,NB
      IM=I-1
  363 WRITE(4,441) IM,(F(I,J),J=2,MM)
      DO 364 I=NB1,NN
      IM=I-1
  364 WRITE(4,442) IM,(F(I,J),J=MA,MM)
      WRITE(4,6) IBC
C     FINAL MOLES IN THE RESERVOIR
      FMOLS=0.0
      DO 370 I=2,NN
      DO 370 J=2,MM
  370 FMOLS=FMOLS+U(I,J)*PZ(I,J)
      FMOLS=FMOLS*DX*DX/(R*T)
      DVOL=(BMOLS-FMOLS)/CU
      PROD=RATE*TIME
      GB(NKK,1)=TIME
      GB(NKK,2)=PROD
      GB(NKK,3)=DVOL
      GB(NKK,4)=PROD/DVOL
      IF(TIME.LT.FDAY) GO TO 350
      NKKT=NKK
      WRITE(6,4)
      WRITE(6,453)
      WRITE(6,701) ((GB(NKK,NJJ),NJJ=1,4),NKK=1,NKKT)
  701 FORMAT(1H ,F7.0,2E14.5,F8.4)
      IBC=IBC+1
 2000 CONTINUE
      STOP
      END
```

```
C                                                   APPENDIX D   ANNX 4
C
C
C                      ********
C      SIMULATION OF TWO-PHASE FLOW IN OIL RESERVOIRS
C      SOLVED BY ADEP
C                      ********
C
C
C      THIS PROGRAM IS DESIGNED TO STUDY SIMULATION OF A
C      WATER-INJECTED  OIL RESERVOIR BY A TWO-DIMENSIONAL
C      MATHEMATICAL MODEL. A FIVE-SPOT MODEL IS CONSIDERED
C      FOR WATER INJECTION. COMPRESSIBILITIES OF WATER AND
C      OIL ARE TAKEN INTO ACCOUNT.
C      NEWTON-RAPHSON  ITERATIVE PROCEDURE IS USED TO SOLVE
C      THE SIMULTANEOUS NONLINEAR ALGEBRAIC EQUATIONS
C                                              ************
C                                              NOMENCLATURE
C                                              ************
C A( , )              ELEMENTS OF COEFFICIENT MATRIX OF
C                     SIMULTANEOUS EQUATIONS
C AO,AL,MO,SMAXKO     CONSTANTS OF RELATION BETWEEN RELATIVE
C                     PERMEABILITY OF MEDIUM TO OIL AND WATER
C                     SATURATION
C BO,B1,MW,RSW        CONSTANTS OF RELATION BETWEEN RELATIVE
C                     PERMEABILITY OF MEDIUM TO  WATER AND
C                     WATER SATURATION
C BS,BZERO            FORMATION VOLUME FACTORS AT PS,PZERO
C CA1,CA2,CA3,PSMIN    $     0.14 .LE. SW .LE. 0.20
C CB1,CB2,CB3          *     0.20 .LE. SW .LE. 0.50
C CC1,CC2              *     0.50 .LE. SW .LE. 0.76
C CD1,CD2,CD3          *     0.76 .LE. SW .LE. 1.0
C CO,CW               COMPRESSIBILITYIS OF OIL AND WATER RESPY.
C      CONSTANTS IN CAPILLARY PRESSURE-SATURATION  RELATION
C DT                  TIME STEP  (DAYS)
C DX                  =DY= SPACE STEP (FT)
C FACTK               PERMEABILITY CAN BE CHANGED BY FACTK
C FDAY                PERIOD FOR WHICH RESERVOIR HISTORY IS
C                     DESIRED, DAYS
C HX( , ),HY( , )     X- AND Y-DIRECTIONAL AVERAGES OF
C                     PERMEABILITY*FORMATION HEIGHT
C                     FORMATION HEIGHT
C IPO,IPW,ISW         INITIAL VALUES OF PO,PW,SW
C NW                  NUMBER OF CYCLES
C NT                  NUMBER OF DATA SETS
C C1,C2               CONSTANTS IN PRESSURE-VOLUME RELATION
C                     FOR OIL
C PC                  CAPILLARY PRESSURE (PSIA)
C PO( , )             OIL PRESSURE (PSIA)
C PW( , )             WATER PRESSURE (PSIA)
C PS,PZERO            SATURATION AND ORIGINAL PRESSURES OF OIL
C QO( , )             PRODUCTION RATE OF OIL FROM A WELL (BBLS
C                     AT IPO PSIA
C QW( , )             INJECTION RATE OF WATER INTO A WELL(BBLS
C                     AT IPO PSIA
```

```
C QOIL                 TOTAL PRODUCTION RATE OF FLUIDS
C                      (BBLS AT IPO PSIA/DAY)
C QWATER               TOTAL INJECTION RATE OF WATER INTO
C                      RESERVOIR  BBLS AT IPO PSIA/DAY)
C RSW                  SATURATION OF WATER BELOW WHICH KRW=0.0
C SMAXKO               SATURATION OF WATER ABOVE WHICH KRO=0.0
C SW( , )              WATER SATURATION (FRACTION)
C TIME                 TIME (DAYS)
C VISO,VISW            VISCOSITIES OF OIL AND WATER (CP)
C VO                   TOTAL OIL VOLUME INITIALLY IN RESERVOIR
C                       BBLS
C VPT                  TOTAL PORE VOLUME OF RESERVOIR (BBLS)
C VW                   TOTAL WATER VOLUME INITIALLY IN RESERVOIR
C                       BBLS
C W1,W2                CONSTANTS IN PRESSURE-VOLUME RELATION
C                      FOR WATER
C WOR                  RATIO OF (WATER INJECTED/OIL PRODUCED)
C XD,YD,ZD             CHANGES IN PW,PO,SW CALCULATED IN EACH
C                      ITERATION
C      THE FOLLOWING FACTORS ARE DEFINED SO THE EQUATIONS
C      MAY BE GENERALIZED FOR ALL POINTS INCLUDING BOUNDARY
C      POINTS.
C WF(I),XF(I),YF(I)        VOLUME FACTORS TO BE USED FOR
C                          FRACTIONAL CELLS AROUND BOUNDARY
C                          GRID POINTS
C BX1(I),BX2( )            * WEIGHTING FACTORS ON FLUX TERMS
C                          IN X-DIRECTION
C BY1( ),BY2( )            * WEIGHTING FACTORS ON FLUX TERMS
C                          IN Y-DIRECTION
C
       COMMON PW(19,19),PO(19,19),SW(19,19),QW(19,19),QO(19,1
      119),
      A FW(19,19),FWC(19,19),FO(19,19),FOC(19,19),HX(19,19),
      B HY(19,19),A(4,4),AZ(4),X1(9),X2(9),Y1(9),Y2(9),WF(9)
       COMMON HKWIM,HKWJM,HKWIP,HKWJP,
      A HKOIM,HKOJM,HKOIP,HKOJP,SIM,SIP,SJM,SJP,X,Y,Z,IJ,I,J,
      1RSW,
      B RSW2,XD,YD,ZD,VISW,VISO,QOIL,QWATER,VO,V12,UO,U12,IT,
      1ITM
      C,JT,JTM,JTP,ITP,M,VF,PCGRAD,B0,B1,B2,B3,B12,B24,A0,A1,
      D A2,A3,A4,AC2,A22,A34,A04,D0,D1,D2,D3,D4,D02,BZERO,CA1
      1,CA2
      E ,CA3,CB1,CB2,CB3,CC1,CC2,CD1,CD2,CD3,PSMIN,SMAXKO,CD3
      12,
      F CA12,CB12,SMAXK2,MO,MOM,MW,MWM
       DIMENSION KJB(9),KJE(9),KIB(9),KIE(9),ITER(19,19),STOR
      1(19
      A ,19) ,XF(9),YF(9),BX1(9),BX2(9),BY1(9),BY2(9)
       DIMENSION LJB(9),LJE(9),LIB(9),LIE(9)
       DIMENSION WP5(1000),WP10(1000),WP15(1000)
       REAL IPO,IPW,ISW,IMASSW,IMASSO,INJECT,ITERX,ITERY,ITER
      1Z
       REAL PC,DENSW,DENSO,T
C      WATER-FLOODING OF OIL RESERVOIRS
C      SOLUTION OF SIMULTANEOUS NON-LINEAR PARTIAL DIFFERENTI
      1AL EQUATIONS
```

```
 1 FORMAT(1H ,9F8.5)
 2 FORMAT(1H ,8F8.5)
 3 FORMAT(1HJ,39HNO. OF ITERATIONS EXCEEDS 100 , I , J =,
  12I4)
 4 FORMAT(1H1,96H IT JT     IPO       IPW       ISW       RSW   B
  1BL(STD) QOIL
  C, QWATER AT 2700PSIA WOR  DX      DT        FDAY)
 5 FORMAT(1X,2I3,F8.1,2F8.4,F6.1)
 6 FORMAT(1X,2I3,2F8.1,2F8.4, 3F11.1                ,F5.2,2F6
  1.1,F8.0)
 7 FORMAT(1X,5E13.5)
 8 FORMAT(1HJ,89H    VISW     VISO        CW               CO
  1        UO
  C          U1             VO              V1)
 9 FORMAT(1H/,10E13.5)
10 FORMAT(1HJ,74H     DELX            DELY            DELZ
  1    ITERX
  C      ITERY        ITERZ)
11 FORMAT(1HK,13X,93HM A T R I X   O F   P H I * H   ( P
  1O R O S I T
  CY * F O R M A T I O N   H E I G H T)  ( F T ))
12 FORMAT(1H ,I3,1X,17F7.3)
13 FORMAT(1HL,8X,110HM A T R I X   O F   K * H   ( P E R
  1M E A B I L
  CI T Y * F O R M A T I O N   H E I G H T )  ( D A R C Y
  1 - F T ))
14 FORMAT(1X,9F8.3)
15 FORMAT(1H ,8F8.3)
16 FORMAT(1HS, 7X,18I4)
17 FORMAT(1HJ,124H      A0               A1               A2
  1    A3
  C      B0               B1               B2              CA1
  1    CA2
  C      CA3)
18 FORMAT(1H',2H  )
19 FORMAT(1H/,I3,1X,17F7.0)
20 FORMAT(1H ,32X,55HW A T E R   P R E S S U R E   M A T
  1R I X   ( P
  CS I A ))
21 FORMAT(1HK,34X,51HO I L   P R E S S U R E   M A T R I
  1X   ( P S I
  CA ))
22 FORMAT(1HK,24X,73HM A T R I X   O F   F R A C T I O N
  1A L   W A T
  CE R   S A T U R A T I O N)
23 FORMAT(1H/,I3,1X,17F7.3)
24 FORMAT(1HL,34X,20HMATRIX OF ITERATIONS)
25 FORMAT(1HS,2F8.4,6E13.4)
26 FORMAT(1HS,3E14.5)
27 FORMAT(1H  ,46X,11HT I M E  = ,F6.0,10H   D A Y S)
28 FORMAT(1X,18I3)
29 FORMAT(1X,8E16.5)
30 FORMAT(1X,2I3)
31 FORMAT(1H2,9H    ERRW=,F8.2,9H    ERRO=,F8.2)
32 FORMAT(1X,4E15.7)
```

```
   33 FCRMAT(1HJ,2H   )
   34 FORMAT(1HK,2H   )
   35 FORMAT(1HL,2H   )
   36 FCRMAT(1H2,2H   )
   37 FORMAT(1H1,2H   )
   38 FCRMAT(1HL,11X,68HJ    1    2    3    4    5    6    7    8    9
     1   1C    11    12
     C   13    14    15    16    17)
   39 FCRMAT(1HS,10X,2HI  )
   4C FORMAT(1HJ,4X,I17H  J   1        2        3        4        5
     1   6        7
     C        8        9        1C       11       12       13       14       1
     15        16
     C  17)
   41 FCRMAT(1HS,3H   I)
   42 FCRMAT(1H/,2H   )
   43 FCRMAT(1X,F6.3,3F6.0,F7.0,F5.1)
   44 FORMAT(1HJ,10X,I4,3F7.0,2X,I4,3F7.C,2X,I4,3F7.0)
   45 FCRMAT(1HL,19X,4HERRW,32X,4HERRO)
   56 FCRMAT(1HJ,15X,5E12.4)
   57 FCRMAT(1H2,15X,56H    INJECT          BKW           WATER
     1    PRODUC
     C      CIL  )
   58 FCRMAT(1HJ,15X,3E13.4)
   59 FCRMAT(1HL,15X,37H  VPT (BBL)     VC (BBL)      VW (BBL)
     1 )
   6C FCRMAT(1H2,45X,9HPW(9,9) =,E14.3)
 3711 FCRMAT(1H3,56X,17HT A B L E   C - 1)
 3712 FORMAT(1H3,56X,17HT A B L E   C - 2)
 3713 FORMAT(1HS,I3,1X,17F7.3)
C     READ IN RANGES CF I AND J FOR FORWARD SWEEP
      READ(5,28) (KJB(M),KJE(M),KIB(M),KIE(M),M=1,9)
      WRITE(6,28)(KJB(M),KJE(M),KIB(M),KIE(M),M=1,9)
C     READ IN RANGES CF I AND J FOR REVERSE SWEEP
      READ(5,28) (LJB(M),LJE(M),LIB(M),LIE(M),M=1,9)
      WRITE(6,28)(LJB(M),LJE(M),LIB(M),LIE(M),M=1,9)
C     READ IN VOLUME FACTORS AND WEIGHTING FACTORS ON FLUX T
     1ERMS
C     M=5 REPRESENTS INTERIOR POINTS AND THE REST BOUNDARY P
     1CINTS
      READ(5,14) (WF(M),M=1,9)
      READ(5,14) (XF(M),M=1,9)
      READ(5,14)(BX1(M),M=1,9)
      READ(5,14)(BX2(M),M=1,9)
      READ(5,14) (YF(M),M=1,9)
      READ(5,14)(BY1(M),M=1,9)
      READ(5,14)(BY2(M),M=1,9)
      DC 1CC3 M=1,9
      X1(M)=XF(M)*BX1(M)
      X2(M)=XF(M)*BX2(M)
      Y1(M)=YF(M)*BY1(M)
 1C03 Y2(M)=YF(M)*BY2(M)
      WRITE(6,14)  (X1(M),M=1,9)
      WRITE(6,14)  (X2(M),M=1,9)
      WRITE(6,14)  (Y1(M),M=1,9)
```

```
      WRITE(6,14)  (Y2(M),M=1,9)
      READ(5,5) IT,JT,IPO,ISW,RSW,DX
      READ(5,7) CW,VISW,VISO,VWIPO,VOIPO,PZERO,PS,BZERO,BS,D
     1ELX,DELY,
     C DELZ,ITERX,ITERY,ITERZ
      READ(5,32) CA1,CA2,CA3,CB1,CB2,CB3,CC1,CC2,CD1,CD2,CD3
     1,
     C PSMIN,SMAXKO
      RSW2=RSW*2.0
      SMAXK2=SMAXKO*2.0
      ITM=IT-1
      ITP=IT+1
      JTM=JT-1
      JTP=JT+1
      PC(T)=CA3 +(CA1/(T-PSMIN) +CA2)/(T-PSMIN)
      IPW=IPO-PC(ISW)
      DD=DX*DX
      CO=(BS-BZERO)*2.0/((BS+BZERO)*(PZERO-PS))
      UO=1.0+CO*IPO
      VO=1.0+CW*IPO
      V1=-CW
      U1=-CO
      W1=1.0-CW*IPO
      W2=CW
      O1=1.0-CO*IPO
      O2=CO
      VOLW(T)=W1+W2*T
      VOLO(T)=O1+O2*T
      CA12=CA1*2.0
      CB12=CB1*2.0
      CD32=CD3*2.0
      U12=U1/2.0
      V12=V1/2.0
      READ(5,30) NT
C     CYCLE FOR EACH DATA SET BEGINS
      DO 3700 NSET=1,NT
      READ(5,43) WOR,BBL,DT,PRINT,FDAY,FACTK
      DT2=DT*2.0
      RATIO=DD/DT
      RW=RATIO*VISW
      RO=RATIO*VISO
      QOIL=BBL*BZERO
      QWATER=QOIL*WOR/4.0
      READ(5,30) MW,MO
      MWM=MW-1
      MOM=MO-1
      AO=1.0/SMAXK2**MO
      A1=-FLOAT(MO)/((2.0**MOM)*SMAXKO**MO)
      BO=1.0/(2.0-RSW2)**MW
      B1= FLOAT(MW)/((2.0**MWM)*(1.0-RSW)**MW)
C     READ IN K (DARCY), H (FT) , PHI (FRACTION)
      READ(5,1) ((HX(I,J),I= 2,10),J= 2,JT)
      READ(5,2) ((HX(I,J),I=11,IT),J=2,JT)
      READ(5,14) ((QC(I,J),I= 2,10),J= 2,JT)
      READ(5,15) ((QC(I,J),I=11,IT),J=2,JT)
```

```
      READ(5,1) ((QW(I,J),I= 2,10),J=2,JT)
      READ(5,2) ((QW(I,J),I=11,IT),J=2,JT)
C     STOR=PHI*H*DX*DX TO BE USED IN MATERIAL BALANCE
C     COMPUTE K*H(DARCY-FT), PHI*H (FT)
      DO 62 J=2,JT
      DO 62 I=2,IT
      HX(I,J)=HX(I,J)*QO(I,J)*FACTK
      FW(I,J)=QW(I,J)*QO(I,J)
   62 STOR(I,J)=FW(I,J)*DD
C     INITIAL OIL AND WATER IN THE RESERVOIR (IN STD BBLS AT
     1 IPO PSIA)
      SUM=0.0
      DO 82 J=3,JTM
      DO 82 I=3,ITM
   82 SUM=SUM+STOR(I,J)
      DO 85 J=3,JTM
   85 SUM=SUM+(STOR(2,J)+STOR(IT,J))/2.0
      DO 87 I=3,ITM
   87 SUM=SUM+(STOR(I,2)+STOR(I,JT))/2.0
      SUM=SUM+(STOR(2,2)+STOR(2,JT)+STOR(IT,2)+STOR(IT,JT))/
     14.0
      VPT=SUM/5.6146
      VO=VPT*(1.0-ISW)
      VW=VPT*ISW*VOLW(IPW)
      WRITE(6,36)
      WRITE(6,59)
      WRITE(6,58) VPT,VO,VW
      WRITE(6,37)
      WRITE(6,3711)
      WRITE(6,11)
      WRITE(6,33)
      WRITE(6,40)
      WRITE(6,41)
      DO 63 I=2,IT
      IM=I-1
   63 WRITE(6,3713) IM,(FW(I,J),J=2,JT)
      WRITE(6,37)
      WRITE(6,3712)
      WRITE(6,13)
      WRITE(6,33)
      WRITE(6,40)
      WRITE(6,41)
      DO 64 I=2,IT
      IM=I-1
   64 WRITE(6,3713) IM, (HX(I,J),J=2,JT)
C     X- AND Y-DIRECTIONAL AVERAGES OF K*H WITH CONVERSION C
     1CNSTANT 6.32
      DO 65 I=2,IT
   65 HY(I,JT)=HX(I,JT)*6.32
      DO 67 I=2,IT
      DO 67 J=3,JT
   67 HY(I,J-1)=(HX(I,J-1)+HX(I,J))*3.16
      DO 69 J=2,JT
      DO 69 I=3,IT
   69 HX(I-1,J)=(HX(I-1,J)+HX(I,J))*3.16
```

```
      CC 71 J=2,JT
   71 FX(IT,J)=FX(IT,J)*6.32
C     (PHI*H)*RW  ANC  (PHI*H)*RC  VALUES
      CW2=CW/2.
      CC2=CC/2.
      CC 73 I=2,IT
      CC 73 J=2,JT
      FC(I,J)=FW(I,J)*RC
      FW(I,J)=FW(I,J)*RW
      FWC(I,J)=FW(I,J)*CW2
   73 FCC(I,J)=FC(I,J)*CO2
      WRITE(6,4)
      WRITE(6,6) IT,JT,IPC,IPW,ISW,RSW,BBL,QCIL,QWATER,WOR,D
     1X,CT,FDAY
      WRITE(6,8)
      WRITE(6,25) VISW,VISC,CW,CC,UO,U1,VO,V1
      WRITE(6,17)
      WRITE(6,9) A0,A1,A2,A3,B0,B1,B2,CA1,CA2,CA3
      WRITE(6,1C)
      WRITE(6,9) CELX,DELY,CELZ,ITERX,ITERY,ITERZ
      WRITE(6,9) CC1,CC2,CD1,CD2,CC3,PSMIN,SMAXKO,DWIPO,DOIP
     1C,
     C PZERC,PS,BZERC,BS,W1,W2,C1,C2
C     INITIALIZE PRESSURES AND SATURATICN
      CC 8C I=2,IT
      CC 8C J=2,JT
      SW(I,J)=ISW
      PW(I,J)=IPW
      PC(I,J)=IPC
      GW(I,J)=C.C
   8C GC(I,J)=0.C
C     PRCDUCTICN WELLS , CONSTANT- C.FT./STD. BBL
      GC(1C,10)=QCIL*VISC*5.6146
C     INJECTICN WELLS
      GW( 5, 5)=GWATER*VISW*5.6146
      GW( 5,15)=GWATER*VISW*5.6146
      GW(15, 5)=GWATER*VISW*5.6146
      GW(15,15)=GWATER*VISW*5.6146
      FINJ=C.C
      FRCDUC=0.C
      INJECT=C.C
      PRCDUC=C.C
      BK=C.C
      NW=C
      NWB=1
      TIME=C.C
 1CCC CONTINUE
      CUT=C.C
 1234 TIME=TIME+CT2
      CUT=CUT+CT2
      NW=NW+1
C     FORWARD SWEEP
      CC 7CCC M=1,9
      VF=WF(M)
      JB=KJB(M)
```

```
      JE=KJE(M)
      IB=KIB(M)
      IE=KIE(M)
      CC 7CCC J=JB,JE
      CC 7CCC I=IB,IE
      X=PW(I,J)-CELX
      Y=PC(I,J)-CELY
      Z=SW(I,J)+CELZ
      IJ=C
  2CC IJ=IJ+1
      IF(IJ.GE. 3C0) GC TC 701
      CALL PREPF
      CALL MULTY
      CALL MATRIF
      CALL GAUSS
      IF(ABS(XC).LE.ITERX .AND. ABS(YC).LE.ITERY .AND.ABS(ZD
     1).LE.ITERZ)G
     1C TC 4CC
      X=X+XC
      Y=Y+YC
      Z=Z+ZC
      GC TC 2C0
  4CC PW(I,J)=X+XC
      PO(I,J)=Y+YC
      SW(I,J)=Z+ZC
 7CC0 ITER(I,J)=IJ
      BK=BK-QW(1C,10)*DT
      FINJ=FINJ     +(GW(5,5)+QW(5,15)+QW(15,5)+GW(15,15))*DT
      FRCDLC=FRCDLC+GC(1C,1C)*DT
C     REVERSE SWEEP
      CC 8CCC M=1,9
      VF=WF(M)
      JB=LJB(M)
      JE=LJE(M)
      IB=LIB(M)
      IE=LIE(M)
      CC 8CCC JF=JB,JE
      J=JB+JE-JF
      CC 8C0O IC=IB,IE
      I=IB+IE-IC
      X=PW(I,J)-CELX
      Y=PC(I,J)-CELY
      Z=SW(I,J)+CELZ
      IJ=C
  6C0 IJ=IJ+1
      IF(IJ.CE. 3C0) GC TC 7C1
      CALL PREPR
      CALL MULTY
      CALL MATRIR
      CALL GAUSS
      IF(ABS(XC).LE.ITERX .AND. ABS(YC).LE.ITERY .AND.ABS(ZD
     1).LE.ITERZ)G
     1C TC 65C
      X=X+XC
      Y=Y+YC
```

```
      Z=Z+ZC
      GO TC 6CO
  65C PW(I,J)=X+XC
      PC(I,J)=Y+YC
      SW(I,J)=Z+ZC
 8CCC ITER(I,J)=IJ
      FINJ=FINJ     +(QW(5,5)+QW(5,15)+QW(15,5)+QW(15,15))*DT
      FRCDUC=FRCDUC+QC(10,1C)*DT
      BK=BK-QW(1C,10)*DT
      WP5(NW)=PC(5,5)
      WP1C(NW)=PC(1C,10)
      WP15(NW)=PC(15,15)
      IF(PW(1C,1C) .LT. 100.C)  GC TO  703
      IF(CLT .LT. PRINT) GO TC 1234
      WRITE(6,18)
      WRITE(6,34)
  749 WRITE(6,27) TIME
      WRITE(6,35)
      WRITE(6,2C)
      WRITE(6,4C)
      WRITE(6,41)
      CC 75C I=2,IT
      IM=I-1
  75C WRITE(6,19) IM,(PW(I,J) ,J=2,JT)
      WRITE(6,21)
      WRITE(6,42)
      CC 752 I=2,IT
      IM=I-1
  752 WRITE(6,19) IM,(PC(I,J) ,J=2,JT)
      WRITE(6,22)
      WRITE(6,42)
      CC 754 I=2,IT
      IM=I-1
  754 WRITE(6,23) IM,(SW(I,J) ,J=2,JT)
      WRITE(6,36)
      WRITE(6,24)
      WRITE(6,38)
      WRITE(6,39)
      CC 756 I=2,IT
      IM=I-1
  756 WRITE(6,16) IM,(ITER(I,J),J=2,JT)
C     FINAL AMOUNT OF WATER AND CIL IN THE RESERVOIR
      FVW=C.0
      FVC=C.C
      CC 666 J=3,JTM
      CC 666 I=3,ITM
      FVW=FVW       +STOR(I,J)*SW(I,J)*VCLW( PW(I,J))
  666 FVO=FVC       +STOR(I,J)*(1.0-SW(I,J))*VOLC( PO(I,J))
      CC 667 J=3,JTM
      FVW=FVW       + (STOR(2,J)*SW(2,J)*VCLW( PW(2,J))+STOR(
     1IT,
     C J)*SW(IT,J)*VCLW( PW(IT,J)))/2.0
  667 FVC=FVC       + (STOR(2,J)*(1.0-SW(2,J))*VOLC( PC(2,J))
     1 +
     C STCR(IT,J)*(1.C-SW(IT,J))*VOLO( PC(IT,J)))/2.0
```

```
      DO 668 I=3,ITM
      FVW=FVW        + (STOR(I,2)*SW(I,2)*VOLW( PW(I,2)) +STOR
     1(I,
     C JT)*SW(I,JT)*VOLW( PW(I,JT)))/2.0
  668 FVO=FVO        + (STOR(I,JT)*(1.0-SW(I,JT))*VOLO( PO(I,J
     1T))
     C + STOR(I,2)*(1.0-SW(I,2))*VOLO( PO(I,2)))/2.0
      FVW=FVW        + (STOR(2,2)*SW(2,2)*VOLW( PW(2,2)) +STOR
     1(2,
     C JT)*SW(2,JT)*VOLW( PW(2,JT)) +STOR(IT,2)*SW(IT,2)*VOL
     1W(
     D PW(IT,2))+ STOR(IT,JT)*SW(IT,JT)*VOLW( PW(IT,JT)))/4.
     10
      FVO=FVO         +(STOR(2,2)*(1.0-SW(2,2))*VOLO( PO(2,2))
     1 +
     C STOR(IT,2)*(1.0-SW(IT,2))*VOLO( PO(IT,2)) + STOR(2,JT
     1)*
     E (1.-SW(2,JT))*VOLO( PO(2,JT)) +STOR(IT,JT)*(1.-SW(IT,
     1JT))
     F *VOLO( PO(IT,JT)))/4.0
      FVW=FVW/5.6146
      FVO=FVO/5.6146
      WATER=FVW-VW
      OIL=VO-FVO
      INJECT=FINJ/(VISW*5.6146)
      PRODUC=PRODUC/(VISO*5.6146)
      BKW=BK/(VISW*5.6146)
      ERRW= 100.0*(INJECT-BKW-WATER)/INJECT
      ERRO= 100.0*(PRODUC-OIL)/PRODUC
      WRITE(6,57)
      WRITE(6,56) INJECT,BKW,WATER,PRODUC,OIL
      WRITE(6,45)
      WRITE(6,58) ERRW,ERRO
      NWT=NW
C     TIME VS PRESSURE AT TWO INJECTING WELLS AND PRODUCING
     1WELL
      WRITE(6,44 ) (NO,WP5(NO),WP10(NO),WP15(NO),NO=NWB,NWT)
      NWB=NWT+1
      IF(TIME .LT. FDAY) GO TO 1000
      GO TO 702
  701 CONTINUE
      ITER(I,J)=IJ
      WRITE(6,3) I,J
      DO 680 I=2,IT
      IM=I-1
  680 WRITE(6,19) IM,(PW(I,J),J=2,JT)
      DO 682 I=2,IT
      IM=I-1
  682 WRITE(6,19) IM,(PO(I,J),J=2,JT)
      DO 684 I=2,IT
      IM=I-1
  684 WRITE(6,23) IM,(SW(I,J),J=2,JT)
  702 CONTINUE
      GO TO 3700
  703 WRITE(6,60) PW(10,10)
```

```
      WRITE(6,18)
      WRITE(6,34)
      WRITE(6,27) TIME
      WRITE(6,35)
      WRITE(6,20)
      WRITE(6,40)
      WRITE(6,41)
      DO 850 I=2,IT
      IM=I-1
  850 WRITE(6,19) IM,(PW(I,J) ,J=2,JT)
      WRITE(6,21)
      WRITE(6,42)
      DO 852 I=2,IT
      IM=I-1
  852 WRITE(6,19) IM,(PC(I,J) ,J=2,JT)
      WRITE(6,22)
      WRITE(6,42)
      DO 854 I=2,IT
      IM=I-1
  854 WRITE(6,23) IM,(SW(I,J) ,J=2,JT)
      WRITE(6,36)
      WRITE(6,24)
      WRITE(6,38)
      WRITE(6,39)
      DO 856 I=2,IT
      IM=I-1
  856 WRITE(6,16) IM,(ITER(I,J),J=2,JT)
 3700 CONTINUE
      STOP
      END
```

```
      SUBRCUTINE PREPF
C
C
C     CALCULATES CERTAIN INTERMEDIATE FUNCTIONS PREPARATORY
C     TC THE COMPUTATION OF THE COEFFICIENT MATRIX IN THE
C     SET CF NONLINEAR ALGEBRAIC EQUATIONS---FORWARD SWEEP
C
C
      COMMON PW(19,19),PO(19,19),SW(19,19),QW(19,19),QO(19,1
     19),
     A FW(19,19),FWC(19,19),FO(19,19),FOC(19,19),HX(19,19),
     B HY(19,19),A(4,4),AZ(4),X1(9),X2(9),Y1(9),Y2(9),WF(9)
      COMMON HKWIM,HKWJM,HKWIP,HKWJP,
     A HKOIM,HKOJM,HKOIP,HKOJP,SIM,SIP,SJM,SJP,X,Y,Z,IJ,I,J,
     1RSW,
     B RSW2,XO,YO,ZO,VISW,VISO,QOIL,QWATER,VO,V12,UO,U12,IT,
     1ITM
     C,JT,JTM,JTP,ITP,M,VF,PCGRAD,BO,B1,B2,B3,B12,B24,AO,A1,
     D A2,A3,A4,AO2,A22,A34,A04,DO,D1,D2,D3,D4,DO2,BZERO,CA1
     1,CA2
     E ,CA3,CB1,CB2,CB3,CC1,CC2,CD1,CD2,CD3,PSMIN,SMAXKO,CD3
     12,
     F CA12,CB12,SMAXK2,MO,MOM,MW,MWM
      REAL KW,KO,T
      KW(T)=BO*(T-RSW2)**MW
      KO(T)=AO*(SMAXK2-T)**MO
  999 SIM=SW(I-1,J)+Z
      SJM=SW(I,J-1)+Z
      SIP=SW(I+1,J)+SW(I,J)
      SJP=SW(I,J+1)+SW(I,J)
  301 IF(Z.GT.RSW.AND.I.EQ.10.AND.J.EQ.10) GO TO 888
      RETURN
C     AFTER WATER BREAK - THROUGH
  888 FKW=KW(Z+SW(10,10))
      IF( (Z+SW(10,10)) .GT.  SMAXK2) GO TO 810
      FKO=KO(Z+SW(10,10))
  889 QW(10,10)=QOIL/(1.+(FKO/FKW)*(VISW/VISO)*((VO+V12*(X+P
     1W(
     C 10,10)))/(UO+U12*(Y+PO(10,10)))))
      QO(10,10) =5.6146*VISO*(QOIL-QW(10,10))
      QW(10,10)=-QW(10,10)*VISW*5.6146
      RETURN
  810 FKO=0.0
      GO TO 889
    2 RETURN
      END
```

```
      SUBROUTINE PREPR
C
C
C     CALCULATES CERTAIN INTERMEDIATE FUNCTIONS PREPARATORY
C     TO THE COMPUTATION OF THE COEFFICIENT MATRIX IN THE
C     SET OF NONLINEAR ALGEBRAIC EQUATIONS---REVERSE SWEEP
C
C
      COMMON PW(19,19),PO(19,19),SW(19,19),QW(19,19),QO(19,1
     19),
     A FW(19,19),FWC(19,19),FO(19,19),FOC(19,19),HX(19,19),
     B HY(19,19),A(4,4),AZ(4),X1(9),X2(9),Y1(9),Y2(9),WF(9)
      COMMON HKWIM,HKWJM,HKWIP,HKWJP,
     A HKOIM,HKOJM,HKOIP,HKOJP,SIM,SIP,SJM,SJP,X,Y,Z,IJ,I,J,
     1RSW,
     B RSW2,XD,YD,ZD,VISW,VISO,QOIL,QWATER,VO,V12,UO,U12,IT,
     1ITM
     C,JT,JTM,JTP,ITP,M,VF,PCGRAD,BO,B1,B2,B3,B12,B24,AO,A1,
     D A2,A3,A4,AO2,A22,A34,AO4,DO,D1,D2,D3,D4,DO2,BZERO,CA1
     1,CA2
     E ,CA3,CB1,CB2,CB3,CC1,CC2,CD1,CD2,CD3,PSMIN,SMAXKO,CD3
     12,
     F CA12,CB12,SMAXK2,MO,MOM,MW,MWM
      REAL KW,KO,T
      KW(T)=BO*(T-RSW2)**MW
      KO(T)=AO*(SMAXK2-T)**MO
  999 SIM=SW(I-1,J)+SW(I,J)
      SJM=SW(I,J-1)+SW(I,J)
      SIP=SW(I+1,J)+Z
      SJP=SW(I,J+1)+Z
  645 IF(Z.GT.RSW.AND.I.EQ.10.AND.J.EQ.10) GO TO 888
      RETURN
C     AFTER WATER-BREAK-THROUGH
  888 FKW=KW(Z+SW(10,10))
      IF( (Z+SW(10,10)) .GT.  SMAXK2) GO TO 810
      FKO=KO(Z+SW(10,10))
  889 QW(10,10)=QOIL/(1.+(FKO/FKW)*(VISW/VISO)*((VO+V12*(X+P
     1W(
     C 10,10)))/(UO+U12*(Y+PO(10,10)))))
      QO(10,10) =5.6146*VISO*(QOIL-QW(10,10))
      QW(10,10)=-QW(10,10)*VISW*5.6146
      RETURN
  810 FKO=0.0
      GO TO 889
    2 RETURN
      END
```

```
      SUBROUTINE MATRIF
C
C
C     CALCULATES THE MATRIX OF COEFFICIENTS IN THE SET OF
C     NONLINEAR ALGEBRAIC EQUATIONS---FORWARD SWEEP
C
C
      COMMON PW(19,19),PO(19,19),SW(19,19),QW(19,19),QO(19,1
     19),
     A FW(19,19),FWC(19,19),FO(19,19),FOC(19,19),HX(19,19),
     B HY(19,19),A(4,4),AZ(4),X1(9),X2(9),Y1(9),Y2(9),WF(9)
      COMMON HKWIM,HKWJM,HKWIP,HKWJP,
     A HKOIM,HKOJM,HKOIP,HKOJP,SIM,SIP,SJM,SJP,X,Y,Z,IJ,I,J,
     1RSW,
     B RSW2,XO,YO,ZO,VISW,VISO,QOIL,QWATER,VO,V12,UO,U12,IT,
     1ITM
     C,JT,JTM,JTP,ITP,M,VF,PCGRAD,BO,B1,B2,B3,B12,B24,AO,A1,
     D A2,A3,A4,AO2,A22,A34,AO4,DO,D1,D2,D3,D4,DO2,BZERO,CA1
     1,CA2
     E ,CA3,CB1,CB2,CB3,CC1,CC2,CD1,CD2,CD3,PSMIN,SMAXKO,CD3
     12,
     F CA12,CB12,SMAXK2,MO,MOM,MW,MWM
      REAL PC,DPCSWM,DKRO,DKRW,T
      DKRW(T)=B1*(T-RSW2)**MWM
      DKRO(T)=A1*(SMAXK2-T)**MOM
      ZP=Z-PSMIN
      IF(Z .LE. 0.20) GO TO 2
      IF(Z .LE. 0.50) GO TO 3
      IF(Z .LT. 0.712) GO TO 4
      PCZ= CD1+ (CD2+CD3*Z)*Z
      DPCZ= CD2+CD32*Z
      GO TO 6
    2 PCZ= CA3+ (CA1/ZP +CA2)/ZP
      DPCZ= (-CA12/ZP -CA2)/ (ZP*ZP)
      GO TO 6
    3 PCZ= CB3 +(CB1/Z +CB2)/Z
      DPCZ= (-CB12/Z -CB2)/ (Z*Z)
      GO TO 6
    4 PCZ= CC1+CC2*Z
      DPCZ= CC2
    6 IF(SIM .LE. RSW2) GO TO 10
      DWSIM=DKRW(SIM)
      GO TO 15
   10 DWSIM=0.0
   15 IF(SJM .LE. RSW2) GO TO 17
      DWSJM =DKRW(SJM)
      GO TO 18
   17 DWSJM=0.0
   18 IF(SIM .GT. SMAXK2) GO TO 20
      DOSIM=DKRO(SIM)
   21 IF(SJM .GT. SMAXK2) GO TO 22
      DOSJM=DKRO(SJM)
      GO TO 25
   20 DOSIM =0.0
      GO TO 21
```

```
   22 DCSJM=C.C
C     MATRIX CF CCEFFICIENTS IN SET OF LINEAR EQUATIONS
   25 A(1,2)=C.C
      A(2,1)=C.C
      A(3,1)=-1.C
      A(3,2)= 1.C
      A(3,3)=-DPCZ
      A(3,4) =PCZ-Y+X
      IF(M.EQ. 5) GO TO 7C01
      A(1,1)=VF*FWC(I,J)*(Z+SW(I,J))+X1(M)*HKWIM+Y1(M)*HKWJM
     A -QW(I,J)*V1
      A(1,3)= VF*(FWC(I,J)*(X-PW(I,J))+FW(I,J))-X1(M)*HX(I-1
     1,J)
     C *DWSIM     *(PW(I-1,J)-X)- Y1(M)*HY(I,J-1)*DWSJM     *(
     1PW(
     C I,J-1)-X)
      A(1,4)= -(VF*(FWC(I,J)*(Z+SW(I,J))*(X-PW(I,J))+FW(I,J)
     C *(Z-SW(I,J))) -X1(M)*HKWIM*(PW(I-1,J)-X)-X2(M)*HKWIP*
     1(
     C PW(I+1,J)-PW(I,J))- Y1(M)*HKWJM*(PW(I,J-1)-X)- Y2(M)*
     1HKW
     C JP*(PW(I,J+1)-PW(I,J)) -QW(I,J)*(VO+V12*(X+PW(I,J))))
      A(2,2)= VF*FOC(I,J)*(2.-SW(I,J)-Z)+X1(M)*HKOIM+Y1(M)*H
     1KOJ
     C M +QO(I,J)*U1
      A(2,3)= VF*( FCC(I,J)*(PO(I,J)-Y)-FO(I,J)) -X1(M)*HX(I
     1-1,
     C J)*DOSIM     *(PO(I-1,J)-Y) -Y1(M)*HY(I,J-1)*DOSJM
     1*
     C (PO(I,J-1)-Y)
      A(2,4)= -(VF*( FOC(I,J)*(2.-SW(I,J)-Z)*(Y-PO(I,J))-FO(
     1I,J)
     C *(Z-SW(I,J))) -X1(M)*HKOIM*(PO(I-1,J)-Y) -X2(M)*HKOIP
     1*(PC
     C (I+1,J)-PC(I,J)) -Y1(M)*HKOJM*(PO(I,J-1)-Y)- Y2(M)*HK
     1OJP
     C *(PO(I,J+1)-PO(I,J)) +QO(I,J)*(UO+U12*(Y+PO(I,J))))
      RETURN
 7C01 A(1,1)=FWC(I,J) *(Z+SW(I,J))+ HKWIM+ HKWJM -QW(I,J)*V1
      A(1,3)=FWC(I,J)*(X-PW(I,J))+ FW(I,J)-HX(I-1,J) *DWSIM
     C *(PW(I-1,J)-X)- HY(I,J-1)*DWSJM     *(PW(I,J-1)-X)
      A(1,4)= -( FWC(I,J)*(Z+SW(I,J))*(X-PW(I,J))+FW(I,J)*(Z
     1-SW
     C (I,J)) -HKWIM*(PW(I-1,J)-X)- HKWIP*(PW(I+1,J)-PW(I,J)
     1) -
     C HKWJM*(PW(I,J-1)-X)- HKWJP*(PW(I,J+1)-PW(I,J)) -QW(I,
     1J)*
     C (VC+V12*(X+PW(I,J))))
      A(2,2)= FCC(I,J)*(2.-SW(I,J)-Z)+HKOIM+HKOJM+QO(I,J)*U1
      A(2,3)=  FCC(I,J)*(PO(I,J)-Y)-FO(I,J)-HX(I-1,J)*DOSIM
     C *(PC(I-1,J)-Y)-HY(I,J-1)*DOSJM     *(PO(I,J-1)-Y)
      A(2,4)= -(FOC(I,J)*(2.-SW(I,J)-Z)*(Y-PO(I,J))-FO(I,J)*
     1(Z-
     C SW(I,J)) -HKOIM*(PO(I-1,J)-Y) -HKOIP*(PO(I+1,J)-PO(I,
     1J))
```

```
C -HKCJM*(PC(I,J-1)-Y) -HKCJP*(PC(I,J+1)-PC(I,J)) +QC(I
1,J)
C *(UC+U12*(Y+PC(I,J))))
 RETURN
 END
```

```
      SUBRCUTINE MATRIR
C
C
C     CALCULATES THE MATRIX OF COEFFICIENTS IN THE SET OF
C     NONLINEAR ALGEBRAIC EQUATIONS---REVERSE SWEEP
C
C
      COMMON PW(19,19),PC(19,19),SW(19,19),QW(19,19),QC(19,1
     19),
     A FW(19,19),FWC(19,19),FO(19,19),FOC(19,19),HX(19,19),
     B HY(19,19),A(4,4),AZ(4),X1(9),X2(9),Y1(9),Y2(9),WF(9)
      COMMON HKWIM,HKWJM,HKWIP,HKWJP,
     A HKOIM,HKOJM,HKOIP,HKOJP,SIM,SIP,SJM,SJP,X,Y,Z,IJ,I,J,
     1RSW,
     B RSW2,XO,YO,ZO,VISW,VISO,QOIL,QWATER,VO,V12,UO,U12,IT,
     1ITM
     C,JT,JTM,JTP,ITP,M,VF,PCGRAD,B0,B1,B2,B3,B12,B24,A0,A1,
     D A2,A3,A4,A02,A22,A34,A04,D0,D1,D2,D3,D4,D02,BZERO,CA1
     1,CA2
     E ,CA3,CB1,CB2,CB3,CC1,CC2,CD1,CD2,CD3,PSMIN,SMAXKO,CD3
     12,
     F CA12,CB12,SMAXK2,MO,MOM,MW,MWM
      REAL PC,DPCSWM,DKRO,DKRW,T
      DKRW(T)=B1*(T-RSW2)**MWM
      DKRO(T)=A1*(SMAXK2-T)**MOM
      ZP=Z-PSMIN
      IF(Z .LE. 0.20) GO TO 2
      IF(Z .LE. 0.50) GO TO 3
      IF(Z .LT. 0.712) GO TO 4
      PCZ= CD1+ (CD2+CD3*Z)*Z
      DPCZ= CD2+CD32*Z
      GO TO 6
    2 PCZ= CA3+ (CA1/ZP +CA2)/ZP
      DPCZ= (-CA12/ZP -CA2)/ (ZP*ZP)
      GO TO 6
    3 PCZ= CB3 +(CB1/Z +CB2)/Z
      DPCZ= (-CB12/Z -CB2)/ (Z*Z)
      GO TO 6
    4 PCZ= CC1+CC2*Z
      DPCZ= CC2
    6 IF(SIM .LE. RSW2) GO TO 10
      DWSIP=DKRW(SIP)
      GO TO 15
   10 DWSIP=0.0
   15 IF(SJM .LE. RSW2) GO TO 17
      DWSJP =DKRW(SJP)
      GO TO 18
   17 DWSJP=0.0
   18 IF(SIP .GT. SMAXK2) GO TO 20
      DOSIP=DKRO(SIP)
   21 IF(SJP .GT. SMAXK2) GO TO 22
      DOSJP=DKRO(SJP)
      GO TO 25
   20 DOSIP =0.0
      GO TO 21
```

```
   22 CCSJP=C.C
C      MATRIX CF COEFFICIENTS)NEWTCN-RAPHSCN ,MULTI-VARIABLE
      1SYSTEM
   25 A(1,2)=C.C
      A(2,I)= 0.
      A(3,1)= -1.C
      A(3,2)= 1.C
      A(3,3)=-CPCZ
      A(3,4) =PCZ-Y+X
      IF(M.EQ. 5) GO TO 8CO1
      A(1,1)= VF*FWC(I,J)*(Z+SW(I,J))+X1(M)*HKWIP+Y1(M)*HKWJ
     1P-
     C QW(I,J)*V1
      A(1,3)=VF*(FWC(I,J)*(X-PW(I,J))+FW(I,J))-X1(M)*HX(I,J)
     1*
     C DWSIP  *(PW(I+1,J)-X) -Y1(M)*HY(I,J)*DWSJP    *(PW(I,
     1J+1)
     C -X)
      A(1,4)= -(VF*( FWC(I,J)*(Z+SW(I,J))*(X-PW(I,J))+FW(I,J
     1)*
     C (Z-SW(I,J))) -X1(M)*HKWIP*(PW(I+1,J)-X)- X2(M)*HKWIM*
     1(PW
     C (I-1,J)-PW(I,J)) -Y1(M)*HKWJP*(PW(I,J+1)-X)-Y2(M)* HK
     1WJM
     C *(PW(I,J-1)-PW(I,J)) -QW(I,J)*(VO+V12*(X+PW(I,J))))
      A(2,2)=VF*FOC(I,J)*(2.-SW(I,J)-Z)+X1(M)*HKOIP+Y1(M)*HK
     1OJP
     C + QC(I,J)*U1
      A(2,3)= VF*( FCC(I,J)*(PO(I,J)-Y)-FC(I,J)) -X1(M)*HX(I
     1,J)
     C *DCSIP    *(PC(I+1,J)-Y)-Y1(M)*HY(I,J)*DOSJP    *(PO(
     1I,J+
     E 1)-Y)
      A(2,4)= -(VF*(FCC(I,J)*(2.-SW(I,J)-Z)*(Y-PO(I,J))-FO(I
     1,J)
     C *(Z-SW(I,J))) -X1(M)*HKOIP*(PO(I+1,J)-Y) -X2(M)*HKOIM
     1*
     C (PC(I-1,J)-PO(I,J)) -Y1(M)*HKOJP*(PC(I,J+1)-Y) -Y2(M)
     1*
     E HKOJM*(PC(I,J-1)-PC(I,J)) +QO(I,J)*(UO+U12*(Y+PO(I,J)
     1)))
      RETURN
 8CO1 A(1,1)= FWC(I,J)*(Z+SW(I,J))+HKWIP+HKWJP-QW(I,J)*V1
      A(1,3)= FWC(I,J)*(X-PW(I,J))+FW(I,J)-HX(I,J)*DWSIP
     1*
     C (PW(I+1,J)-X)-HY(I,J) *DWSJP    *(PW(I,J+1)-X)
      A(1,4)= -(FWC(I,J)*(Z+SW(I,J))*(X-PW(I,J))+FW(I,J)*(Z-
     1SW
     C (I,J)) -HKWIP*(PW(I+1,J)-X) -HKWIM*(PW(I-1,J)-PW(I,J)
     1)-
     C HKWJP*(PW(I,J+1)-X) -HKWJM*(PW(I,J-1)-PW(I,J))-QW(I,J
     1)
     E *(VC+V12*(X+PW(I,J))))
      A(2,2)=FOC(I,J)*(2.-SW(I,J)-Z)+HKCIP+HKOJP+QO(I,J)*U1
      A(2,3)= FCC(I,J)*(PC(I,J)-Y)-FC(I,J)-HX(I,J)*DOSIP*(
```

```
C PO(I+1,J)-Y)- HY(I,J)*DOSJP      *(PO(I,J+1)-Y)
 A(2,4)=-(FOO(I,J)*(2.-SW(I,J)-Z)*(Y-PO(I,J))-FO(I,J)*(
1Z-
C SW(I,J)) -HKOIP*(PO(I+1,J)-Y) -HKOIM*(PO(I-1,J)-PO(I,
1J))
C -HKOJP*(PO(I,J+1)-Y) -HKOJM*(PO(I,J-1)-PO(I,J))+QO(I,
1J)
E *(UO+U12*(Y+PO(I,J))))
 RETURN
 END
```

```
      SUBROUTINE MULTY
C
C
C     CALCULATES INTERMEDIATE FUNCTIONS INVOLVING KR
C     H, ETC.
C
C
      COMMON PW(19,19),PO(19,19),SW(19,19),QW(19,19),QO(19,1
     119),
     A FW(19,19),FWC(19,19),FO(19,19),FOC(19,19),HX(19,19),
     B HY(19,19),A(4,4),AZ(4),X1(9),X2(9),Y1(9),Y2(9),WF(9)
      COMMON HKWIM,HKWJM,HKWIP,HKWJP,
     A HKOIM,HKOJM,HKOIP,HKOJP,SIM,SIP,SJM,SJP,X,Y,Z,IJ,I,J,
     1RSW,
     B RSW2,XD,YD,ZD,VISW,VISO,QOIL,QWATER,VO,V12,UO,U12,IT,
     1ITM
     C,JT,JTM,JTP,ITP,M,VF,PCGRAD,BO,B1,B2,B3,B12,B24,AO,A1,
     D A2,A3,A4,AO2,A22,A34,AO4,DO,D1,D2,D3,D4,DO2,BZERO,CA1
     1,CA2
     E ,CA3,CB1,CB2,CB3,CC1,CC2,CD1,CD2,CD3,PSMIN,SMAXKO,CD3
     12,
     F CA12,CB12,SMAXK2,MO,MOM,MW,MWM
      REAL  KW,KO,T
      KW(T)=BO*(T-RSW2)**MW
      KO(T)=AO*(SMAXK2-T)**MO
      IF(SIM .LE. RSW2) GO TO 201
      HKWIM=KW(SIM)*HX(I-1,J)
      GO TO 202
  201 HKWIM=0.0
  202 IF(SJM .LE. RSW2) GO TO 203
      HKWJM=KW(SJM)*HY(I,J-1)
      GO TO 204
  203 HKWJM=0.0
  204 IF(SIM .GE. SMAXK2) GO TO 215
      HKOIM= KO(SIM)*HX(I-1,J)
  211 IF(SJM .GE. SMAXK2) GO TO 216
      HKOJM= KO(SJM)*HY(I,J-1)
      GO TO 220
  215 HKOIM=0.0
      GO TO 211
  216 HKOJM=0.0
  220 IF(SIP .LE. RSW2) GO TO 205
      HKWIP=KW(SIP)*HX(I,J)
      GO TO 206
  205 HKWIP=0.0
  206 IF(SJP .LE. RSW2) GO TO 207
      HKWJP=KW(SJP)*HY(I,J)
      GO TO 208
  207 HKWJP=0.0
  208 IF(SIP .GE. SMAXK2) GO TO 235
      HKOIP=KO(SIP)*HX(I,J)
  231 IF(SJP .GE. SMAXK2) GO TO 236
      HKOJP=KO(SJP)*HY(I,J)
      GO TO 237
  235 HKOIP=0.0
```

```
      GC TC 231
  236 FKCJP=C.C
  237 CCNTINUE
      RETURN
      END
```

```
      SUBROUTINE GAUSS
C
C
C     SOLUTION OF LINEAR ALGEBRAIC EQUATIONS IN EACH
C     ITERATION BY GAUSSIAN ELIMINATION
C
C
      COMMON PW(19,19),PO(19,19),SW(19,19),QW(19,19),QO(19,1
     19),
     A FW(19,19),FWC(19,19),FO(19,19),FOC(19,19),HX(19,19),
     B HY(19,19),A(4,4),AZ(4),X1(9),X2(9),Y1(9),Y2(9),WF(9)
      COMMON HKWIM,HKWJM,HKWIP,HKWJP,
     A HKOIM,HKOJM,HKOIP,HKOJP,SIM,SIP,SJM,SJP,X,Y,Z,IJ,I,J,
     1RSW,
     B RSW2,XO,YO,ZO,VISW,VISO,QOIL,QWATER,VO,V12,U0,U12,IT,
     1ITM
     C,JT,JTM,JTP,ITP,M,VF,PCGRAD,B0,B1,B2,B3,B12,B24,A0,A1,
     D A2,A3,A4,A02,A22,A34,A04,D0,D1,D2,D3,D4,D02,BZERO,CA1
     1,CA2
     E ,CA3,CB1,CB2,CB3,CC1,CC2,CD1,CD2,CD3,PSMIN,SMAXKO,CD3
     12,
     F CA12,CB12,SMAXK2,MO,MOM,MW,MWM
      L=0
   70 L=L+1
      PIVOT=A(L,L)
      IZ=L
      DO 71 K=L,2
      IF(ABS(PIVOT) .GE. ABS(A(K+1,L))) GO TO 71
      PIVOT=A(K+1,L)
      IZ=K+1
   71 CONTINUE
      IF(IZ.EQ.L) GO TO 74
      DO 73 KK=L,4
      AZ(KK)=A(L,KK)
      A(L,KK)=A(IZ,KK)
   73 A(IZ,KK)=AZ(KK)
   74 LP=L+1
      DO 75 KK=LP,4
   75 A(L,KK)=A(L,KK)/A(L,L)
      DO 76 IK=LP,3
      DO 76 JK=LP,4
   76 A(IK,JK)=A(IK,JK)-A(L,JK)*A(IK,L)
      IF(L.EQ.2) GO TO 77
      GO TO 70
   77 ZO=A(3,4)/A(3,3)
      YO=A(2,4)-A(2,3)*ZO
      XO=A(1,4)-A(1,2)*YO-A(1,3)*ZO
      RETURN
      END
```